

Operation, Monitoring and Control Technology of Power Systems

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Marek Zima and Marija Bočkarjova
EEH - Power Systems Laboratory
ETH Zürich

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Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

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Preface

The primary intention behind this compendium is to provide a study material for the part *Operation, Monitoring and Control Technology of Power Systems* of lectures within the course *Systemdynamik und Leittechnik der elektrischen Energieversorgung* given at ETH Zurich in the eighth semester of studies at the Department of Information Technology and Electrical Engineering.

A wider spectrum of topics is covered, focusing on understanding principles and relations between the topics. Further details can be found in corresponding textbooks.

This compendium starts with an overview of both technological and organizational context of power systems and their structure. Technology, which is used to build up monitoring and control systems is outlined. Power systems operation principles and activities are described as well as particular tools - algorithms' principles and computations.

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Marek Zima and Marija Bočkarjova

Chapter 1

Introduction

This chapter first introduces the objective of power system operation. Then, the description is provided how power systems are structured and which entities participate in their operation.

Electricity supply is one of critical infrastructures influencing functioning of a modern society. Its correct performance strongly influences other infrastructures dependent on it, such as communication systems, traffic and transportation systems, gas and water delivery, financial operations etc.

Electricity supply to final consumers (industry, commercial and residential subjects and entities) is provided by power systems. Main criteria for assessing power systems performance, are:

- Reliability
- Economy
- Quality
- Environmental impact

Environmental impact on the nature and humans shall be kept minimal, e.g. minimal pollution, noise, radiation and taken space. Quality, or more frequently used term *power quality*, expresses how the supplied energy in form of current and voltage waveform differs from the regular sinusoidal shape. Power quality problems have usually a local nature, i.e. significantly influencing only certain limited area. Typical examples of power quality problems are distortions of the voltage shape by power electronics equipment, significant short-term drops of the voltage etc. Economy has an obvious meaning of minimal cost related to assuring power supply under the three other criteria (environmental friendliness, power quality and reliability).

Widely accepted definition of reliability, according to [3], is

probability of the power system satisfactory operation over the long run. It denotes the ability to supply adequate electric service on a nearly continuous basis, with few interruptions over an extended time period.

Another important term is security:

Security of a power system refers to the degree of risk in its ability to survive imminent disturbances (contingencies) without interruption of customer service. It relates to robustness of the system to imminent disturbances and, hence, depends on the system operating condition as well as the contingent probability of disturbances.

Relations between reliability and security are:

Security and stability are time-varying attributes which can be judged by studying the performance of the power system under a particular set of conditions. Reliability, on the other hand, is a function of the time-average performance of the power system; it can only be judged by consideration of the systems behavior over an appreciable period of time.

Note that the term stability is discussed later in chapter 3. In the context of this chapter it is not relevant.

Reliability is the overall objective in power system design and operation. To be reliable, the power system must be secure most of the time.

When the security is jeopardized and a power system is subjected to a disturbance, which is not eliminated, it can lead to catastrophic scenarios having a very serious impact on the society. If this happens, a complicated and complex restoration procedure must take place.

This compendium focuses on the aspect of security, however, the other aspects have to be considered too. Finding an appropriate balance and compromise between above listed criteria, which are often in contradiction (for example, a cheaper power plant may cause more severe pollution etc.), is generally very challenging in most of power systems design problems.

1.1 Technological Structure of Power Systems

Components, which together build up a complete power system, can be divided to two groups:

- Primary Equipment
- Secondary Equipment

Primary equipment conducts large currents or is stressed by high voltages, in other words, components, which carry a transfer of energy. This section lists and discusses some examples of primary equipment. Thus, when talking about power systems in this section, it is primary equipment, which is understood by that.

Secondary equipment are components and systems comprising tools for monitoring, protection and control. Chapter 2 is dedicated to secondary equipment.

Basic principle structure of power systems is shown in figure 1.1. The picture is oriented, so that power flow is in top-down direction.

The top part represents power **generation** in power plants, which inject their production into transmission systems via step-up transformers, as terminal voltage of large generators is typically in the range 15 - 22 kV¹ (given by

¹It is a common practice when mentioning voltage in kV in this type of context, it refers to phase-to-phase RMS voltage of three-phase systems.

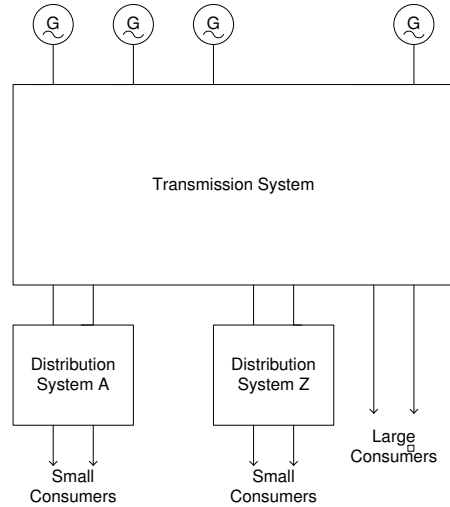


Figure 1.1: Traditional technological structure of power systems.

stator insulation capabilities). Generators are synchronous machines, where their power production is controlled by the mechanical torque delivered to their shaft and their terminal voltage is controlled by the excitation current in their rotor. Control circuits are then governor controller and AVR (Automatic Voltage Regulator), which are discussed in the other part of this course.

Nuclear and thermal units have usually larger rating (200 - 1 600 MVA) at which their efficiency is highest. Because of economical reasons, they are not so flexible in modification of their production², therefore they cover mainly base load.

Hydro units can be found in a very wide rating range - from several MVA to thousands MVA (Itaipu in Paraguay/Brasil, Three Gorges in China). Hydro units can be divided into two categories - with or without storage capacity. For plants, with storage capacity (i.e. having a basin capable of accommodating water for several hours of operation, even when not having any water inflow), start-up procedure is usually relatively simple and fast and their turbines are quite robust, thus they typically participate in production during peak demand periods, whereas outside of peak hours they accumulate water in their basin. The plants without storage capacity (or rather minimal storage capacity) are constructed on rivers, where a permanent water flow is guaranteed.

Transmission system is a backbone of power systems and its main purpose is to transport energy in large volumes over large distances, originally from production to consumption centers. Therefore transmission systems are operated on a very high and extra high voltage levels (in Europe 110 - 400 kV), also in order to minimize resistive losses, thus rather increasing voltage than current level (recall that resistive losses are $R.I^2$). On the other hand, isolation capabilities of components have to be much higher, what makes them much more expensive. Therefore advantages of very high voltage levels pay off only for large transmission distances.

²Start-up of a thermal or a nuclear unit is a procedure taking several hours.

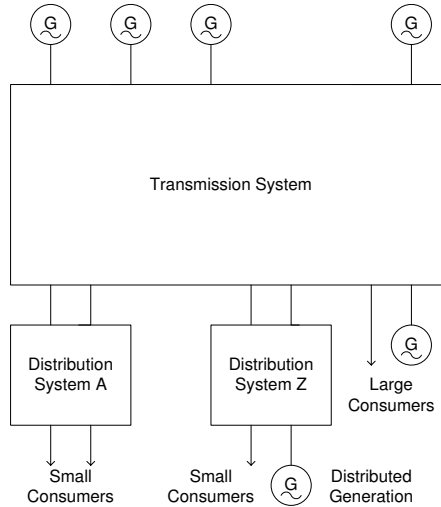


Figure 1.2: Recent technological structure of power systems.

The structure of transmission systems is meshed, in order to preserve a system integrity even in a case of a line outage. A connection between different power systems (for example between two countries) is done on the transmission system level.

The task of **distribution systems** is to deliver electrical energy to the end consumer. Their structure can be meshed, but they are operated as radial systems, i.e. there is a path of lines and transformers connected in series between the final consumer and the point of connection to the transmission system. Distribution systems employ all voltage levels between 100 kV, down to 0.4 kV, what is the voltage in consumer wall socket.

A large majority of commercial and residential **consumers** are supplied from distribution systems, however, frequently large industrial plants are directly connected to transmission systems.

The traditional concept of unidirectional flow of power (from generators via transmission system to distribution systems and eventually to final consumers) has been changing in the last decade towards structure shown in figure 1.2.

Penetration of distribution (Hence the expression *distributed generation*. Additionally, this expression also refers to the fact of spatial sparse geographical spreading of many units with low power rating.) systems with small generation units has been rapidly increasing because of technological advances in this area and a very strong legislation support in some countries³.

A large portion of distributed generation is wind and solar generation, whose production is weather dependent and thus often volatile, less predictable and its controllability is very limited. Many generators of low ratings are asynchronous machines, which need to be energized from the network side (This implies that they can not be used for so-called *black start* in case of system restoration.)

³As an example, utilities in Germany are obliged to connect new distributed generation and buy energy from distributed generation any time it produces it at predetermined price, i.e. not according to the market value. Similar rules apply also in some other countries.

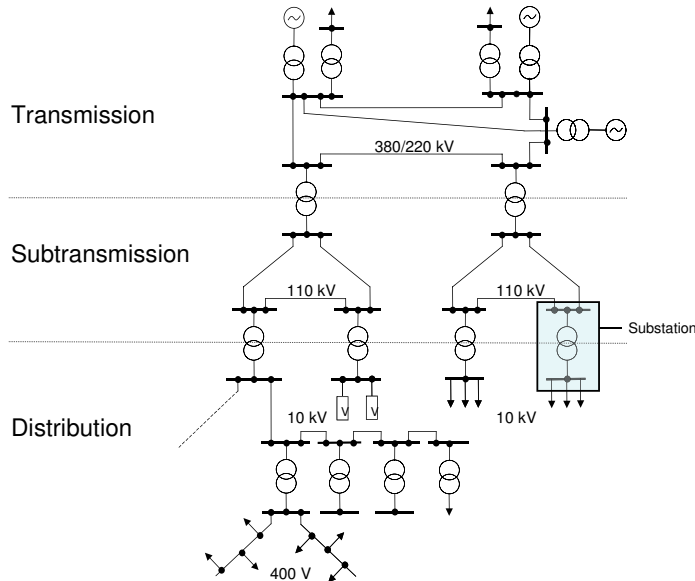


Figure 1.3: Single line diagram example.

and do not offer a voltage support and control. Flows in distribution networks are strongly affected from distribution generation, which is injecting its power directly at this system level.

An additional factor is generation capability of industrial consumers. Their generation can be either real physical unit, installed within their plant, or fictitious one, which is a possibility to withdraw their planned consumption, thus from system point of view appearing as an additional generation injection.

1.2 Primary Equipment Components

Primary equipment components can be modelled with various degrees of detail, according to the purpose of computations and analysis. When studying fast (with time constants in order of milliseconds) transients, dominantly local phenomena, such as short circuit for protection purposes, usually instantaneous quantities (voltage, current) of all three phases are used. When system-wide dynamics (typically having slower time constants, from hundreds of milliseconds to tens of seconds) is of interest, single phase RMS time variant representation is applied, assuming that there is symmetry among phases (i.e. that behavior of one phase accurately represents behavior of all three phases). Topics covered by this compendium (and the whole course) are mostly of system-wide nature, thus from this point on, single-phase RMS modelling is assumed unless stated otherwise.

Here, in this compendium we deal mostly with components of transmission and distribution systems, whereas generators are addressed in the other part of this course. Modelling of primary equipment components (e.g. lines etc.) for analysis purposes is described in section A.2. In this section we discuss only

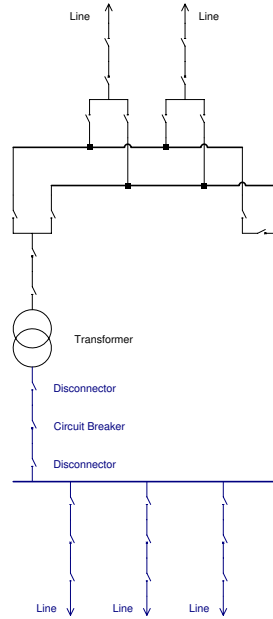


Figure 1.4: Single line diagram of a substation. The top double busbars part corresponds to 110 kV voltage level. The bottom part (under transformer) represents single busbars connection to 10 kV voltage level.

principal issues, much more can be learnt from courses given by the *High Voltage Laboratory*.

Transmission of energy from one to another geographical location is done by transmission and distribution lines, which connect substations, see figure 1.3.

A substation contains all the equipment for transforming one voltage level to another (transformers), to perform switching operations (e.g. circuit breakers to connect or disconnect a line), to obtain measurements (voltage and current measurement transformers) etc. Substation can either be air insulated (AIS) or gas insulated (GIS). The latter ones are encapsulated in pipes containing a gas with good isolation capability, so isolation distances are then much smaller, making the whole arrangement much more compact in size.

Mutual connection of components in substations is done via busbars. Busbars may have various configurations, most common are single busbars, double busbars and one and half circuit breaker busbars. A more detailed configuration of the substation from figure 1.3 is shown in figure 1.4.

Power Transformers Transformers change voltage level by electromagnetic coupling of two (two-winding transformers) or three (three-winding transformers) coils. In case of two-winding transformers, if the primary side has a higher voltage than the secondary side of the transformer, it is referred to as step-down transformer, in opposite case the term step-up transformer is used. Primary side is understood as the side, where the energy is injected. The voltage ratio between primary and secondary side is given by the turns ratio of windings. Some modifications of power transformers are frequently intro-

duced with the objective to add control features. This yields tap-changing and phase-shifting transformers.

Control Devices Control devices may be divided into two groups based on the control objective:

Voltage Control Devices Shunt Capacitor, Shunt Reactor, Static VAR Compensator, Tap-changing Transformer

Power Flow Control Devices Phase-shifting Transformer, Controlled Series Capacitor, Unified Power Flow Controller

Shunt components are connected between the bus and ground (thus shunt) and their objective is to either absorb or produce reactive power in order to compensate for either excess or lack of reactive power in the network. Their advantage is simplicity, on other hand, control possibilities are very limited, they can be either completely switched on, or off. This disadvantages is overcome by Static VAR Compensator (SVC), which with the help of Power Electronics based converter can modify its capacitance in a wide range.

Tap-changing transformers are equipped with tap-changer, which allows slight modification of turns ratio, which is used to compensate for voltage variations on the primary side in order to keep the voltage on the secondary side within the desired operation range.

Phase-shifting transformers have special arrangements, including additional windings connected in series with the primary and/or secondary windings. All windings are equipped with tap-changers. By certain moves of tap-changers, voltage phase angle between primary and secondary side can be controlled, thus having a significant influence on the active power flow.

Most common type of series (series because it is connected in series with a line) control device is Thyristor Controlled Series Capacitor (TCSC). Similarly to SVC, TCSC capacitance can be changed to compensate line reactance, which strongly affects active power flow in the line.

There is a general term FACTS device, which refers to a device containing power electronics elements (mentioned here are SVC, TCSC and Unified Power Flow Controller) allowing a fast control, thus giving a possibility in addition to influence the damping of oscillations, which may occur in the system. Power oscillations are addressed in the other part of this course.

Measurement Transformers Measurement transformers (frequently also referred to as Instrument Transformers) are used to scale down the measured quantity to a level, which is not dangerous for human beings and can be well processed by electronic equipment. In the case of voltage transformers the nominal secondary side is most commonly designed to be 100 Volts and the secondary side current is 1 or 5 Amps.

Usually there are two secondary windings - one is for an accurate measurement under normal conditions, whereas the other serves normally only protection purposes. The measurement core is more accurate but within a small range (recall that magnetic characteristics of transformers is linear only in a small part of the range, then it saturates.) and the protection core is less accurate but it should be able to reproduce also very high fault currents.

The accuracy of measurement transformers is given by accuracy classes. Most common ones are 0.1, 0.2 and 0.5. The measurement class name says what is a maximum percentage deviation of the measured value from the real value at the nominal voltage or current. So for example for a voltage measurement transformer:

$$\varepsilon_V = \frac{(K_n \cdot V_s - V_p)}{V_p} \cdot 100 \quad (1.1)$$

where V_s is secondary voltage, V_p is primary voltage and K_n nominal transformer ratio.

Note that measurements are biased by measurement inaccuracy, what has to be considered in power systems analysis and operation. One of the algorithms described in this compendium targets partial compensation for measurement errors.

Switches There are several types of switches:

- Circuit Breaker
- Disconnecter
- Earth Switch

Only Circuit Breakers (CB) have the capability to interrupt current. This is done on higher voltage levels in a chamber containing a gas with very high isolation capabilities (e.g. SF6). Circuit breakers have to be capable of interrupting both normal load as well as fault current.

All other types of switches can only be opened or closed when the current does not flow through them. The purpose of disconnecter is to additionally isolate a component (or a system part) after a circuit breaker has already been opened. Earth switch connects already isolated (i.e. unenergized) components to ground for safety purposes (so any remaining or induced charge would dissipate in ground and humans could touch the component - e.g. for purpose of maintenance or reparation).

1.3 Organizational Structure and Context of Power Systems

Traditionally, power systems have been administered by a single company, referred to as **Vertically Integrated Utility**, covering the whole chain of power production, transmission, distribution and final delivery (see figure 1.5). This company bears responsibility for the system reliability and thus also secure operation. This includes coordination transmission and generation (both operation as well as long-term development). Simultaneously, it determines the final price of electricity (including production cost, transmission and distribution cost, auxiliary services and profit) subjected to the approval of the regulator.

The **regulator** is an entity, usually under a direct control of government or a corresponding ministry. Besides approving final electricity price, it is also involved in approval procedures for larger investments of the Vertically Integrated Utility, such as acquisition of new transmission assets and extensions of the power system (building of new power plants, transmission lines etc.).

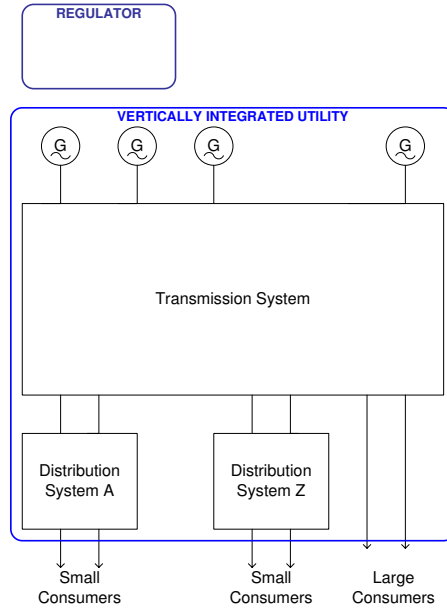


Figure 1.5: Traditional organizational structure of power systems.

Liberalization⁴ of electric industry has brought changes into the organizational framework of power systems, which principle scheme is shown in figure 1.6.

Regulator remains, however, its role is different. Now, instead of directly influencing final price, regulator's task is to create a fair and sustainable environment for the electricity market. This is done by approving transmission pricing policies and long-term monitoring of the market so that a manipulation and misuse of the market by market participants do not take place.

A new entity, not possessing any physical assets, are traders. Trading companies either participate in a power trading (mostly in cross-border trading) or provide financial services, such as electricity price hedging.

In order to introduce competition, Vertically Integrated Utility is split (frequently referred to as unbundling) to several parts - generation, transmission and distribution.

Generation companies (GENCOs) own power plants.

Responsibility for the security of power systems lies then on the System Operator (SO) operating the transmission system. There is no uniform electricity market design. In different power markets the entity bearing responsibility for the system operation have different names, such as Transmission System Operator (TSO), Independent System Operator (ISO), Transmission System Coordinator (TSC), related mainly to the fact if it owns, or only operates transmission assets.

The main complication, brought by the separation of ownership of generation and transmission, is the lack of coordination in the long-term system expansion

⁴The first country undergoing liberalization of its electricity sector in Europe has been United Kingdom (or more specifically only England and Wales) in 1990.

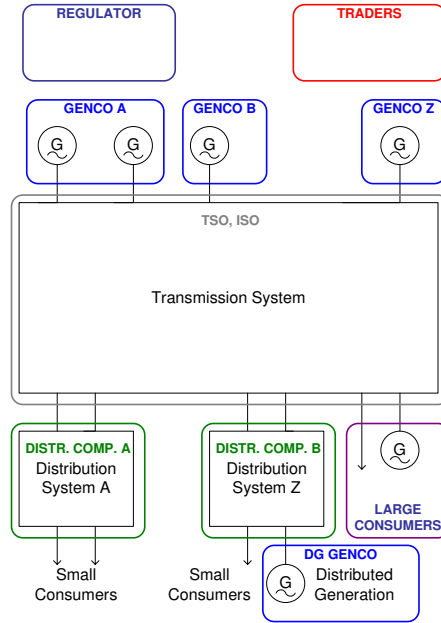


Figure 1.6: Recent organizational structure of power systems.

planning. This results in the very reduced predictability of utilization of transmission assets and correct allocation of controls.

Involvement of many parties in generation and distribution of energy creates a more complex economical environment - many more economical links and large volatility of economical factors. This in turn introduces a strong variation of power flow patterns. Interconnecting links (frequently called tie-lines), which were originally built for power exchange under emergency conditions, are now used for regular electricity trading over long distances, resulting in that tie-lines are often the most stressed/loaded elements in transmission systems.

1.4 Interconnected power systems

Previous sections described power system as an independent self-sufficient system, which is typically covering a country. But most of power systems are interconnected with their neighboring systems. These connections are mostly established on transmission system level. Incentives for interconnection are of both security and economical nature:

1. Coordinated use of power plants
2. Sharing of emergency reserves
3. Higher system security
4. Possibility for energy trading

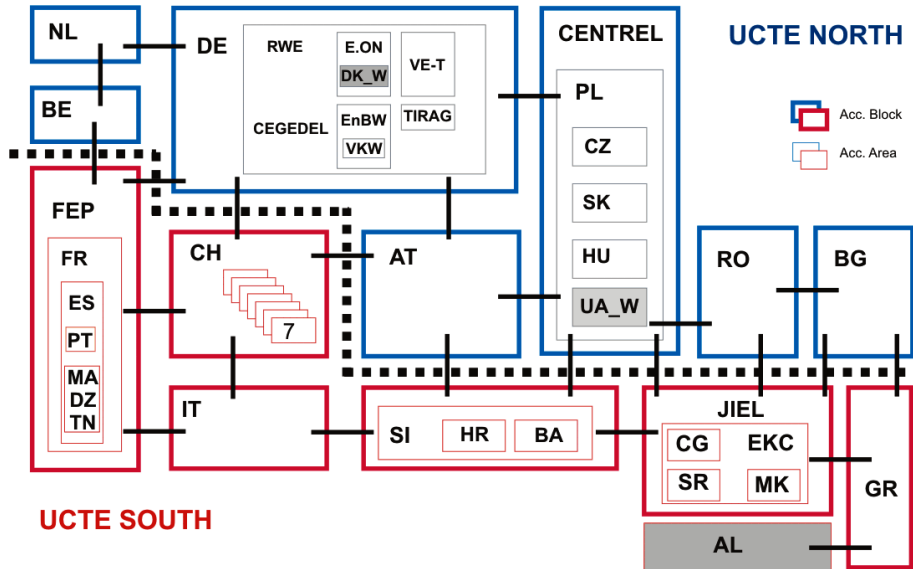


Figure 1.7: Control structure of UCTE. (The source of the picture: www.ucte.org)

Most⁵ of the continental European power systems are interconnected with each other forming a large interconnected power system covering 34 countries with total of 210 000 km of transmission lines, serving a population of around 500 million people from the basis of installed capacity of 530 GW.

In order to coordinate mostly technical and operational issues, organization called UCTE (Union for Coordination of Transmission of Electricity) was founded in 1951. UCTE is controlled in a decentralized manner, although some coordination tasks are assigned to two system operating companies - Swiss ETRANS and German RWE, see figure 1.7. UCTE system operating companies shall follow in their operation practices a document called Operation Handbook. Operation Handbook shall make sure that operation policies and practices of UCTE members are consistent and compliant.

Europe wide electricity trading oriented activities are coordinated by organization ETSO (European Transmission System Operators). ETSO includes several interconnected⁶ systems - UCTE, Nordel (Norway, Sweden, Finland, Denmark), UKTSOA (England, Scotland and Wales) and TSOI (Ireland). The form of coordination is harmonization of conditions to access and use transmission networks.

⁵Nordic countries form a separate interconnected system called Nordel. Baltic countries are connected to the transmission system of Russian federation.

⁶There are DC connections between listed AC interconnected power systems. They do not share technical issues, such as frequency control etc, but they are ideal for trading, since a better controllability is provided and thus physical flows equal contractual ones.

Chapter 2

Monitoring and Control Technology

This chapter deals with secondary technology, which in other words forms a platform and provides tools for operating (i.e. monitoring and control) a power system.

2.1 Hierarchical Concept

The monitoring and control structure (i.e. secondary technology and equipment) of power systems is hierarchical. Although it may have a historical origin, it proved to be a suitable concept for several reasons, such as robustness, scalability and most of all a simple philosophy/principles for coordination of different tasks. The coordination is achieved by different speed of operation and the covered area. Namely, the lower the position of processes and tasks in the hierarchy, the faster execution and more local scope they have.

Within the hierarchical concept, there are two types of tasks seen from the viewpoint of the location:

Tasks crossing hierarchical layers Their purpose is to provide a platform for system-wide supervision. In other words, providing information to operators in Regional and Network Control Centers, as well as a possibility to perform control actions by operators. From technological viewpoint, devices and technology implementing these tasks belong to SCADA.

Local autonomous tasks Their scope is only local¹ and they usually pose very high requirements on the execution performance (mostly speed). Their typical representative is protection.

Hierarchical concept consists of following layers, which will be further described in separate subsections:

- Process Level

¹Note that they are local in terms of collected information they process and control action execution. However, the impact of their control action may be system-wide, which may sometimes play a very negative role, for example in the case of protection malfunction.

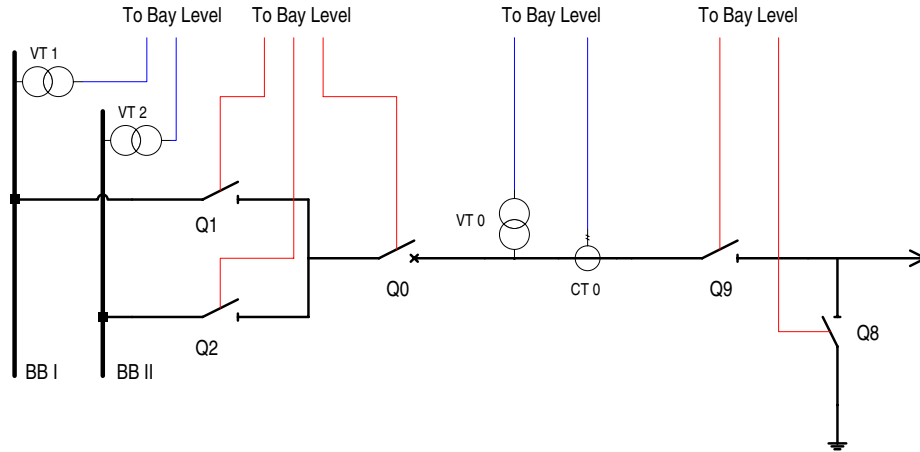


Figure 2.1: Process level connection to double busbar switchyard configuration.

- Bay Level
- Substation Level
- Regional Control Center Level
- Network Control Center Level

2.1.1 Process Level

A simple scheme principally representing the process level is shown in figure 2.1. In this example, primary equipment part represents perhaps the most common transmission system switchyard part - a connection of a high voltage line to a double busbar. The switch marked as $Q0$ is circuit breaker, all other switches are disconnectors².

All switches are equipped with a mechanism - a motor, which moves contacts and thus opens or closes the switch when a command in form of a binary signal is sent from bay level. Status of switches is monitored and communicated to the bay level in form of binary signals (on/off). Voltage measurement transformers $VT1$ and $VT2$ measure voltages of busbars $BB I$ and $BB II$, respectively, and transformer $VT0$ measures line end voltage. Line current is measured by the current measurement transformer $CT0$.

To summarize, interface signals between process and bay level can be listed as follows (from the process level perspective):

- Binary Inputs (Open or close commands to switches.)
- Binary Outputs (Switches' status information.)
- Analog Outputs (Measurements of current and voltage.)

²Recall that circuit breaker is capable of interrupting a flowing current. Disconnectors do not possess this capability. Their purpose is to increase security by isolating parts of switchyard after a circuit breaker is already open.

and interface between primary and secondary equipment is realized by means of:

- Sensors (*VTs*, *CTs*, and switches' status sensors)
- Actuators (switches' trip coils and contacts' movement motors)

Most common connection between process and bay level is a hard-wired direct connection. Some manufacturers pursue a philosophy of Remote Input/Output terminals, which are placed in a marshaling kiosk (it can be imagined as a small box next to the switchyard) and are hard wired to sensors and actuators and connected via fibre optics to bay level, in order to minimize potential electromagnetic compatibility problems. However, most recent trends aim at introduction of sensors possessing a transducer and a direct communication interface.

2.1.2 Bay Level

Devices, integrating bay level tasks, are usually placed in two separate cubicles according to the type of functionalities they serve:

- Protection
- Control

Cubicles are shown on the left side of figure 2.2. They have slots with standardized dimensions, able to accommodate protection and control devices from different manufacturers. Typical set of devices placed inside of cubicles, corresponding to the chosen example of the line connection to double busbars, is shown in the right part of the figure. The shaded bottom part represents area usually reserved for optional devices.

Devices hosting protection and control functions are frequently referred to as IED - Intelligent Electronic Devices. Frequently a manufacturer uses essentially the same hardware platform both for devices for protection and control. The distinction (between protection and control) is then made by the corresponding software and minor hardware modules. A simple scheme capturing the principal functional IED architecture is shown in figure 2.3.

IED possesses capabilities to receive and process measurements, issue control commands and communicate with higher level systems (e.g. substation automation).

Referring to figure 2.3, functional blocks of IED architecture are commonly as follows:

CPU , Central Processing Unit, interprets instructions and processes data contained in programs incorporating protection, control and communication functionalities.

PCMCIA stands for Personal Computer Memory Card International Association, which originally specified standards for Portable Computer (PC) Cards. PC cards are peripherals, which shall enable additional hardware functions. In this particular case it is communication to higher level systems, such as Station Level computer (to be described later).

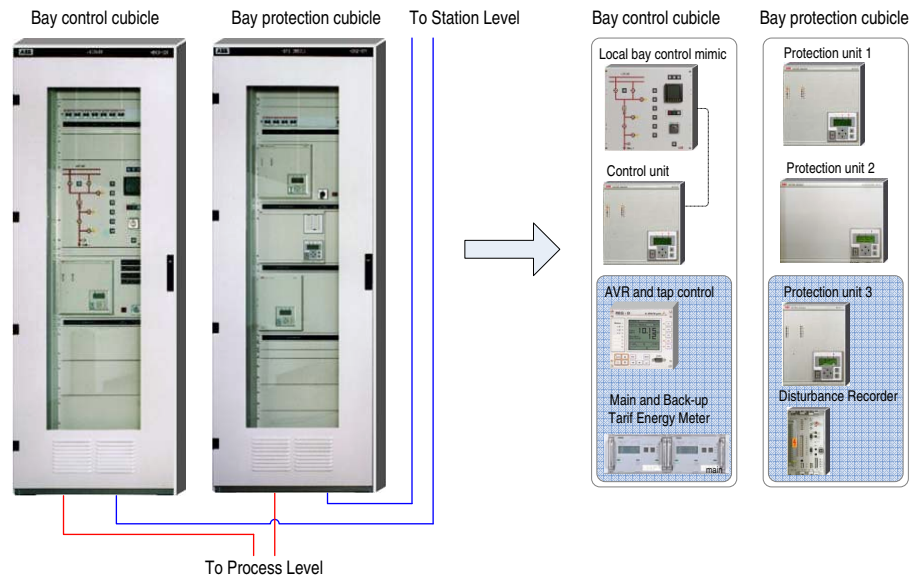


Figure 2.2: Bay level architecture example.

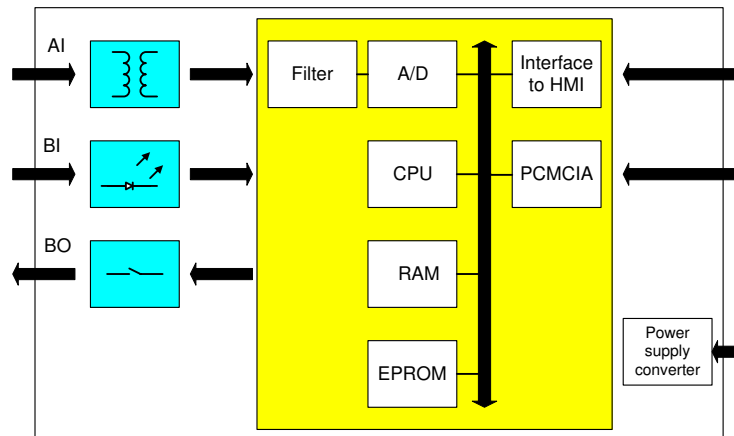


Figure 2.3: Intelligent Electronic Device functional blocks.

RAM , Random-Access Memory, holds data only as long as a power supply to it is applied. In IEDs, RAM is used for storing data related to real-time computations - such as measurements samples and instructions computing protection functions.

EPROM Data stored in Erasable Programmable Read-Only Memory remain untouched even when power supply is off. Data can be erased only by exposing EPROM to strong ultraviolet light. Thus, in IEDs, EPROM serves as a storage medium for programs.

Filter, A/D Converter Incoming analog measurements often contain noise, which is removed by low-pass filter. Then A/D converter samples analog signal into a digital form suitable for processing by IED. An usual value of sampling frequency is 600 Hz, i.e. 12 samples per voltage or current period (being 20 milliseconds in case of 50 Hz nominal system frequency).

Interface to HMI Most of IEDs can be configured and controlled via both local and remote HMI.

Interface to Process Level A galvanic isolation is used between signals coming from process level and an IED. For example, analog measurements are isolated by a transformer before further processing.

Power supply module Substations are equipped by a network of a secure (i.e. redundant) DC power supply, typically 110 or 220 V. The voltage level is lowered to appropriate level(s) needed by the IED.

Bay level tasks can be divided into two groups:

- Protection
- Control

Protection is autonomous task dedicated only to bay level (except busbar protection) and is further described more in detail in section 2.3.

Control oriented functionalities are essentially of two types:

- Monitoring
- Switching operations

Thus, the term *control* in the context of the technological structure of power systems does not directly refer to the control theory, rather to the control "tools" - data acquisition and control execution.

Monitoring

Monitoring represents supervision of data coming from the process level and sending them to substation level as soon as certain criteria are met. For binary inputs it is a change of their value - state. When such a situation is detected, a GPS time stamp is added to the signal. The purpose is e.g. to determine an exact time of a line trip. Analog inputs are continuously scanned and values are compared to the last value sent to station level. As soon as a value of analog input deviates by a defined dead-band either directly from the last value, or accumulated/integrated deviation from the last value. This implies that most of the time in vast majority of systems, measurements (voltage magnitude, active

and reactive power) are collected in irregular time intervals. An alternative option, however very rarely employed, is to pre-set a regular time interval in which the analog measurement should be converted to a discrete value and sent further up.

Switching operations

Switching operations can be initiated either locally (via a local bay level mimic) or remotely (from any other hierarchically higher level). Switching means sending a binary signal to the process level, which shall open or close a switch (circuit breaker, disconnecter or earth switch). A feature called interlocking is implemented, which blocks closing or opening a switch a way, that could lead to the equipment damage or endangering human health. A typical example is an attempt to open a disconnecter which both sides are energized. That would lead to an arc - a current flowing in the air, which would heat up contacts and eventually destroy them.

2.1.3 Substation Level

An architecture of a modern larger substation secondary equipment is shown in figure 2.4. Common elements are:

Gateway Bidirectional communication interface to higher level systems, which allows remotely triggered (e.g. by network operator) execution of control commands in the substation as well as remote collection of measurements taken in the substation.

Time synchronization is achieved either by a time server and mutual communication or by a GPS receiver and master clock, which serve as a source of the time reference signal, which is used both on station as well as bay level.

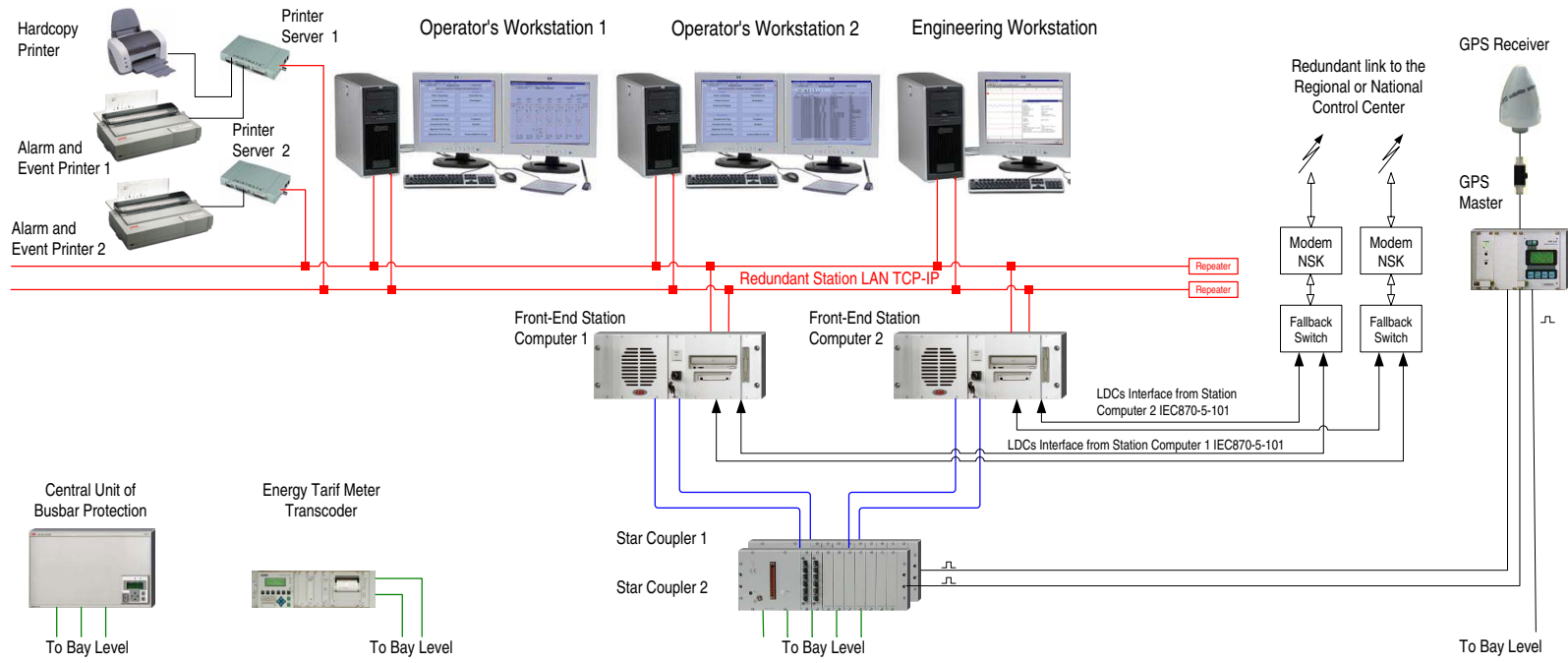
Busbar and breaker failure protection cover the whole substation, therefore they are placed on the substation level and collect measurements coming from all bays.

Tariff metering Some substations (especially the ones located on the interface to other power system, or country) possess also an independent system for collection of measurements related to the pricing of transmission services. Measured signals are active and reactive powers, typically in time slots of the length 15 minutes.

Substation Automation consists of station computer, operators' workplaces (i.e. HMI), peripheral devices (e.g. event printers) and communication among all listed components.

Substation Automation is a system, which provides a set of substation-wide functionalities. Such as monitoring possibilities of the whole substation, components condition monitoring (a typical example is number of switching operations of a circuit breaker, which has a strong impact on the circuit breaker lifetime), data archiving and switching operations between bays. Switching operations can be done using a sequencer functionality obeying inter-bay interlocking rules, which are similar to the ones on the bay level, but here relations between different bays are considered.

Figure 2.4: Station level architecture.



2.1.4 Regional and Network Control Center Level

The major task of the control center is to provide information to an operator, based on which he makes a decision whether he should interfere and introduce some control actions or not.

An example of a typical control center is shown in figure 2.5. It receives data from substations and depending on system architecture, also from other control centers (from regional control centers and/or national control centers of surrounding countries). Data are then either directly displayed, or further processed by Energy Management System (EMS) applications, as discussed in section 2.2, together with signal acquisition data chain and components in a control center.

2.2 SCADA/EMS

SCADA stands for Supervisory Control and Data Acquisition. SCADA is a technology allowing collection of data (typically to a central location) from remote (often distributed over large geographical area) facilities and sending control commands to those facilities. In other words, SCADA represents a tool, which an operator uses for a supervision of a large process/system. When talking about a particular process/system supervised by SCADA, expression 'SCADA system' is mostly used.

It is quite difficult to determine a starting point of SCADA. Most often it is linked to the introduction of remote telemetry and bidirectional communication (i.e. remote execution of control commands) in the beginning of 1960-ties by oil and gas production companies, having facilities in areas inhospitable for human operators' presence. This period also coincides with computers development and their inclusion in SCADA. In the 1970-ties, SCADA was becoming more mature technology, boosted by improved HMI (Human Machine Interface) allowed by higher computers' performance.

Present SCADA installations range from small, municipality level installations, to large systems covering several thousands kilometers. SCADA has spread to several application fields:

process industry

transport systems infrastructure

utilities water, gas and electricity

A basic structure of a modern SCADA system consists of three groups of components:

- Distributed Data Acquisition and Control Execution - RTUs and Actuators and Sensors
- Communication
- Central Processing

Central Processing usually takes place in a so called Master Station (sometimes referred to as Master Terminal Unit) and provides a direct access point for an

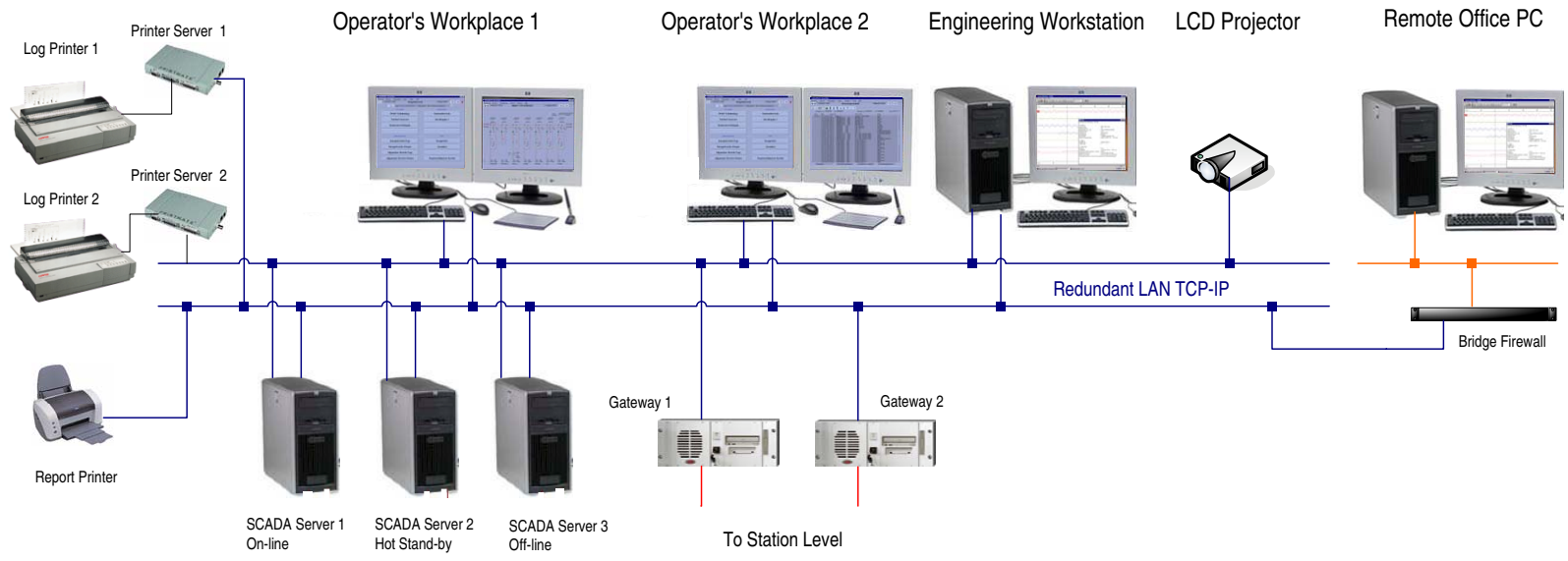


Figure 2.5: Control center architecture.

operator - HMI³. Master Station may consist of various hardware platforms and software modules. In small SCADA installations, hardware may be a single PC. In larger SCADA installations, hardware may include several servers, operators' and maintenance working places. Operator's working place is usually represented by a keyboard and one or several screens. Sometimes and additional mimic is provided, being either a large dimensioned on-wall projection, a large LCD panel, or a special wall representing the supervised system. Frequently a set of printers is provided in order to produce a hard-copy list of events. Operating systems are usually Unix or similar. Windows is sometimes used for graphical interface part. Software modules implement functions as communication, data storage and archiving, HMI etc.

In terms of hardware, SCADA communication systems employ all types of media, e.g. radio and direct serial or modem connections, frequently combinations of them within one SCADA system installation. Standard SCADA communication protocols are IEC 60870-5-101 or 104, Profibus and DNP3. However, a large number of legacy⁴ protocols exist, e.g. Modbus, RP-570 etc. Many protocols feature extensions, enabling them to operate via TCP/IP. However, a usual practice is to keep SCADA communication system free of connection to internet in order to avoid possible security⁵ problems.

A major component in data acquisition and control execution is RTU (Remote Telemetry Unit, sometimes also interpreted as Remote Terminal Unit). Typically, there is one RTU per each geographical location of a remote facility⁶. Usually, RTUs have a modular structure allowing them to accommodate arbitrary large number of inputs, which may be either analog or binary, and binary outputs. Essentially, RTU is a communication interface to the process level, i.e. measurements (analog inputs), switches (e.g. circuit breakers, disconnectors etc.) statuses (binary inputs) and opening or closing switches (binary outputs). Thus, the measurement itself is taken by a sensor converting it to a form suitable for RTU input. In power systems this is done usually in two separate stages, i.e. first the measured quantity is converted to the safe level (e.g. voltage is measured by the voltage measurement transformer and converted to the value around 100 Volts) and then it is rectified by a transducer to a RMS (Root Mean Square) value. Similarly, RTU output goes to an actuator, which is executing the desired control command.

In power systems context, central processing takes place in a control center, typically looking as in 2.5. Frequently, a back-up control center is established, which should take over all vital functions in a case of the main control center failure (e.g. a physical damage in war conditions). In some systems, one or several regional control centers may serve as a back-up of national control center.

SCADA usually delivers only 'raw' data, which are further processed by Energy Management System (EMS). EMS is a set of programs/tools, which adds an interpretation to measurements, allowing an operator to assess the system security better and even to compute suitable control actions.

RTUs are normally placed on the Substation Level, one RTU per substation. However, an alternative trend to RTUs in recent years is, that in many

³The HMI is usually implemented in a form of a mimic representing the supervised plant.

⁴These are either vendor or user specific.

⁵Note that security in this context refers to a risk of a cybor attack, not power system operation security traditional definition.

⁶In power systems context, this means one RTU per substation.

countries/systems data acquisition is carried out by Bay Level control IED and Substation Automation on Substation Level.

2.3 Power Systems Protection

Objective of the protection in power systems is to eliminate faults or unacceptable operating conditions for a component and related effects on the network. Fault elimination is usually done by isolation of the affected component. There are various types of faults, a special category are faults on generators (e.g. underexcitation). But all other common transmission components are exposed mostly to faults as overvoltage and most of all overcurrent, which is more commonly referred to as short-circuit. A short-circuit is an unintentional and undesired conductive connection between two places having a different electrical potential (most common type of a failure is a connection between a phase and ground), which results in an excessive electric current flow. Negative consequences of a short circuit current are mechanical (as a large current induces significant forces) and thermal (losses being dissipated by a short circuit heat up exposed components). If a short circuit is not eliminated, it may, with a high probability, damage an exposed component. Origins of short circuits are normally very difficult to predict as they may involve natural phenomena (as a lightning strike during a thunderstorm), a human error (e.g. hitting an underground cable during digging works) or aging of the equipment (e.g. isolation material degradation).

Requirements on protection are:

Reliability Assurance the protection will perform correctly. This can be described by a compromise between two contradictory objectives:

Dependability The degree of certainty that a relay or relay system will operate correctly (sensitivity: ability to determine fault conditions).

Security The degree of certainty that a relay or relay system will not operate incorrectly (selectivity: maximum continuity of service with minimum system disconnection).

Speed of operation Minimum of fault duration and consequent equipment damage

Simplicity Minimum protective equipment and associated circuitry to achieve protection objective

Economics Maximum protection at minimal total costs

Protection operates essentially in stages:

- Measuring - receiving analog values, their sampling, i.e. A/D conversion.
- Fault Type Detection - detecting the type of fault (e.g. phase to ground or 3-phase short circuit) and selecting corresponding algorithm for detailed computations.
- Fault Computation - performing detailed computation of fault conditions.
- Decision - evaluation of fault conditions, if trip conditions are met.

Failure Type	Description
G1	Short-circuit between a stator winding and ground.
G2	Short-circuit between two stator windings.
G3	Short-circuit between turns of the same stator winding.
G4	Short circuit between rotor (excitation) winding and ground.
T1	Short-circuit between transformer winding and the oil tank.
T2	Short-circuit between two transformer windings.
T3	Short-circuit between turns of the same transformer winding.
T4	Short-circuit between primary and secondary windings.
B1	Short-circuit between busbar and ground.
B2	Short-circuit between busbars.
L1	Line single-phase-to-ground short-circuit.
L2	Line 2-phases-to-ground short-circuit.
L3	Line 3-phases-to-ground short-circuit.
L4	Line 3-phases short-circuit.
L5	Line phase-to-phase short-circuit.

Table 2.1: Types of common failures. The letter in their name refers to the affected component, i.e. 'G' to generator, 'T' to transformer, 'B' to busbar and 'L' to line.

- Trip - sending out binary signal to open a corresponding circuit breaker.

The first protection devices used electromechanical principles (Current on the secondary side of a measurement transformer flows through a coil. If its value is high - corresponding to a fault current - it induces electromagnetic field, which interacts with the field of a permanent magnet and initializes a trip signal.), what gave them expression *electromechanical relays*. The term relay is since then often used to refer to protection devices, even for ones, which do not possess any electromechanical features.

In 1960, *electrostatic protection* devices have appeared, which did not have moving parts anymore (thus electrostatic), but rather operation amplifiers, which can be tuned to perform arithmetic operations depending on their tuning by values of connected resistors and capacitors.

Although two previous types of relays may still be found in operation, since 1985 a dominant type of protection devices are *digital* (also referred to as numerical) *relays*. As mentioned in subsection 2.1.2, they are implemented in devices called IEDs. In order to increase dependability, concepts guaranteeing a redundancy are used. In transmission networks, for the protected bay, this means using two IEDs operating simultaneously in parallel. In distribution networks, one IED operates as a main protection, and the second one as a time-delayed back-up. Various types of protections are described in the remaining part of this section.

Protections addressing system aspects (i.e. system protection schemes, underfrequency and undervoltage protection schemes) are described in subsection 3.2.2. Here the emphasis is given on component protection. Most common representatives of types of component failures, are shown in figure 2.6 and further described in table 2.1.

Note that most common type of failure is single-phase-to-ground line short-circuit, which may represent perhaps 95 % of failures taking place in power

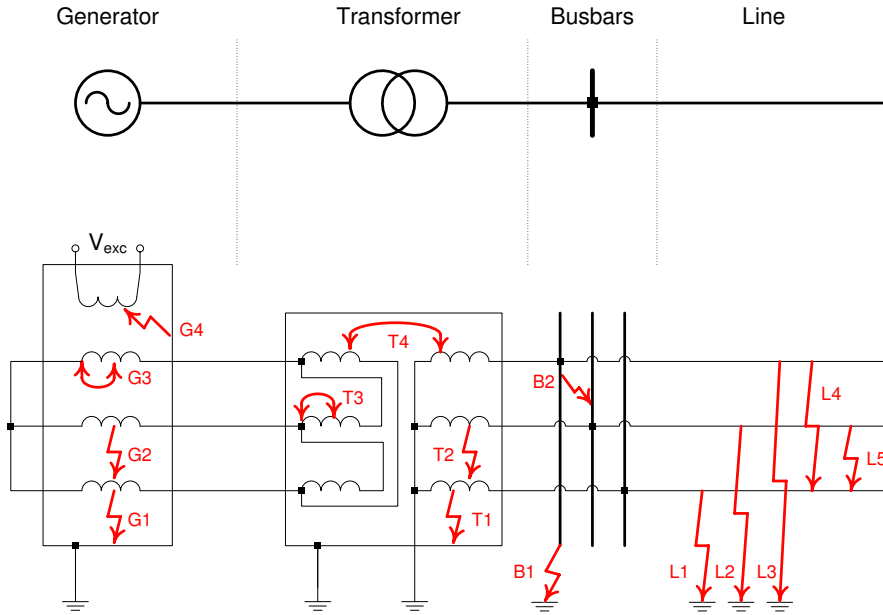


Figure 2.6: Most common type of failures - short circuits in power systems. The top part represents single line diagram equivalent of the bottom 3-phase scheme.

systems' primary equipment.

Very rare, but with very severe consequences, is circuit breaker failure. Usually it is revealed when a circuit breaker fails to interrupt a fault current created by a fault in other element (e.g. line), in other words, circuit breaker remains closed despite of the OPEN command received from a protection device. In such case, all other circuit breakers, connected to the same busbars as the faulty circuit breaker, have to simultaneously open in order to eliminate the fault current. This is very demanding on the busbar/breaker-failure protection and usually results in loss of supply for the whole affected substation.

2.3.1 Overcurrent Protection

Overcurrent protection is perhaps the most used protection. Its principle is basically very simple. The RMS value of the current flowing to (or out of) the protected object, as shown in figure 2.7, is compared with the predefined value of the maximum allowed current. If the measured current is higher, the circuit breaker is opened in order to prevent damaging of the protected object.

Frequently the time duration of the fault current is considered too, mostly in the shape of the characteristic in figure 2.8. So small overcurrents can be tolerated much longer than large currents.

To summarize, overcurrent protection does not suit for really time critical applications, where stability issues are of concern. It offers only very limited selectivity and coordination.

The main advantage of overcurrent protection is its simplicity and thus low

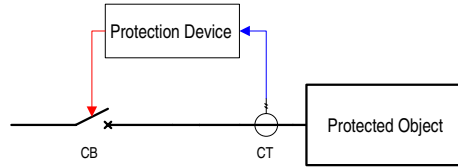


Figure 2.7: Overcurrent protection connection scheme. The protection device is fed with the current measured by current transformer CT. If an overcurrent is detected, circuit breaker CB is open by a trip signal

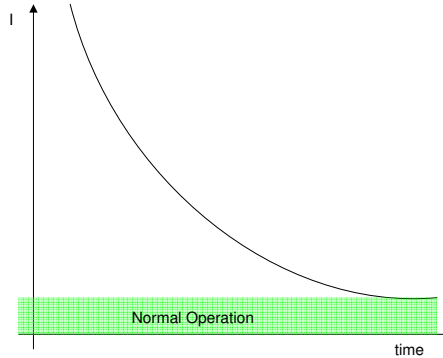


Figure 2.8: Overcurrent protection characteristic.

production and installation cost. Its application area are radial, mainly distribution networks.

2.3.2 Differential Protection

Differential protection, as the name suggests, compares incoming and outgoing currents in order to identify an internal fault in the protected object, see figure 2.9. Theoretically, according to the first Kirchhoff's law, the sum of currents should be zero. Practically, a small tolerance has to be included, to cover measurement errors of current transformers.

Differential protection is very selective and accurate in identifying fault conditions. But it relies on measurements from different locations and their communication to the protection unit. Thus, it is vulnerable to loss of a communication link to one of measurement locations, as well as communication time delay. This imposes high demands on communication channels reliability and speed. (Frequently alternative communication paths are included in the design, in case of primary communication link is interrupted. This creates higher complexity as communication time delay changes in such condition.)

Differential protection is most common type of protection for transformers, busbars and short lines (short lines are considered to be lines up to 40 - 50 kilometers.) on transmission level.

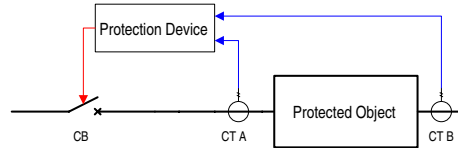


Figure 2.9: Differential protection connection scheme. The protection device is fed with currents measured by current transformers CT A and CT B. If a fault current is detected, circuit breaker CB is open by a trip signal

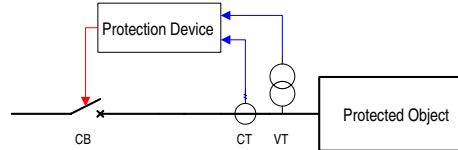


Figure 2.10: Distance protection connection scheme. The protection device is fed with the current measured by current transformer CT and the voltage measured by voltage transformer VT. If a fault current is detected, circuit breaker CB is open by a trip signal

2.3.3 Distance Protection

Distance protection is implemented not as a stand-alone functionality being only at one single location, but as a scheme in which the protected object is covered by two or more distance protections coordinating their actions. Therefore the topic of the distance protection is treated here in two stages: basic principle; and distance protection schemes.

Distance protection is a dominant type of protection of transmission and distribution lines.

Distance Protection Principles Distance protection measures a "distance to ground" in terms of impedance: $\vec{Z} = \frac{\vec{V}}{\vec{I}}$. So if there is a ground fault on a line, the impedance measured based on the current and the voltage on the one line end (see figure 2.10) is lower than the line impedance, thus indicating the fault.

Normal loading as well as fault conditions may vary in a very wide range, therefore complex plane is used to display the impedance in distance protection.

Usually, three zones determine distances along the line, where a fault can occur. As shown in figure 2.12, most common tuning of zones is as follows:

- Zone 1 provides the primary protection of the line and it shall trip immediately when the measured impedance is smaller than 80 % of the line impedance.
- Zone 2 is also a part of the primary protection with the reach to 120 % of the line (i.e. 1.2. multiple of the protected line, covering also the beginning of the next line) and its operation is timed with the time delay 0.3 - 0.5 s.

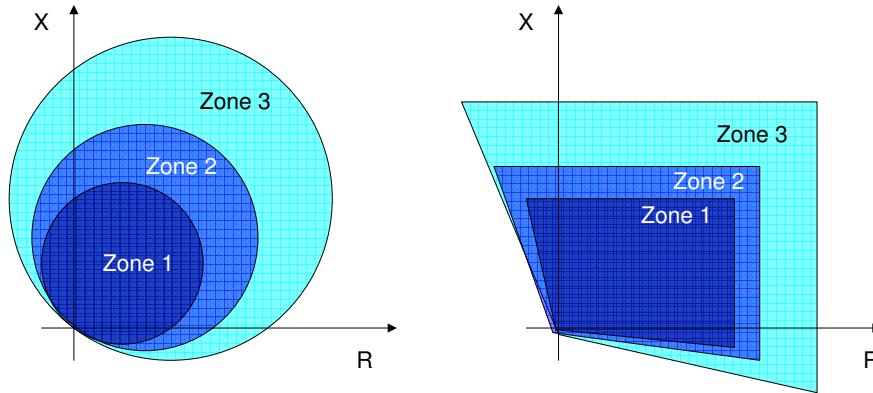


Figure 2.11: Two examples of zones' shapes of distance protection.

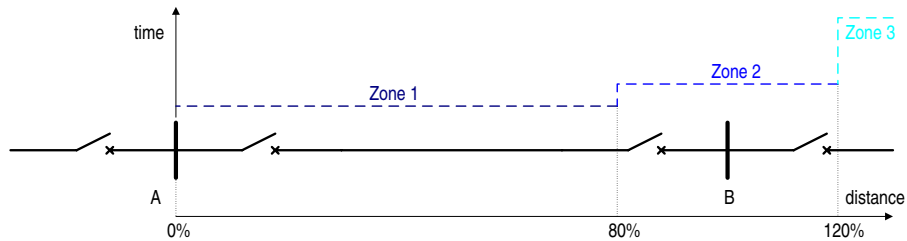


Figure 2.12: Typical zones' settings of distance protection.

- Zone 3 provides a backup functionality (i.e. secondary protection) for the next line, which is covered by the third zone setting, which operation is time delayed 2 seconds, unless a blocking signal is received.

There are many distance protections in operation, having different shapes of zones, figure 2.11 shows an example of two shape types.

Distance Protection Schemes Most common scheme for protection of transmission lines is *Permissive Overreaching Scheme*. Its principle can be explained with the help of figure 2.13 and zones:

Underreaching zone 1 As soon as a fault appears in the first zone, local circuit breaker is opened.

Overreaching zone 2 If a fault is detected within the second zone, permissive signal is sent to the protection terminal on the other end of the line. Local circuit breaker is tripped immediately, if a permissive signal is received from the other end of the line. If no permissive signal is received, but the impedance remains in the second zone longer than pre-set time (typically 0.4 second), circuit breaker is tripped anyway.

So, if fault $F1$ takes place, it is "seen" by underreaching first zones of both protections (both at location A and B).

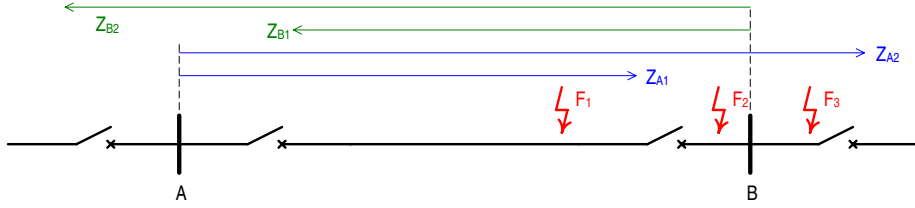


Figure 2.13: Permissive Overreaching distance protection scheme protecting a line.

If fault F_2 occurs, protection B trips its local circuit breaker and sends permissive signal to protection A, which "saw" the fault and after receiving the permissive signal, it trips its local circuit breaker too.

If fault occurs at the location F_3 , protection A detects it in its second zone, but it receives no permissive signal from protection B, since the fault is out of the directly protected line. However, if the neighboring line is not tripped (for example there is a failure of the circuit breaker on the neighboring line, or protection of the neighboring line does not detect the fault), fault F_3 is persisting in the observation of the protection A, protection A trips its local circuit breaker in order to eliminate the fault remotely. So in this case relay A acts as a backup protection.

Chapter 3

Operation Principles

This chapter reviews threats of power system security (such as instability phenomena). The philosophy of power system operation is introduced, including an explanation of its components - operation and planning activities.

3.1 Introduction

Power systems have originally arisen as individual self-sufficient units, where the power production matched the consumption. In the case of a severe failure, a system collapse was unavoidable and meant a total blackout and interruption of the supply for all customers. But the restoration of the whole system and synchronization of its generators were relatively easy due to the small size of the system.

Power systems size and complexity have grown to satisfy a larger and larger power demand. Phenomena, having a system-wide, global nature, endangering the normal operation of power systems have appeared, explicitly:

- Frequency Instability - is inability of a power system to maintain steady frequency within the operating limits. Keeping frequency within the nominal operating range (ideally at nominal constant value) is essential for a proper operation of a power system. A maximal acceptable frequency deviation (usually 2 Hz) is dictated by an optimal setting of control circuits of thermal power plants. When this boundary is reached, unit protection disconnects the power plant. This makes the situation even worse - frequency further decreases and it may finally lead to the total collapse of the whole system. Frequency instability is in its nature rather a tracking than truly a stability control problem.
- Voltage Instability - is the inability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance. A system enters a state of voltage instability when a disturbance, increase in load demand, or change in system conditions causes a progressive and uncontrollable drop in voltage.
- Transient Angular Instability (also called Generator's Out-of-step) - is the inability of the power system to maintain synchronism when subjected to

a severe transient disturbance. The resulting system response involves large excursions of generator angles and is influenced by the nonlinear power-angle relationship. In case of transient angle instability, a severe disturbance is a disturbance, which does not allow a generator to deliver its output electrical power into the network (typically a tripping of a line connecting the generator with the rest of the network in order to clear a short circuit). This power is then absorbed by the rotor of the generator, increases its kinetic energy, which results in the sudden acceleration of the rotor above the acceptable revolutions and eventually damage of the generator.

- Small-signal Angular Instability (also mentioned as Generators' Swinging or Power Oscillations) - is the inability of the power system to maintain synchronism under small disturbances. Such disturbances occur continually on the system because of small variations in loads and generation. The disturbances are considered sufficiently small for linearization of system equations to be permissible for purposes of analysis. Local modes or machine-system modes are associated with the swinging of units at a generating station with respect to the rest of the power system. The term local is used because the oscillations are localized at one station or small part of the power system. Inter-area modes are associated with the swinging of many machines in one part of the system against machines in other parts. They are caused by two or more groups of closely coupled machines being interconnected by weak ties.

Note, that transient phenomena, having purely local nature, such as short-circuits, usually do not have a direct system-wide impact (Although they may be an initiating trigger of instabilities, e.g. a slow short-circuit fault clearing may cause transient instability of a closely located generator etc.), therefore they are excluded from further considerations.

With the rising importance of the electricity for industry (and the entire society), the reliability of supply, and thus power systems security, has become a serious issue. Interconnection of the separated/individual power systems have offered a number of benefits, such as sharing the reserves both for a normal operation and emergency conditions, dividing of the responsibility for the frequency regulation among all generators and a possibility to generate the power in the economically most attractive areas, thus providing a good basis for the power trade. Although this has reduced some negative features mentioned above, on the other hand it has also introduced new problems - such as a potential for spreading of disturbances over large distances and thus paralyzing vast geographical areas etc.

Counteracting (mitigating) above listed phenomena, in order to keep a secure system operation, has become a task of four major activities forming control philosophy of power systems:

- Operation
- Day-ahead Operation Planning
- Short-term Planning
- Long-term Planning

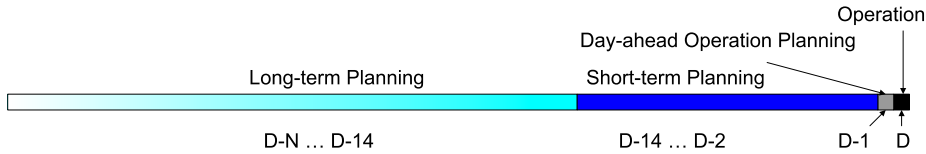


Figure 3.1: Time scale (bottom part of the figure) of Monitoring and Control Activities (upper part of the figure).

Their mutual relation and time scale of their allocation is shown in figure 3.1.

Remaining sections of this chapter individually describe these activities further in detail.

3.2 Operation

Control methods mitigating the listed dangerous phenomena (frequency, voltage, transient and small-signal instability) and keeping the power system in a secure state are mainly based on the classification of power system states. Explicitly, according to [4], these states are:

Normal All system variables are within the normal range and no equipment is being overloaded. The system operates in a secure manner and is able to withstand a contingency without violating any of constraints.

Alert All system variables are still within the acceptable range and all constraints are satisfied. However, the system has been weakened to a level where a contingency may cause an overloading of equipment that places the system in an emergency state.

Emergency Some system variables are outside of acceptable range (e.g. voltages too low, lines overloaded). If no control changes are introduced, system progresses into In Extremis.

In Extremis Cascading spreading of system components outages resulting in partial or system-wide blackout (loss of supplied load).

Restoration Energizing of the system or its parts and reconnection and resynchronization of system parts.

Mutual relations and possible transitions among the operation states are shown in figure 3.2.

With respect to the above categorization of operation states, control approaches for keeping power systems secure are usually applied in two stages:

Normal and preventive control This control is applied in the normal and alert state. Its objective is either to stay in or to return into normal state.

Emergency control This control is applied in emergency or in extremis state to stop the further progress of the failure and to bring the system into normal or alert state.

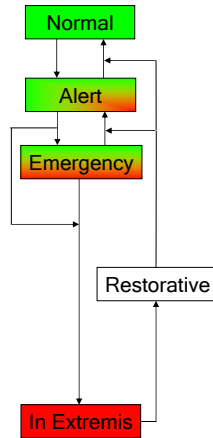


Figure 3.2: Power system operating states. Arrows express possible transitions among them.

3.2.1 Normal and Preventive Control

Typical representatives of normal and preventive control are:

- Hierarchical automatic control:
 - Frequency control
 - Voltage control
- Centralized manual control based on:
 - Contingency screening
 - Operator judgment

Control measures usually include:

- Change of active power generation set-points, i.e. redispatch.
- Change of reference points of flow controlling (FACTS) devices.
- Start-up of generation units.
- Change of voltage set-points of generators and Static VAR Compensators (SVC).
- Switching of shunts elements (reactors, capacitors).
- Change of substation configuration (e.g. splitting of busbars).

The hierarchical frequency control concept has been established in power systems for decades. Its description is given in the other part of this course as well as it can be found in a number of textbooks dealing with power systems control or operation.

On the other hand, hierarchical voltage control has been implemented only in a few countries (e.g. France, Italy). Two additional higher levels - Secondary

Voltage Regulation (SVR) and Tertiary Voltage Regulation (TVR), enrich primary voltage regulation. National TVR shall coordinate SVRs that control the areas voltage profiles. The objective here is to create a system-wide voltage profile minimizing the transmission of reactive power over longer distances and maximizing the reactive power generation reserves. Larger reactive power generation reserves mean that they can be activated in case of a disturbance, i.e. the system is made more robust.

Hierarchical frequency and voltage control concepts are usually fully automated, i.e. the control loop is closed and does not involve any human intervention. Possible negative interactions among layers are minimized by appropriate selection of their time constants, e.g. primary layer dynamics is in order of seconds, secondary layer in tens of seconds to minutes and tertiary layer minutes to hour.

As mentioned above, in a large majority of power systems, hierarchical frequency control is the only fully automated closed loop control in Normal and Alert state. In all other cases, the human factor is involved in form of power system operator. Then a control scheme/loop basically consists of:

Data acquisition and monitoring This stage is usually handled by Supervisory Control and Data Acquisition (SCADA) system (described more in section 2.2) and State Estimation (described in section 4.1).

Operator's decision Operator based on the knowledge about the present state of the system, coming from SCADA system, decides, whether to intervene by modifying actual values of controls. Operator can base his intervention decision either on his experience and judgment or on decision support tools.

Control execution This stage materializes either via link SCADA - Substation Automation, or communication with the personnel executing the control manually in substations or/and power plants.

In this control scheme, the main objective is to keep the power system in a secure state, expressed by the compliance with N-1 criterion. That means that a possible outage of any single component shall not create an unacceptable stress of other component(s) or instability problem. In most power systems, a procedure called Security Assessment is employed for this purpose. Security Assessment is usually implemented as a program belonging to the Energy Management System (EMS) processing present state information given by State Estimation. Security Assessment is then done in a continuous cycle, typically every 5 or 15 minutes, or it is initiated by operator. The consequences of possible component outages, which are then examined, are components operated outside their limits (line overloads and undervoltage or overvoltage), voltage instability, transient instability). Usually, each type of consequences is analyzed by a separate software package, independently of others (i.e. there is a package for voltage instability analysis, a package for components overloads etc.). Security Assessment is often executed in two steps:

1. Complete set of possible contingencies is processed using fast (sometimes only approximate) static analysis, neglecting system dynamics. This procedure is often referred to as Contingency Screening.

2. Reduced set of contingencies, identified as possibly most severe ones in the Contingency Screening, are analyzed in detail in form of time domain simulation considering all relevant dynamics aspects.

For checking of unacceptable components stresses, employment of purely static methods (i.e. Contingency Screening) is sufficient and the whole Security Assessment is then referred to as Static Security Assessment (SSA). Essentially, for each assumed contingency a power flow computation (described in section A.3) is done, followed by a simple comparison of computed post-contingency state and components operational limits. When a time-domain simulation is involved, the term Dynamic Security Assessment (DSA) is used.

The relation between the active power consumed in the monitored area and the corresponding voltages is expressed by so called PV-curves (often referred as "nose curves"). The increased values of loading are accompanied by a decrease of voltage. When the loading is further increased, the maximum loadability point is reached, from which no additional power can be transmitted to the load under those conditions. In case of constant power loads, i.e. loads whose power consumption is independent of the voltage magnitude, the voltages in the nodes become uncontrollable and rapidly decrease, resulting in voltage collapse. However, the voltage level close to this point is sometimes very low, what is not acceptable under normal operating conditions, although it is still within the stable region. But in emergency cases, some utilities accept it for a short period. There are also other alternative graphical representations, e.g. QV-curves (amount of needed reactive power to keep a certain voltage at a given active power loading). PV-curves (or QV-curves) starting from actual system state are computed for considered set of contingencies. If any of the resulting PV-curves indicates that the system would be unstable (or would have an unacceptably low voltage profile) after a contingency, the operator takes preventive measures, such as switching shunt capacitors, generation redispatch etc.

3.2.2 Emergency Control

Typical representatives of emergency control in today power systems are:

- Protection based systems:
 - Under frequency load shedding (UFLS) schemes
 - Under voltage load shedding (UVLS) schemes
 - System Protection Schemes (SPS)
- Damping control

Emergency control measures may include:

- Tripping of generators
- Fast generation reduction through fast-valving or water diversion
- Fast HVDC power transfer control
- Load shedding

- Controlled opening of interconnection to neighboring systems to prevent spreading of frequency problems
- Controlled islanding of local system into separate areas with matching generation and load
- Blocking of tap changer of transformers
- Insertion of a braking resistor

The main challenge in emergency control is the urgency, in which it has to be applied. Since historically very high demands for high performance communication system and control decision logic could not be met, emergency control strategy relies on devices reacting to their local measurements based on their setting determined off-line by simulations of assumed dangerous scenarios.

Under frequency load shedding (UFLS) schemes Local devices used for UFLS schemes are UFLS relays. UFLS schemes and relays might be sorted in various categories, but their functionality is essentially the same. They are usually triggered when frequency drops to a predefined level and/or with a predefined rate of frequency change. Their action is disconnection of the load in several steps (5 - 20 % each) from the feeders they supervise. However, their effectiveness is strongly dependent on their careful tuning based on prestudies, since there is no on-line coordination between them.

Under voltage load shedding (UVLS) schemes Under voltage load shedding relays are a conventional local solution to prevent voltage instability. The criterion triggering the load shedding action is a predefined voltage level in the supervised node (For example 88 % and 86 % of the nominal voltage in one particular isolated network.).

System Protection Schemes (SPS) The abbreviation SPS is often also referred to as Special Protection Schemes. However, throughout this thesis the System Protection Scheme interpretation is assumed due to more appropriate descriptive value (expression 'special' may be misleading due to large room left for its interpretation).

SPS differ from UFLS and UVLS schemes and relays essentially in two aspects:

1. SPS use in addition to (or instead of) measurements also a particular topology change (i.e. contingency) information to detect a dangerous system state.
2. SPS consist of several relays, which often use an information from a remote location (e.g. measurement taken by one relay is sent to other relay, which processes it and executes a control action).

In literature, SPS are defined as:

a protection scheme that is designed to detect a particular system condition that is known to cause unusual stress to the power system and to take some type of predetermined action to counteract the

observed condition in a controlled manner. In some cases, SPSs are designed to detect a system condition that is known to cause instability, overload, or voltage collapse. The action prescribed may require the opening of one or more lines, tripping of generators, ramping of HVDC power transfers, intentional shedding of load, or other measures that will alleviate the problem of concern. Common types of line or apparatus protection are not included in the scope of interest here.

Damping Control Some power systems lack a "natural" damping of oscillations, which may occur, and they would be unstable when subjected to any minor disturbance and sometimes even under normal operation conditions if no measures increasing the damping were introduced.

A traditional way of introducing an additional damping in the system is using of Power System Stabilizer (PSS)¹, which modulates the output voltage of the generator.

Besides generators, PSS can be installed and used for modulation of FACTS devices control.

The coordinated tuning of PSSs is a complex task, since they should be robust - work in the wide range of operation conditions and provide the best possible performance. This process is done off-line. Data needed for tuning of PSSs are usually obtained by modal analysis of power system model linearized around its operating point.

3.3 Day-ahead Operation Planning

The objective of day-ahead operation planning is to prepare a plan for a secure and economical operation of the system for the coming next day, which was described in the previous section.

3.4 Short-term Planning

The main subject of short-term planning are activities having time constants in order of several days to several weeks. Typical example is an approval of maintenance of generators and transmission components in a coordinated way, guaranteeing that there will be sufficient resources and system capability to execute a secure operation at the *day D*.

3.5 Long-term Planning

Long-term planning activities can be divided into the groups:

- Computations and Analysis
- Preparation of Operation Procedures
- Preparation of Operation Rules

¹Note that PSS is the subject of the exercise given in the other part of this course.

- System Extensions
- System Maintenance
- Experimental Tasks

Computations and Analysis A long-term *load forecast* focuses mainly on a peak annual consumption values and its location within the network, not a particular load profile.

Load forecast, together with expected new generation additions (in context of liberalized markets, where a new generation connection to the network has to be announced for an approval to the entity responsible for the system operation) are used for procedure called *adequacy assessment*. Adequacy assessment measures capability of power system to supply load in all steady states under all normal conditions and situations, which may occur. Adequacy assessment can be seen as a forecast employing statistical tools and methodology. Transmission adequacy assessment is performed very rarely² in contrast to generation adequacy assessment, which is a standard type of system assessment.

Generation adequacy assessment targets quantification of generators' capability to meet the peak load demand, considering possible generation units' outages. Standard quantification indices are:

LOLP *Loss of Load Probability* is number of hours in a period when there is not enough capacity to supply the load. LOLP is usually expressed in hours/year or % of the period.

UE *Unserviced Energy* is the amount of energy that can not be delivered depending on deficit of capacity. UE is expressed in MWh/year or % of load.

Note that in pure generation adequacy assessment, only ratio between load and generation, as well as generation reliability is considered, transmission and distribution system is assumed to be 100 % reliable. estimated indices (LOLP, UE etc.) are higher than limit being considered as acceptable, more generation shall be added to the system in order to guarantee³ adequate supply availability.

Having forecasted load and generation patterns, studies and simulations of dangerous scenarios, possibly having an impact on the system stability and security, are conducted. This procedure is called *security assessment*. Dangerous scenarios are usually created for peak load conditions and all single and chosen multiple contingencies. When unsatisfactory system performance is revealed, mitigation measures have to take place. In a short-term perspective, operation rules are modified, in a long-term outlook, corresponding system extensions are stimulated.

²Transmission system is strongly meshed and outages of transmission components are very rare (e.g. once every 5 - 7 years), thus the analysis employing statistical methodology does not usually yield satisfactory results and requires an excessive computational effort. Distribution systems have a simpler topology and a higher frequency of outages, thus providing reliable input data for the analysis. There are many indices describing distribution systems adequacy in use.

³Technically, there is no guarantee that real network operation will directly correspond to estimated and targeted values of adequacy.

Short-circuit analysis processes forecasted load and generation patterns and system topology, in order to determine short-circuit powers throughout the network and subsequently, potentially highest short-circuit currents. After that it is investigated if they can be cleared (if installed circuit breakers have sufficient rating) and how fast (if angular instability of nearby generators can be encountered).

Operation limits are determined if some operation scenarios would endanger the system. However, this can be sometimes seen only as a short term solution, until the limitation is removed or minimized by an investment into new primary or secondary equipment. They are of course needed to map capabilities of the transmission system e.g. from the viewpoint of market participants.

Preparation of Operation Procedures Network operators are trained how to handle various types of situations and which procedures they shall apply. An emphasis is dominantly given on *alert* and *emergency* states. First on recognition signs of a dangerous situation and then on identification of effective employment of necessary controls. Results of *security assessment* studies is utilized, often in form of look-up tables and instruction manuals.

A separate category is system *restoration* after a major part of the system has suffered a blackout. In each utility there are plans and procedures prepared, starting from small power plants, having a black start capability (usually small hydro power plants, having an autonomous excitation system based on batteries, not auxiliary connection o the main grid as it is common nowadays) and good controllability and then connecting loads in appropriate amounts to avoid large frequency excursions and overvoltages. This is done gradually feeder by feeder.

Preparation of Operation Rules Previous paragraph dealt with internal operation procedures to be applied by network operators. This paragraph discusses how operation rules for external subjects are determined by a transmission operation company. In particular, what are conditions for an usage of the transmission network. This is an especially important subject in context of liberalized electricity markets. There are essentially three subtopics:

- Ancillary services pricing
- Transmission pricing
- Transmission capacity allocation rules

Ancillary services usually refer to control functionalities to be supplied by generators not for a particular customer but for the benefit of overall system security. Here belongs secondary frequency control, voltage control, emergency reserves etc. So in liberalized markets, there are various pricing schemes to provide an incentive to generators to participate in ancillary services.

There are various methods of transmission pricing, but that is out of scope of this course. From technological perspective, most important features, they consider, are coverage of induced transmission losses and transmission system limitations.

Transmission system limitations often impose constraints on desired power transmission among certain places (i.e. nodes) in the system. For such 'cuts' of the network, transmission capacity is computed and allocated to entities

interested in power transfer. Transmission capacity is most often restricted by thermal capacity of lines, but sometimes also by stability (voltage or small-signal) either on internal places in the network, but most often on interfaces between systems. In Europe, coordination in cross-border trading and thus also questions related to transmission capacity, are addressed by the association ETSO (European Transmission System Operators). According to ETSO, the following terms describe transmission capacity (note that all listed terms refer to an interface between two systems, which may consist of several lines):

TTC *Total Transmission Capacity* is the maximum exchange program between two areas compatible with operational security standards applicable at each system if future network conditions, generation and load patterns were perfectly known in advance.

TRM *Transmission Reliability Margin* is a security margin taking into account uncertainties of the computed TTC values arising from: Unintended deviations of physical flows during operation due to the load-frequency regulation, emergency exchanges between TSOs due to unexpected unbalanced situations in real time, inaccuracies, e.g. in data collection and measurements.

NTC *Net Transfer Capacity* is the maximum exchange program between two areas compatible with security standards applicable in both areas and taking into account technical uncertainties on future network conditions. ($NTC = TTC - TRM$) NTC considers all stability and components overloads limits under N-1 criteria assumption, except transient stability dependent on clearing time. NTC is time dependent (it is becoming more accurate as the system approaches the given state and generation and load patterns estimates are more accurate) and represents only a theoretical value (e.g. in case of a group of several lines, a physical line flow does not have to follow optimal redistribution of flows among lines).

AAC *Already Allocated Capacity* is a total amount of allocated transmission rights, whether they are capacity or exchange programs depending on the allocation method.

ATC *Available Transmission Capacity* is the part of NTC that remains available, after each phase of the allocation procedure, for further commercial activity. $ATC = NTC - AAC$

From side of TSO, there are two phases of activities related to transmission capacities:

- planning phase: computation of NTC
- allocation phase: market mechanism

System Maintenance A maintenance of both primary and secondary assets is conducted on regular basis. The equipment aging is checked and performance tests are carried out (e.g. if transformer isolation and losses are still adequate etc.). An important aspect in case of secondary equipment is software maintenance, e.g. update of databases containing models of power system components

used for system studies etc (Databases updates are caused by adding new components in the network, new composition of loads, new flow patterns, updates of models after new measurements are available etc.).

When maintenance has to take place, it is necessary to make consequent preparations considering:

- Secure conditions to perform maintenance (e.g. light loading)
- Coordination with all involved parties (neighboring TSOs)
- Announcing sufficiently in advance
- Modification of operation rules (e.g. NTC reduced etc.)

System Extensions If system performance (from technical or economical perspective) could be improved by an installation of an additional equipment, system operating company carries out feasibility studies, design computations and commissioning activities. An expression, which describes the whole set of activities related to it, is *Asset Management*, defined as:

Process for acquisition and use of an operation component for a maximal economical profit and management of risks and cost during the entire component lifetime.

Experimental Tasks New technologies, tools and procedures are first thoroughly tested and examined before fully applying in system operation.

Chapter 4

Algorithms and Computations

This chapter shall address theory and execution of two important in power system control algorithms, namely, state estimation and optimal power flow. The description below assumes knowledge of basic power system modelling techniques and power flow computation, as discussed in the course **227-0526-00: Modelling and Analysis of Power Networks**. A brief review of the necessary basics is provided in appendix A.

Employment of these computations was discussed in the previous chapters dealing with hierarchical monitoring concept and system operation tasks.

4.1 State Estimation

Power networks are extremely complex systems, containing thousands of elements. A large number of measurement devices is installed across the system and provides measurement data to the control center. However, this data might be inaccurate, even faulty, inconsistent, an incomplete. Simultaneously it is critical for the ability of power system operator to take the appropriate actions in power system control.

State estimation is a group of methods that provide reliable real time model of the system and includes several stages, as follows:

Topology processor determines the structure of power system and connectivity of the parts of the system from the logical statuses of the switches.

Observability analysis determines whether from the obtained analog measurement set the system state can be determined.

State estimation solution determines most likely state of the system, composed of complex bus voltages in the entire power system as well as best estimates for line flows, loads, generation outputs. It is based on the system model and the obtained measurements.

Bad data processing detects presence of the gross measurement errors and eliminates them.

Parameter and structural error processing estimates network parameters and structure, and eliminates the errors.

Better knowledge of the power system state would result also in more accurate transmission pricing for the market participants. Thus, state estimation is an important a technique to improvement the quality of the measurement data.

4.1.1 Weighted Least Square estimation

Assumptions: steady state balanced conditions - loads, powers flows are three phase and balanced, transmission lines are fully transposed and other devices are symmetrical. This allows single phase positive sequence modelling of power system.

Notations

Several notation that will be used in the text are provided below, for the quick reference:

x - states

z - measurements

e - measurement errors, Gaussian probability distribution

μ - mean (or expected) value

σ - standard deviation

W_{ii} - weight describing measurement accuracy, $W = R^{-1}$

r_i - residuals of measurement from expected value

$h(x)$ - measurement function (function relating state x to measurement z)

$H(x)$ - measurement Jacobian (partial derivative of $h(x)$ by x)

$J(x)$ - optimization objective function

$g(x)$ - partial derivative of $J(x)$ by x

$G(x)$ - the Gain matrix (partial derivative of $g(x)$ by x)

Problem formulation

Usually, when we talk about *state variables* in power system state estimation, we mean *voltage magnitudes* and *angles* of the system nodes.

Bus power injections P_i, Q_i , branch power flows P_{ij}, Q_{ij} and other quantities in power system are uniquely determined through some function¹ $h(x)$ by the state vector:

$$x^T = (\theta_1 \dots \theta_n, V_1 V_2 \dots V_n). \quad (4.1)$$

However, mapping the states x to measurements z or vice versa is biased by measurement errors e :

$$z_i - h_i(x) = e_i, \quad (4.2)$$

¹see equations A.47 in the appendix

where $h(x)$, known as *measurement function* relates the system state vector x to the measured quantities, such as voltage magnitudes V_i , bus power injections P_i, Q_i , branch power flows P_{ij}, Q_{ij} .

The errors in (4.2) are assumed to be stochastic and to have the following properties:

- Gaussian probability distribution with the mean value μ and the standard deviation σ^2 :

$$f(e) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\frac{z-\mu}{\sigma}\right)$$
- mathematical expectation of the measurement error is zero:

$$E(e_i) = \mu = 0, \quad i = 1 \dots M$$
- measurement errors are independent, i.e.

$$E[e_i e_j] = 0, \text{ thus } Cov(e) = E[e e^T] = R = diag\{\sigma_1^2, \sigma_2^2 \dots \sigma_M^2\}$$

The equation (4.2) is probabilistic, thus the most likely rather than exact values of the state variables can be determined.

Joint probability density function of obtaining set of m measurements with the particular independent errors, can be written as:

$$f_m(e) = f(e_1) \cdot f(e_2) \cdots f(e_m) \quad (4.3)$$

By maximizing the value of this function (also called joint maximum likelihood function) or its logarithm we say that the obtained set of measurements should have the most likely errors.

$$\max \log f_m(e) = \sum_{i=1}^m \log f(e_i) = \quad (4.4)$$

$$= -\frac{1}{2} \sum_{i=1}^m \left(\frac{e_i}{\sigma_i}\right)^2 - \frac{m}{2} \log 2\pi - \sum_{i=1}^m \log \sigma_i \quad (4.5)$$

or sequentially:

$$\min \sum_{i=1}^m \left(\frac{e_i}{\sigma_i}\right)^2 \quad (4.6)$$

in accordance with (4.2) it can be rewritten:

$$\min \sum_{i=1}^m \left(\frac{z_i - h_i(x)}{\sigma_i}\right)^2 \quad (4.7)$$

Assigning residuals $r_i = \Delta z_i = z_i - h_i(x)$ and denoting the weights assigned to the square of the residuals r_i^2 as $W_{ii} = \frac{1}{\sigma_i^2}$, the residual formulation Weighted Least Square Estimator (WLS) for x becomes:

$$\begin{aligned} \min \sum_{i=1}^m W_{ii} \cdot r_i^2 \\ \text{s.t. } z_i = h_i(x) + r_i, \quad i = 1 \dots M \end{aligned} \quad (4.8)$$

the solution of this optimization problem is the weighted least squares estimation.

²Standard deviation σ_i refers to the measurement signal chain accuracy of the measurement z_i , e.g. CT/PT, amplifier, RTU

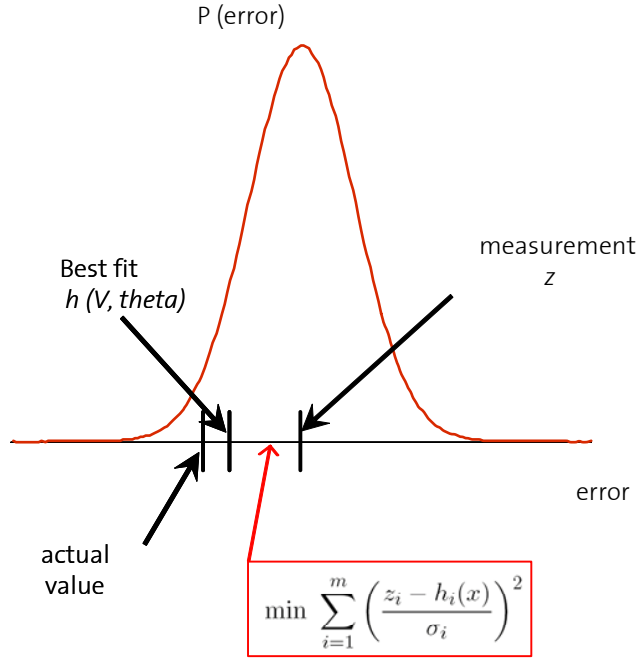


Figure 4.1: Formulation of the state estimation problem.

Problem solution

The WLS estimator will minimize the following objective function $J(x)$:

$$J(x) = [z - h(x)]^T \cdot W \cdot [z - h(x)]. \quad (4.9)$$

As a minimum first order optimality conditions should be met:

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) \cdot W \cdot [z - h(x)] = 0, \quad (4.10)$$

where $H(x) = \frac{\partial h(x)}{\partial x}$ is measurement Jacobian, having the following structure:

$$H = \begin{bmatrix} 0 & \frac{\partial V_{mag}}{\partial V} \\ \frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V} \end{bmatrix}, \quad (4.11)$$

where each of the partial derivative denotes $n \cdot m$ matrix, n is the number of measurements and the m number of state variables θ or V , it is important to note that $n > m$.

The elements corresponding to active and reactive power injection measurements are the same as power flow problem provided in the appendix (A.54) - (A.55), while the other elements are given below:

- Elements corresponding to active power flow measurements:

$$\frac{\partial P_{ij}}{\partial \theta_i} = V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \quad (4.12)$$

$$\frac{\partial P_{ij}}{\partial \theta_j} = -V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \quad (4.13)$$

$$\frac{\partial P_{ij}}{\partial V_i} = -V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) + 2(g_{ij} + g_{ij}^{sh}) V_i \quad (4.14)$$

$$\frac{\partial P_{ij}}{\partial V_j} = -V_i (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \quad (4.15)$$

- Elements corresponding to reactive power flow measurements:

$$\frac{\partial Q_{ij}}{\partial \theta_i} = -V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \quad (4.16)$$

$$\frac{\partial Q_{ij}}{\partial \theta_j} = V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \quad (4.17)$$

$$\frac{\partial Q_{ij}}{\partial V_i} = -V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) - 2(b_{ij} + b_{ij}^{sh}) V_i \quad (4.18)$$

$$\frac{\partial Q_{ij}}{\partial V_j} = -V_i (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \quad (4.19)$$

- Elements corresponding to the voltage magnitude measurements:

$$\frac{\partial V_i}{\partial V_i} = 1, \quad \frac{\partial V_i}{\partial V_j} = 0, \quad \frac{\partial V_i}{\partial \theta_i} = 0, \quad \frac{\partial V_i}{\partial \theta_j} = 0. \quad (4.20)$$

Expanding non-linear function $g(x)$ in Taylor series around state vector x^k , and neglecting second and higher order terms:

$$g(x^k) + \frac{\partial g(x^k)}{\partial x} (x - x^k) = 0, \quad (4.21)$$

where $\frac{\partial g(x^k)}{\partial x}$ is called a gain matrix $G(x^k) = H^T(x^k) \cdot W \cdot H(x^k)$.

This leads to iterative solution scheme known as Gauss-Newton method, with the updates of $\Delta x^{k+1} = x^{k+1} - x^k$ for the next iteration $k + 1$ being obtained by solving the equation:

$$G(x^k) \Delta x^{k+1} = -g(x^k). \quad (4.22)$$

Substituting $g(x^k)$ as in (4.10) the following equations are obtained known as Normal equations:

$$G(x^k) \Delta x^{k+1} = H^T(x^k) \cdot W \cdot [z - h(x^k)]. \quad (4.23)$$

Solving this equation for the updates Δx^{k+1} and iterating until the required accuracy ε is reached: $\Delta x < \varepsilon$, will provide the solution of SE.

Alternative solutions of WLS

Usually, for the solution of (4.23) the gain matrix $G(x^k)$ is not inverted, instead some numerical techniques are used: matrix decomposition, followed by the back-forward substitution. Besides, it has been discovered that Normal equations are prone to numerical instabilities, thus causing divergence of the solution.

The alternative formulations of WLS method are seeking for numerical substitutes of Normal Equations. Some of the methods are summarized in the table below:

Method	Basis Equation	Description
Normal Equation	$G\Delta x^{k+1} = H^T W \Delta z$	inversion of matrix G
Cholesky decomposition $G = L^T L$	$L^T L \Delta x^{k+1} = H^T W \Delta z$	solving first for $L \Delta x^{k+1}$ then for Δx^{k+1}
Orthogonal factorization $\tilde{H} = QR$	$U \Delta x^{k+1} = Q_n^T \Delta \tilde{z}$	Q_n, U obtained by QR decomposition
Hybrid method $G = U^T U$	$U^T U \Delta x = \tilde{H}^T \Delta \tilde{z}$	instead of Cholesky the orthogonal decomposition
Peters and Wilkinson $\tilde{H} = LU$	$(L^T L) \Delta y = L^T \Delta \tilde{z}$ $U \Delta x = \Delta y$	LU decomposition, Cholesky of $(L^T L)$ to solve for Δy , then for Δx

where

- $\Delta \tilde{z} = W^{1/2} \Delta z$, z is residual vector $\Delta z = z - h(x)$
- $\tilde{H} = W^{1/2} H$
- $\tilde{H} = QR$, such as R is an $m \cdot n$ upper trapezoidal matrix, Q is $m \cdot m$ orthogonal matrix ($Q^T = Q^{-1}$)
- $QR = [Q_n \quad Q_0] \begin{bmatrix} U \\ 0 \end{bmatrix}$, U is $n \cdot n$ triangular matrix
- $\tilde{H} = LU$, L is lower trapezoidal matrix and U is upper triangular

All these methods target the use of triangular matrices and thus, forward/back substitution methods.

State Estimation general iteration procedure

1. Choose initial value of x^k (usually "flat start")
2. Update iteration number $k = k + 1$
3. Compute $H(x^k)$, $h(x^k)$
4. Compute matrix $G(x^k) = H^T(x^k) \cdot W \cdot H(x^k)$
5. Compute right hand side $g(x^k) = H^T(x^k) \cdot W \cdot [z - h(x^k)]$
6. Solve for Δx^k , decomposing $G(x^k)$
7. Test convergence: $\max |\Delta x^k| \leq \varepsilon$?
if no, $x^{k+1} = x^k + \Delta x^k$ and go to step 2.

State Estimation Orthogonal decomposition iteration procedure

1. Choose initial value of x^k (usually "flat start")
2. Update iteration number $k = k + 1$
3. Compute $H(x^k)$, $h(x^k)$
4. Compute Q_n, U as QR decomposition elements of $\tilde{H} = W^{1/2}H$
5. Compute vector $\Delta z_q = Q_n^T W^{1/2} \Delta z$
6. Solve $U \Delta x = \Delta z_q$ for Δx
7. Test convergence: $\max |\Delta x^k| \leq \varepsilon$?
if no, $x^{k+1} = x^k + \Delta x^k$ and go to step 2.

4.1.2 Observability Analysis

Observability analysis shall determine whether the location and number of the measurements is sufficient to estimate the power system state. It has to be carried out during the installation of state estimator and during the online operation, since the topology of the system may change as well as communication link may be lost or the meter failure occurs.

The particular methods for determining the observability are behind the scope of this course. However, provided below measurement classification will be used in the following explanations.

Classification of the measurements

Power system may contain various types of the measurement devices, installed at the different locations. These may have different impact on the state estimation results, and shall be summarized as follows:

- **Critical measurement:** elimination of that measurement from the measurement set result in an unobservable power system;
- **Redundant measurement:** In case of removal of this measurement the power systems would not be observable;
- **Critical pair:** Two redundant measurements whose simultaneous removal from the measurement set would result in an unobservable power system;
- **Critical k-tuple:** A critical k-tuple contains k redundant measurements whose simultaneous removal from the measurement set would result in an unobservable power system; None of those k measurements belong to a critical tuple of lower order.

4.1.3 Bad Data Identification

Measurements may contain random errors due to various reasons: the finite accuracy of the measurement devices, telecommunications, unexpected interference. Bad data is a measurement that contains large error. Some of the errors, such as negative voltage value might be easily identified by pre-screening. However, the ability to filter out biased measurements should be expected from SE, if a sufficient redundancy exists. The nature of this filter depends on the specific estimation method.

For the WLS estimation method, detection and identification of the bad data can be performed by the analysis of obtained residuals values.

Residual sensitivity matrix

The linearized measurement equation is:

$$\Delta z = H\Delta x + e \quad (4.24)$$

Note that measurement residuals may still be correlated even when the measurement errors are not. The WLS estimator for the linearized model is:

$$\Delta \hat{x} = G^{-1}H^T W \Delta z \quad (4.25)$$

and the estimated value of $\Delta \hat{z}$, also called the *hat* matrix, is:

$$\Delta \hat{z} = H\Delta \hat{x} = HG^{-1}H^T W \Delta z = K\Delta z. \quad (4.26)$$

The matrix K poses some interesting features: a diagonal element being relatively high in relation to off-diagonal elements in the row implies that local redundancy around corresponding meter is poor.

It can be proved that the measurement residuals and the measurement covariance can be expressed, as:

$$r = (I - K)e = Se, \quad (4.27)$$

$$Cov(r) = \Omega = SW^{-1} \quad (4.28)$$

where S is the residual sensitivity matrix, representing the sensitivity of the measurement residual to measurement errors. The residual covariance matrix has important properties and can be used for the identification of the bad data:

- Ω is a real and symmetric matrix
- $\Omega_{ij}^2 \leq \Omega_{ii}\Omega_{jj}$
- $\Omega_{ij} \leq \frac{\Omega_{ii} + \Omega_{jj}}{2}$
- if $\Omega_{ij} \geq \epsilon$ then measurement i and j are said to be strongly interacting. The threshold ϵ depends on the network and the measurement topology, as well as desired level of selectivity among measurements

Use of Normalized residuals for Bad Data Detection

The residue can be normalized by dividing its absolute value by corresponding diagonal entry in the residual covariance matrix Ω , resulting in r_i^N :

$$r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}} = \frac{|r_i|}{\sqrt{W_{ii}^{-1} S_{ii}}} \quad (4.29)$$

If there is no bad data, measurement residuals are relatively small and do not exceed some threshold (that can be determined from the experience and knowledge of the measurement system accuracy). When the analysis of residuals indicates presence of bad data, the largest normalized residue corresponds to the bad measurement. The condition for such detection of the bad measurement that it is single and non-critical (redundant).

4.1.4 Parameters errors

State estimation assumes the parameters of the system to be known, however these may differ/vary during the operation of power system due to loading (tap changers of the transformers, resistance of the line), weather conditions (line susceptance), or incorrectly provided in the manufacturer data sheets or not updated in the system database.

This may have the following consequences:

- significant adverse impact on SE results and loss of the operator confidence in it;
- misleading other applications based on the SE results, such as security assessment;
- correct measurement identified as bad data due to their inconsistency with incorrect network parameters.

Some approaches have been developed dealing with the parameter estimation. These will not be studied in this course.

4.1.5 Topology errors

The actual power system structure is far more complicated than the model used for power flow or state estimation studies. It contains number of switching devices, circuit breakers (CB) and busbars, by switching these the status of the particular equipment and topology can be changed.

The routine that converts the detailed bus section/switch model into the compact bus/branch model is called topology processor (TP). It analyzes the status of the switching devices in order to determine the way physical nodes and transmission elements are interconnected and in order to reduce the number of analyzed nodes.

When the status of circuit breaker(s) is determined incorrectly and results in locally incorrect bus/branch model called **topological error**. It usually causes SE to be significantly biased, and the bad data processing would eliminate several measurements, sequentially resulting convergence problems or in unacceptable state.

Different approaches are required for two types of the topology errors:

- Branch status error - error in status of the network branch (non-zero impedance element). The approaches to overcome this type of error are similar to parameter estimation techniques. The impedances of the branches are explicitly presented in SE, therefore there is no need for a special model.
- Substation configuration error - errors affecting circuit breakers that link busbars (zero impedance elements). This requires specific procedure, when the affected circuit breakers appear explicitly in SE model.

Two essential method for the branch status errors identification are: by means of normalized residuals or state vector augmentation.

State vector augmentation

Let us introduce a variable k that represents a branch status, $k = 1$ represents branch in service and $k = 0$ out of service. Then the idea is to include the variable k into the state vector for any suspected branch (for example large residuals of the measurements in some area could be the reason to suspect the topology errors of one of the branches in that area), so the admittances of the corresponding branch become:

- series: $(g_{ij} + jb_{ij})k$
- parallel: $jb_{ij}^{sh}k$

In WLS estimation, the obtained value of k will be dictated by the analog measurements. If k is close to 1, it is an indication of branch being in service. In practice, however the estimate of k may significantly diverge from 0 or 1 and the rest of the state variables would be determined unprecise. Then, an additional constraint may be introduced ³:

$$k(1 - k) = 0 \quad (4.30)$$

that will force SE to converge to $k = 0$ or $k = 1$.

The inclusion of the new state variable k into a state vector will cause the necessity to modify corresponding elements of the measurement function and the measurement Jacobian as well.

Modifications of the measurement function

While the power flows can be re-written, as:

$$P_{ij}(k) = kP_{ij} = k\{V_i^2(g_{ij}^{sh} + g_{ij}) - V_iV_j(g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij})\} \quad (4.31)$$

$$Q_{ij}(k) = kQ_{ij} = k\{-V_i^2(b_{ij}^{sh} + b_{ij}) - V_iV_j(g_{ij}\sin\theta_{ij} - b_{ij}\cos\theta_{ij})\}, \quad (4.32)$$

³this is the case for the formulations of WLS where the constraints are explicitly modelled, however it may as well be applied to orthogonal factorization formulation being handled as very accurate measurement

the power injections in the busses will only be affected by the power inflows from the corresponding lines:

$$\begin{aligned} P_i(k) &= \sum_{\substack{m=1, \\ m \neq j}}^n P_{im} + kP_{ij} = \\ &= V_i \sum_{\substack{m=1, \\ m \neq j}}^n V_m (G_{im} \cos \theta_{im} + B_{im} \sin \theta_{im}) + kP_{ij} \end{aligned} \quad (4.33)$$

$$\begin{aligned} Q_i(k) &= \sum_{\substack{m=1, \\ m \neq j}}^n Q_{im} + kQ_{ij} = \\ &= V_i \sum_{\substack{m=1, \\ m \neq j}}^n V_m (G_{im} \sin \theta_{im} - B_{im} \cos \theta_{im}) + kQ_{ij} \end{aligned} \quad (4.34)$$

Modifications of the measurement Jacobian

The measurement Jacobian will have the following changes:

- modifications in the rows corresponding to active and reactive power injections in busses i , j and flows through branch ij and ji (4.31) - (4.34);
- additional column containing zeros on irrelevant rows and P_{ij} (or Q_{ij}) elements on affected rows;
- additional row containing only one element (in the additional column), corresponding to (4.30);

The changes related to active power are shown below, as well as the additional row and column:

$$H = \left[\begin{array}{cc|c} \dots & \dots & 0 \\ k \frac{\partial P_i}{\partial \theta} & k \frac{\partial P_i}{\partial V} & \frac{\partial P_i(k)}{\partial k} \\ \vdots & \vdots & 0 \\ k \frac{\partial P_j}{\partial \theta} & k \frac{\partial P_j}{\partial V} & \frac{\partial P_j(k)}{\partial k} \\ \vdots & \vdots & 0 \\ k \frac{\partial P_{ij}}{\partial \theta} & k \frac{\partial P_{ij}}{\partial V} & \frac{\partial P_{ij}(k)}{\partial k} \\ \vdots & \vdots & 0 \\ k \frac{\partial P_{ji}}{\partial \theta} & k \frac{\partial P_{ji}}{\partial V} & \frac{\partial P_{ji}(k)}{\partial k} \\ \dots & \dots & \dots \\ \hline 0 & 0 & 1 - 2k \end{array} \right], \quad (4.35)$$

where $\frac{\partial P_i(k)}{\partial k} = \frac{\partial P_{ij}(k)}{\partial k} = P_{ij}$.

The reactive power Q related elements will be computed in the analogous way. If more than one line status should be verified, then the number of state variables will increase accordingly, as well as number of rows and columns in Jacobian.

The starting value of k

The choice of the initial value of k_0 is very important. When (4.30) is treated as a very accurate measurements, the SE will converge to 1 for the $k_0 > 0.5$ and it will converge to 0 for $k_0 < 0.5$. Therefore, it is important to set initial value of k exactly to $k_0 = 0.5$. Furthermore, the additional state variable k should be added after the first iteration, in order to avoid ill-conditioning of measurement Jacobian.

4.2 Optimal Power Flow

Optimal Power Flow (OPF) is a general term for computations aiming at determining how some controls shall be modified or set in order to obtain a desired power system performance. In the optimization framework, this can be mathematically formulated as follows:

$$\min_u f(x, u) \quad (4.36)$$

$$0 = g(x, u) \quad (4.37)$$

$$0 \leq h(x, u) \quad (4.38)$$

where x is the system state (recall that this is vector consisting of voltage magnitudes and angles at all buses of the considered system) and u available controls (which differ based on the). The desired power system performance is described in the form of the objective function (4.36). The system is subjected to equality and inequality constraints (4.37) and (4.38) respectively⁴.

In power system context, equality constraints (4.37) are power flow equations, which must always hold (see section A.3). Inequality constraints (4.38) typically express security limitations - $N-1$ criterion. In other words, an outage of a single component can not result in an overload of other component, i.e. all post contingency line flows have to be within the respective lines' limits. OPF containing these constraints is frequently referred to as *Security Constrained OPF*.

In addition, both system state x and controls u are usually restricted by lower and upper bounds. Voltage magnitudes, as a subset of system state, shall normally be within the operation range of equipment. Control restrictions are given by physical limits of control equipment - e.g. maximum generator's active power production etc.

The particular content of the cost function determines the objective of OPF. Several representatives⁵ are:

Minimum generation cost System load can be met by various combinations of power plants' productions. OPF can be employed to determine, which is most economically efficient while keeping security criteria (i.e. inequality constraints).

⁴Note, that the above listed equations are based on steady-state representation of power system behavior.

⁵Note that above listed examples of OPF application is used mostly by vertically integrated utilities. In liberalized environment, some system operating companies employ OPF for market clearing procedure yielding nodal coefficients forming base for transmission pricing.

Minimum losses Controls (mainly active power generation) are modified in order to decrease losses induced in transmission system, mostly lines.

Maximization of reactive power reserves This application of OPF is positioned on the top - tertiary layer of the hierarchical voltage control concept. The objective is to minimize reactive power production, so the system would accommodate disturbances without dramatic drop of voltages approaching the system to voltage collapse. Reference voltages for Automatic Voltage Regulators of generators are used as control inputs.

Usually a combination of several objectives is considered, where weighting factors determine the importance of sub-objectives (i.e. terms of the cost function). Additional terms are usually added into the cost function, expressing value of employment of control resources (for example different generation cost of different generators).

OPF computations are nowadays a common integral part of software packages belonging to Energy Management System (for more details, consult section 2.2). OPF is then employed both in day-ahead operation planning as well as in real-time operation, where the initial system state is determined by state estimation.

As equations (4.37) and (4.38) describing power systems are nonlinear, it is quite complex task to solve the optimization problem (especially considering a magnitude of power system description, frequently containing several hundreds to several thousands of variables). A common approach is then starting computations from a certain nominal case (system state), which is linearized, allowing reformulation of constraints into linear equations. That enables employment of quadratic or linear programming solution techniques, which are very robust and efficient.

Appendix A

Review of power systems modelling basics

A.1 Alternating current systems

In this section the basic dependencies between the electrical quantities are briefly reviewed. These dependencies are essential though for the understanding of the algorithms of power system analysis that are derived in the following sections.

A.1.1 Instant power, complex apparent power

The *instant power* $p(t)$ consumed by an impedance \underline{Z} connected to the sinusoidal voltage source can be computed as:

$$\begin{aligned} p(t) &= u(t)i(t) = U_M I_M \cos \omega t \cos(\omega t - \varphi) \\ &= U_M I_M \cos \omega t [\cos \omega t \cos \varphi + \sin \omega t \sin \varphi] \end{aligned} \quad (\text{A.1})$$

where $U_M = U\sqrt{2}$ and $I_M = I\sqrt{2}$ are the peak values of the voltage and current.

Applying trigonometric transformations leading to the functions of double frequency:

$$\begin{aligned} p(t) &= UI[(1 + \cos 2\omega t) \cos \varphi + \sin 2\omega t \sin \varphi] \\ &= P(1 + \cos 2\omega t) + Q \sin 2\omega t, \end{aligned} \quad (\text{A.2})$$

where

$P = UI \cos \varphi$ is the active power consumed by resistance,

$Q = UI \sin \varphi$ and is the reactive power.

The *complex apparent power* \underline{S} consumed by an impedance \underline{Z} is defined as:

$$\underline{S} = \underline{Z}I^2 = \underline{Z}I I^* = \underline{U}I^*, \quad (\text{A.3})$$

where I^* is a complex conjugated of the current \underline{I} .

Assuming that the angle between voltage and current is $(-\varphi)$, we get:

$$\underline{S} = \underline{U}I^* = UIe^{j\varphi} = UI(\cos \varphi + j \sin \varphi) = P + jQ. \quad (\text{A.4})$$

In the expression above the real part of the complex apparent power is $P = \Re(\underline{S})$ active power and the imaginary part is $Q = \Im(\underline{S})$ reactive power.

A.1.2 Three phase systems

The power system is built as a three phase system. One advantage of it over the single phase system is that 3 times more power can be transferred through 3 wires of the three phase system in comparison to 2 wires of the single phase system. Thus, the equipment costs, particularly of the transmission lines, are smaller per unit of the transferred power. Besides, three phases create rotating magnetic field that is necessary for the start-up of the electrical machines.

The sinusoidal voltage sources have same voltage magnitude compared with the ground and therefore are *symmetrical*. The phase angle between the voltages equals to 120° :

$$u_r(t) = U\sqrt{2} \cos \omega t \quad (\text{A.5})$$

$$u_s(t) = U\sqrt{2} \cos(\omega t - 120^\circ) \quad (\text{A.6})$$

$$u_t(t) = U\sqrt{2} \cos(\omega t + 120^\circ). \quad (\text{A.7})$$

The voltage between phases is called phase-to-phase voltage. It can be shown easily with the help of geometry that for example $U_{rs} = \sqrt{3}U_r e^{j30^\circ}$. Assuming that angle between voltage and current equals to $(-\varphi)$ the phase current can be written as:

$$i_r(t) = I\sqrt{2} \cos(\omega t - \varphi) \quad (\text{A.8})$$

$$i_s(t) = I\sqrt{2} \cos(\omega t - 120^\circ - \varphi) \quad (\text{A.9})$$

$$i_t(t) = I\sqrt{2} \cos(\omega t + 120^\circ - \varphi). \quad (\text{A.10})$$

The total power of the three phases is

$$\begin{aligned} p(t) &= u_r(t)i_r(t) + u_s(t)i_s(t) + u_t(t)i_t(t) = \\ &= UI[(1 + \cos 2\omega t) \cos \varphi + \sin 2\omega t \sin \varphi] + \\ &+ UI[(1 + \cos 2(\omega t - 120^\circ)) \cos \varphi + \sin 2(\omega t - 120^\circ) \sin \varphi] + \\ &+ UI[(1 + \cos 2(\omega t + 120^\circ)) \cos \varphi + \sin 2(\omega t + 120^\circ) \sin \varphi]. \end{aligned} \quad (\text{A.11})$$

Proceeding with rearrangements:

$$\begin{aligned} p(t) &= 3UI[\cos \varphi + \cos \varphi \underbrace{(\cos 2\omega t + \cos 2(\omega t - 120^\circ) + \cos 2(\omega t + 120^\circ))}_{=0} + \\ &+ \sin \varphi \underbrace{(\sin 2\omega t + \sin 2(\omega t - 120^\circ) + \sin 2(\omega t + 120^\circ))}_{=0}]. \end{aligned}$$

The emphasized expressions equal to zero. Therefore, the power $p(t)$ is constant:

$$p(t) = 3UI \cos \varphi \quad (\text{A.12})$$

and it does not pulsate as in the case of the single phase. This is also a very important reason for the use of three phase system. The complex apparent power of three phases equals:

$$\underline{S} = \underline{U}_r \underline{I}_r^* + \underline{U}_s \underline{I}_s^* + \underline{U}_t \underline{I}_t^* = 3U_f I e^{j\varphi} = 3U_f I \cos \varphi + j3U_f I \sin \varphi. \quad (\text{A.13})$$

where U_f is a RMS value of a phase voltage and I is a RMS value of a phase current. If the power is expressed in the phase-to-phase voltage values then the expression becomes:

$$\underline{S} = \sqrt{3}UI e^{j\varphi} = \sqrt{3}\underline{U}I^*.$$

A.1.3 Analysis of the symmetric three-phase system

If the three-phase system is operated in symmetrical conditions the processes in each of the phases will be similar, differing only by the time delay due to 120° shift. Therefore the analysis can be carried out by studying only one phase.




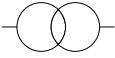


A.2 Steady State Modelling of Power Systems

Power system is subjected to the continuous changes of the operating conditions - loads variations, switching, generation patters, power flows. If the variation of the parameters are not significant and the time dependance can be neglected, this operation condition of power system is referred as **steady state**. In this case the equations describing the state are algebraic in contrast to differential equations that consider time dependance of the parameters.

A.2.1 Components representation

In this section we will discuss modelling or mathematical description of the primary power system equipment for the steady state analysis: *load, generation, line, transformer, shunt*. The usual graphical symbols of these components are summarized in table A.1.

Table A.1: Symbols of the power system primary equipment

Component	Symbol	Component	Symbol
Load		Line	
Generation		Transformer	
Shunt (capacitor)		Shunt (reactor)	

Loads

Power systems serve to satisfy the electrical power demand of the consumers - loads. Some of the loads might be purely active such as: boilers, heating, bulbs; while in most of the cases both active and reactive power is needed for motors and other appliances. The mathematical representation of the loads generally depends on the type of consumers, studied phenomena and the analyzed network.

For the steady state analysis, the static representation of the load, i.e. description by the linear equations is sufficient. These models consider load de-

pendency on the voltage and in general form might be described as follows:

$$P = P_0 \left(\frac{U}{U_0} \right)^\alpha \quad (\text{A.14})$$

$$Q = Q_0 \left(\frac{U}{U_0} \right)^\beta \quad (\text{A.15})$$

where P and Q are active and reactive power at the considered voltage U , P_0 and Q_0 are active and reactive power at the nominal voltage U_0 , α and β are some coefficients.

Three most frequently used models are summarized below:

- Constant power: $\alpha = 0$, $\beta = 0$,

$$S = P_0 + jQ_0,$$

- Constant current: $\alpha = 1$, $\beta = 1$,

$$S = P_0 \frac{U}{U_0} + jQ_0 \frac{U}{U_0} = I_0^* U$$

- Constant impedance: $\alpha = 2$, $\beta = 2$,

$$S = R_0 \frac{U_0^2}{R_0^2 + X_0^2} \left(\frac{U}{U_0} \right)^2 + jX_0 \frac{U_0^2}{R_0^2 + X_0^2} \left(\frac{U}{U_0} \right)^2 = \frac{U^2}{R_0 - jX_0}$$

The symbol for the representation of the load on the single line diagram of power system is shown in table A.1.

Generation

The demand for active power and the losses in the lines transferring this power to the consumer is covered by the power plants generators. In addition, the generators are capable of producing some reactive power, thus keeping the voltage in system within the allowed limits.

The active power production of the generator is determined by the turbine control. The reactive power production is controlled by the excitation system of the generator: the exciter feeds the field winding the current that creates the main flux. The reactive power capability is limited by currents magnitude in several components of the generator:

- **Stator winding heating limit** usually determines the reactive power production limit at high loading of a generator: active power losses and therefore heating are proportional to winding current $P_{losses} = I_t R_t^2$.

Since $I_t = \sqrt{(P^2 + Q^2)}/U_t$, the current limitation is a circle in complex plane of powers for the constant voltage U_t (fig. A.1).

- **Rotor winding heating limit** usually limits reactive power production of a generator at low loads and also has a circular shape.

- **Stator end region heating limit** determines limit of reactive power consumption in underexcitation mode due to increased armature end leakage flux and, thus, eddy currents in the laminations.

Thus the reactive power generation capability depends on the type, design characteristics etc of each generator specifically.

For the steady state analysis generators are modelled as a constant power injections and maintain constant bus voltage (within the reactive power capability limits).

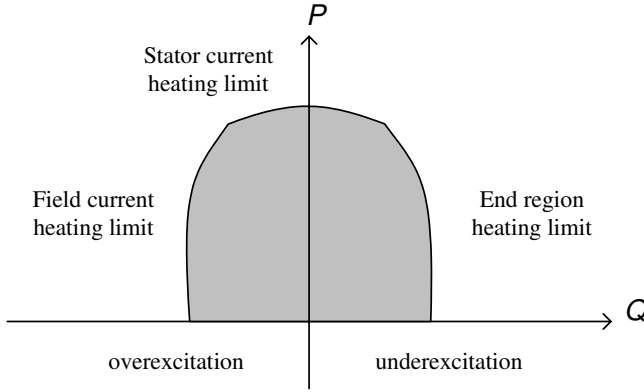


Figure A.1: Generator reactive power production limits

Line

Power lines deliver electrical power produced by generators to the consumers. Consider the mathematical model of the line as shown in Fig. A.2. Current flowing in the conductor creates certain thermal losses that are modelled by the resistance r of the line. jx is a representation of the reactance that creates an electromagnetic field around the line.

In addition, as the line operates under certain voltage, there is a difference of the potentials between the different phases and the ground creating a capacitive element - susceptance jb . Moreover, small parts of current are being lost in the isolators of the line and due to ionization of the air around the conductor causing corona phenomenon. This is modelled by the conductance g .

The currents \underline{I}_{ij} in the beginning and \underline{I}_{ji} in the end of the line can be expressed through the voltages at the line ends (see Fig. A.2):

$$\underline{I}_{ij} = y_{ij}(\underline{V}_i - \underline{V}_j) + y_{ij}^{sh} \underline{V}_i \quad (\text{A.16})$$

$$\underline{I}_{ji} = y_{ij}(\underline{V}_j - \underline{V}_i) + y_{ji}^{sh} \underline{V}_j \quad (\text{A.17})$$

where

$y_{ij} = 1/(r_{ij} + jx_{ij})$ is the series admittance of the line.

$y_{ij}^{sh} = y_{ji}^{sh} = g_{ij} + jb_{ij} = g_{ji} + jb_{ji}$ is the shunt admittance of the line connected at the busses i, j ;

Transformer

Due to the losses it is profitable to operate transmission lines at high voltage. However, generators and loads are designed for lower voltages. The transformation of the power to higher/lower voltage level is made by the transformers.

Let us examine Fig. A.3, voltage e_1 applied to primary winding of the transformer results in terminal current i_1 that creates magnetizing flux ϕ_m and leakage flux ϕ_1 . The magnetizing flux ϕ_m induces current in the secondary winding, that creates leakage flux ϕ_2 . To summarize:

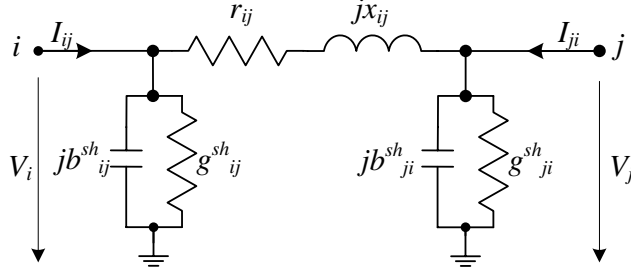


Figure A.2: Equivalent transmission line circuit

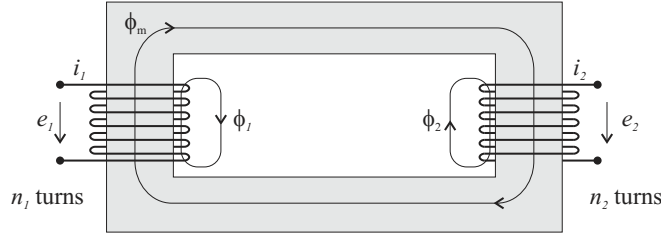


Figure A.3: Flux linkages of the transformer windings

$$\phi_1 = k_1 n_1 i_1 \quad (\text{A.18})$$

$$\phi_2 = -k_2 n_2 i_2 \quad (\text{A.19})$$

$$\phi_m = k_m (n_1 i_1 - n_2 i_2) \quad (\text{A.20})$$

where k_1, k_2 and k_m are the permeance of the corresponding flux paths.

The leakage fluxes do not contribute to energy transfers, thus transformers design is usually targeting to minimize these.

In the sinusoidal operation conditions, the application of Maxwell law gives the voltage relations:

$$\begin{aligned} e_1 &= \omega n_1 (\phi_1 + \phi_m) \\ e_2 &= \omega n_2 (\phi_2 + \phi_m) \end{aligned} \quad (\text{A.21})$$

Taking into account (A.18) - (A.21) and expressing e_1 :

$$\begin{aligned} e_1 &= \omega n_1 (k_1 n_1 i_1 + k_m (n_1 i_1 - n_2 i_2)) = \\ &= \omega ([k_1 + k_m] n_1^2 i_1 - k_m n_1 n_2 i_2) = ([x_1 + x_m] i_1 - x_m \frac{n_2}{n_1} i_2) \end{aligned} \quad (\text{A.22})$$

where $x_1 = k_1 n_1^2$ - is the leakage inductance of the winding 1 and $x_m = k_m n_1^2$ is the magnetizing inductance;

Similarly e_2 can be expressed. This leads to transformer equivalent circuit, as depicted in Fig. A.4.

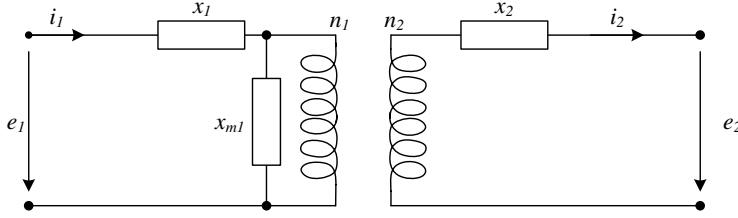


Figure A.4: Transformer equivalent circuit

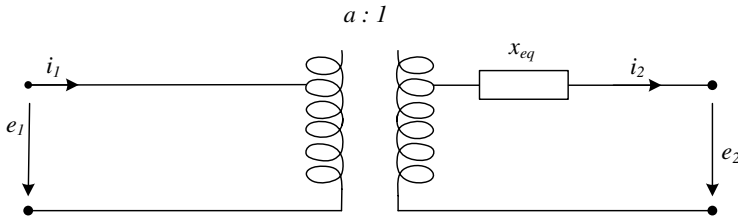


Figure A.5: Standard transformer equivalent circuit

As the reactance x_m is very large in contrast to x_1 and x_2 it is frequently ignored, resulting in open circuit.

Equivalent impedance of the transformer

Let us examine first the ideal transformer in Fig. A.4. If the leakage fluxes are ignored, then from (A.21) it can be written:

$$\frac{v'_1}{v'_2} = \frac{n_1}{n_2} \quad (\text{A.23})$$

Moreover taking into account power equality $v'_1 i'_1 = v'_2 i'_2$:

$$\frac{i_1}{i_2} = \frac{n_2}{n_1} \quad (\text{A.24})$$

In voltage drop across the leakage reactance of primary winding is $x_1 i_1$, transforming it to the secondary side (fig.A.5) we obtain:

$$x_1 i_1 \frac{n_1}{n_2} = x_1 i_2 \left(\frac{n_1}{n_2} \right)^2 \quad (\text{A.25})$$

Therefore equivalent transformer impedance can be obtained:

$$x_{eq} = x_1 \left(\frac{n_1}{n_2} \right)^2 + x_2. \quad (\text{A.26})$$

Off-nominal tap ratio The overall voltage profile in the transmission network may vary within certain limits. One of the means to keep the voltage

in the sub-transmission and distribution network at permissible level and to provide reactive power support to transmission network is changing the turns ratio of the transformers. For this purpose frequently transformers supplying consumers are equipped with the tap-changers.

To transform the equivalent representation of the transformer to off-nominal tap ratio, the similar approach can be applied as for the primary to secondary side transformation.

In case of the off-nominal tap ratio, the additional "ideal-transformer" is modelled and the reactance becomes:

$$x'_{eq} = a^2 x_{eq}. \quad (\text{A.27})$$

In the similar manner one could derive expressions for the transformer equivalent impedance, taking into account resistive losses in the windings.

Shunts

Voltage control is mainly performed by the generators. Additional devices helping to regulate the voltage in power systems are capacitors and reactors. Capacitors produce reactive power in the low voltage conditions, while the reactors consume reactive power, lowering the voltage. Usually, these devices are modelled by the shunt reactance (inductive or capacitive) connected to the bus.

Summary of the parameter relation

Let us consider the model of transmission line. Formula (A.2) gives the current values at the ends of the line depending on the connected bus voltages. The power flow through the line can be obtained, as :

$$\begin{aligned} \underline{S}_{ij} &= \underline{V}_i \underline{I}_{ij}^* = \underline{V}_i [y_{ij}(\underline{V}_i - \underline{V}_j) + y_{ij}^{sh} \underline{V}_i]^* = V_i^2 (y_{ij}^* + y_{ij}^{sh*}) - y_{ij}^* \underline{V}_i \underline{V}_j^* = \\ &= V_i^2 [g_{ij} + g_{ij}^{sh} - j(b_{ij} + b_{ij}^{sh})] - (g_{ij} - jb_{ij}) V_i V_j e^{j(\theta_i - \theta_j)} \end{aligned} \quad (\text{A.28})$$

Introducing notation $\theta_{ij} = \theta_i - \theta_j$ and separating the real and imaginary part, it can be obtained:

$$P_{ij} = V_i^2 [g_{ij} + g_{ij}^{sh}] - V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij}) \quad (\text{A.29})$$

$$Q_{ij} = -V_i^2 [b_{ij} + b_{ij}^{sh}] - V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij}) \quad (\text{A.30})$$

Continuing with the analysis in the similar way for different system components and summarizing results of the previous section, we can obtain consumed or produced powers, as in table A.2.

A.2.2 Per unit system

A usual practise in electric network computations is to express voltages, powers, impedances and currents in per-unit (or relative) of a certain reference (base) value:

$$\text{Per-unit value} = \frac{\text{value in physical units}}{\text{base value of the quantity}}. \quad (\text{A.31})$$

Table A.2: Consumed/produced system components power

Component	Quantity	Expression
Load	$P = P_0$	$P_l = P_0$
	$Q = Q_0$	$Q_l = Q_0$
	$P(V)$	$P_l = \Re(\underline{I}_0^* \underline{V})$
$Q(V)$	$Q_l = \Im(\underline{I}_0^* \underline{V})$	
$.I = const$	$P(V)$	$P_l = rV^2/(r^2 + x^2)$
	$Q(V)$	$Q_l = xV^2/(r^2 + x^2)$
$.Z = const$	P	P_g
	Q	Q_g according to maximal heating
Line	flow, P_{ij}	$P_{ij} = V_i^2(g_{ij}^{sh} + g_{ij}) - V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij})$
	flow, Q_{ij}	$Q_{ij} = -V_i^2(b_{ij}^{sh} + b_{ij}) - V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij})$
	losses, P_w	$P_w = 3rI^2 = r[P_{ij}^2 + (Q_{ij} + b_{ij}^{sh} V_i^2)^2]/V_i^2$
	losses, Q_w	$Q_w = 3xI^2 = x[P_{ij}^2 + (Q_{ij} + b_{ij}^{sh} V_i^2)^2]/V_i^2$
Transformer	flow, P_{ij}	$P_{ij} = a^2 V_i^2 g_{ij} - a V_i V_j (g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij})$
	flow, Q_{ij}	$Q_{ij} = -a^2 V_i^2 b_{ij} - a V_i V_j (g_{ij} \sin \theta_{ij} - b_{ij} \cos \theta_{ij})$
Shunt	P_{sh}	losses are neglected
	Q_{sh}	$Q = V_i^2 b_i^{sh}$

Table A.3 shows all the necessary base values and the relation between those. Per-unit system can be applied to a network with several voltage levels, thus having several base voltages and, consequently, several base impedances and base currents. The representation of all components but the transformers in the per unit system is quite straightforward.

Table A.3: Base quantities in per unit system

Quantity	Notation	Value
Base Voltage	V_b	phase-to-phase voltage, kV
Base Power	S_b	three-phase power, MVA
Base Current	$I_b = S_b/(V_b \sqrt{3})$	current/phase, kA
Base Impedance	$Z_b = V_b^2/S_b$	impedance, Ohm

Transformers representation in per-unit system

Let us consider transformer model in Fig. A.4. The leakage impedances of the primary and the secondary site are connected through an ideal transformer with the ratio 1 : a . If the base voltages on each side are related by the transformation ratio:

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{V_{2b}}{V_{1b}} = a \quad (\text{A.32})$$

and the base power S_b is the same on both sides, then:

$$\frac{I_{1b}}{I_{2b}} = \frac{S_b}{\sqrt{3}V_{1b}} \frac{\sqrt{3}V_{2b}}{S_b} = a. \quad (\text{A.33})$$

Therefore,

$$\frac{I_1}{I_{1b}} = \frac{I_2}{I_{2b}} \quad (\text{A.34})$$

and in per-unit system the current on both sides are equal:

$$I_{1pu} = I_{2pu} = I_{pu}. \quad (\text{A.35})$$

From the Fig. A.4 follows that the voltage on the secondary side can be expressed as:

$$V_2 = (V_1 - \sqrt{3}I_1Z_p)a - \sqrt{3}I_2Z_s. \quad (\text{A.36})$$

Rewriting in pu and the base values, the expression becomes:

$$V_{2pu} = \frac{aV_{1b}V_{1pu} - a\sqrt{3}I_{1pu}I_{1b}Z_{ppu}Z_{1b} - \sqrt{3}I_{2pu}I_{2b}Z_{spu}Z_{2b}}{V_{2b}} = \quad (\text{A.37})$$

$$= V_{1pu} - I_{1pu}Z_{ppu} \frac{a\sqrt{3}I_{1b}Z_{1b}}{V_{2b}} - I_{2pu}Z_{spu} \frac{\sqrt{3}I_{2b}Z_{2b}}{V_{2b}} = \quad (\text{A.38})$$

$$= V_{1pu} - I_{pu}(Z_{ppu} + Z_{spu}). \quad (\text{A.39})$$

Thus, the transformer can be modelled simply by a single impedance.

The impedance values in pu can be derived easily:

$$Z_{ppu} + Z_{spu} = \frac{Z_p}{Z_{1b}} + \frac{Z_s}{Z_{2b}} = \frac{Z_p + Z_s/a^2}{Z_{1b}} = \frac{a^2Z_p + Z_s}{Z_{2b}}. \quad (\text{A.40})$$

If the both windings of transformer are operated at the reference (nominal) voltages, it does not matter to which of the sides the values of impedances are transformed as these are resulting in the same values of the per-unit system.

In overall, the use of per-unit system has several advantages:

- Increase of the numerical accuracy of the results;
- Possibility to analyze network with different voltage levels simply;
- The impedances at different voltage levels can be compared directly;

A.2.3 Single Line diagram

The single line diagram is the graphical representation of the power system components and the interconnections between those. The analysis the the symmetrical three-phase system is possible and simpler computing all the values for a single phase. Similarly, there is no necessity in presenting all the three phases of a power system, as a single line diagram is easier to perceive.

The complexity of the single line diagram depends on the carried analysis - only the components of interest are to be included. For example, the substation structure frequently is not considered in power flow studies, thus the substation

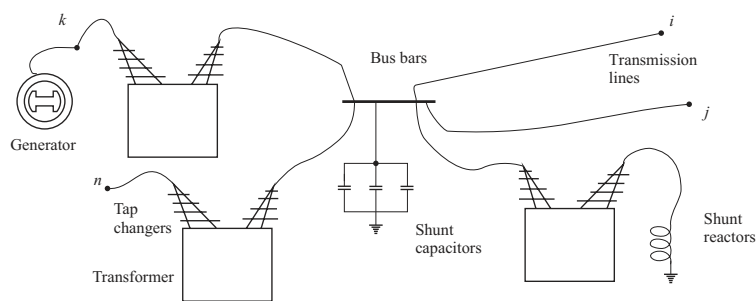


Figure A.6: Schematic representation of the equipment connected at bus

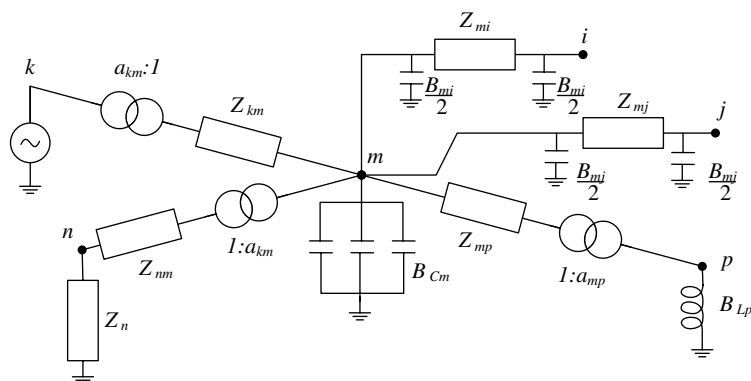


Figure A.7: Equivalent circuit of the equipment connected at bus

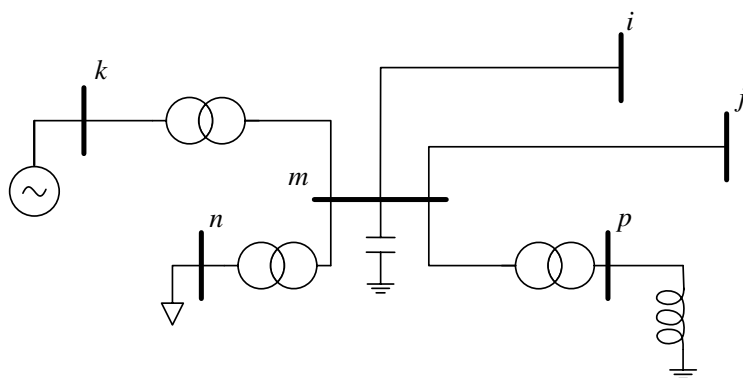


Figure A.8: Single-line diagram of a power system bus with connected equipment

can be represented by a single bus to which all the equipment is connected. The representation of the main components was introduced in the previous subsections. Fig. A.8 shows the example of the single line diagram for the part of the system in Fig. A.6. The equivalent electric circuit for the same system is shown in Fig. A.7.

Depending on the system, various approaches might be used in structuring the diagram. The bussed can be preferred to be grouped by the approximate geographical locations and/or importance or the certain transmissions, emphasizing for example the important transits, or loops. Frequently, for the better visualization the different voltage levels are distinguished by the different colors of the drawing. The coloring can also be used for the representation the result of computations.

A.3 Power Flow Computations

Values of the voltage magnitude and angle as well as the flows through the lines are changing continuously due to load variations, changes of the generation pattern, switching changes etc.

The transmission network and bulk loads are modelled by the set of simultaneous linear algebraic equations expressing Kirchhoff's laws for the network, and the voltage/current/power characteristics of the loads.

A.3.1 Nodal method

Let us consider node m in a power system with the different types of equipment connected to it, as in Fig. A.8.

According to Kirchhoff law, the sum of all currents flowing into the node and flowing out of the node equals to zero:

$$\underline{I}_{sh} + \Sigma \underline{I}_{line} + \underline{I}_{transf} - \underline{I}_{generator} + \underline{I}_{load} = 0, \quad (\text{A.41})$$

rearranging the expression and noting \underline{I}_i as load-generation imbalance (or net current injections) in bus, we obtain:

$$\underline{I}_i = \underbrace{\underline{I}_{generator} - \underline{I}_{load}}_{\text{loads and generation}} = \underbrace{\underline{I}_{sh} + \Sigma \underline{I}_{line} + \underline{I}_{transf}}_{\text{net injections}}. \quad (\text{A.42})$$

As clarified in section A.2 the current injections from the network equipment can be expressed as some functions of considered bus and connected bus voltages and the impedances of the equipment:

$$\underline{I}_i = \underline{I}_{sh} + \Sigma \underline{I}_{line} + \underline{I}_{transf} = \Sigma f(\underline{V}_i, \underline{V}_j \dots \underline{V}_k). \quad (\text{A.43})$$

Similarly, equations can be written for all the nodes in the system. Hence, we obtain system of n equations:

$$\begin{pmatrix} \underline{I}_1 \\ \vdots \\ \underline{I}_n \end{pmatrix} = \begin{pmatrix} Y_{11} & \dots & Y_{1n} \\ \vdots & \ddots & \vdots \\ Y_{n1} & \dots & Y_{nn} \end{pmatrix} \cdot \begin{pmatrix} \underline{V}_1 \\ \vdots \\ \underline{V}_n \end{pmatrix} \quad (\text{A.44})$$

where

- n is a number of busses
- Y_{ij} - the coefficients forming bus admittance matrix (discussed below)

Nodal admittance matrix of the system

In (A.44) the complex admittances \underline{Y} relate the total net injection current and the bus voltages. When the system of equations is written for the whole power system, these admittances form the bus admittance matrix or Y -bus matrix.

Analysis of the (A.44) leads to derivation of some bus admittance matrix forming rules:

- It can be uniquely determined when the parameters and the interconnection of the elements are known;
- Diagonal ii -elements equals to the sum of all the admittances connected to the bus i , including line shunts: $Y_{ii} = y_i^{sh} + \sum_{j=1, j \neq i}^n (y_{ij}^{sh} + y_{ij})$
- Non-diagonal ij -element equals to the equivalent admittance between busses i and j , with the negative sign: $Y_{ij} = -y_{ij}$;
- 0 - elements imply that there is no direct connection between the busses;
- this matrix is symmetric along main diagonal¹: $Y_{ij} = Y_{ji}$;

The real and imaginary parts of the matrix Y can be separated:

$$Y = G + jB, \quad (\text{A.45})$$

as will be used in the following sections.

This system of linear equations uniquely determines the state of power system, and it would be sufficient to solve it in order to find all the quantities in the analyzed system. However, as we have found out in the previous section, load current may be dependent on voltage (constant power loads), similar can be applicable to generation. Thus, it becomes impossible to model some of the elements as current injections or impedances and the necessity for other methods arises.

A.3.2 Power flow calculations

We clarified that current injection into busses can be computed as (A.44). Multiplying complex conjugated of both parts of these equations by the corresponding bus voltage, we obtain complex apparent power injections S_i . Decomposing it into real and imaginary part, we obtain *active and reactive power injection* in busses from the network, as follows:

$$\begin{aligned} P_i &= V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ Q_i &= V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}). \end{aligned} \quad (\text{A.46})$$

Introducing notations:

¹except when the system includes e.g. phase-shifting transformers

- $S_{Gi} = P_{Gi} + jQ_{Gi}$ - complex power produced by the generator
- $S_{Li} = P_{Li} + jQ_{Li}$ - complex power consumed by the load,

we get power balance expressions, which are similar to current balance (A.42):

$$\begin{aligned} P_i &= P_{Gi} - P_{Li} = V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ Q_i &= Q_{Gi} - Q_{Li} = V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}). \end{aligned} \quad (\text{A.47})$$

This formulation of the power system balance, allows us to consider all the necessary parameters. For example when loads are defined by impedances, these can be included in the admittance matrix similarly as shunts. Solution of these equations determines values of voltage magnitudes and angles. Out of these parameters all the flows can be determined (see table A.2).

Categories of buses

For the formulation (A.47) several types of busses shall be identified in the system:

PQ-bus, Load bus. Represents the bus, where active and reactive power injection is known and independent from the voltage. However, bus voltage magnitude V and angle θ are unknown. This is most frequently pure constant load busses - a suitable model for loads connected to low voltage side of the regulating transformers, which keep voltage constant irrespectively on the upper side variations. As a particular case, this bus can be also without load and generation $P_i = 0$, $Q_i = 0$;

PV-bus, Generation. Represents the bus, where active power injection P_i and the voltage magnitude V_i are known. Correspondingly, bus voltage magnitude angle θ and the reactive power injection Q_i are the unknowns. That means in this bus some voltage regulating device is installed to keep the voltage constant, for example synchronous generator or controllable shunt capacitors or controllable shunt reactors. Synchronous generator regulates the voltage by the magnetizing current and thus produced reactive power, while controllable shunts devices change reactance and thus generated reactive power to keep voltage constant.

$U\theta$ -bus, Slack. Only one bus in each system. At the slack bus the voltage angle is a reference usually chosen as 0-degrees and the voltage magnitude is assumed to be known. Unknown quantities are the active and reactive power. As the active power is allowed to vary that implies existence of the power generation at this bus. Moreover, since it is the only bus where the active power is not defined - it covers the imbalance in power system (due to, for example, losses). Constant voltage implies presence of the voltage regulating device.

The table A.4 summarizes the characteristics of the different types of busses.

Table A.4: Bus types in power flow computation

Bus type	Number of busses	Defined quantities	Unknowns
Slack, $V\theta$ -bus	1	V_i, θ_i	P_i, Q_i
Generator, PV -bus	m	P_i, V_i	Q_i, θ_i
Load, PQ -bus	$n - m - 1$	P_i, Q_i	V_i, θ_i

A.3.3 Solution of power flow problem

The system of equations (A.47) is non-linear as the it includes squared voltages as well as trigonometric expressions. For such a system one needs to employ and iterative solution technique. The technique will be first applied to the simplified one dimensional case followed by the multidimensional formulation for the power systems.

General Newton-Raphson algorithm for nonlinear equations

The generalized Newton-Raphson method is an iterative algorithm for solving set of nonlinear equations in an equal number of unknowns. At each iteration of the N-R method, the nonlinear set of equations is approximated by the linear ones. For example, in the single dimensional case equation:

$$f(x) = 0 \quad (\text{A.48})$$

must be solved. Expanding function f in Taylor series around point x^k and ignoring second and higher order terms, we obtain:

$$f(x) = f(x^k) + (x - x^k) \frac{\partial f(x^k)}{\partial x} \quad (\text{A.49})$$

since $f(x) = 0$ and denoting Jacobian $J = \frac{\partial f(x^k)}{\partial x}$ we obtain update of the variable x :

$$x - x^k = \frac{-f(x^k)}{J} \quad (\text{A.50})$$

Since the function is not linear, the expression above would not give an exact value for x . However, by repeating this procedure until the necessary precision is reached, one can obtain solution for the nonlinear equation. The graphical interpretation of the algorithm is shown in Fig. A.9.

Newton-Raphson algorithm for power flow equations

The method presented above can be extended to a multidimensional system of nonlinear equations, including the power flow equations.

The number of equations equals to number of unknowns and can be categorized in accordance with the bus types:

- (m) equations are formed for the active power P_i injections in PV-busses;
- ($2[n - m - 1]$) equations are formed for the active power P_i and reactive power Q_i injections in for PQ-busses.

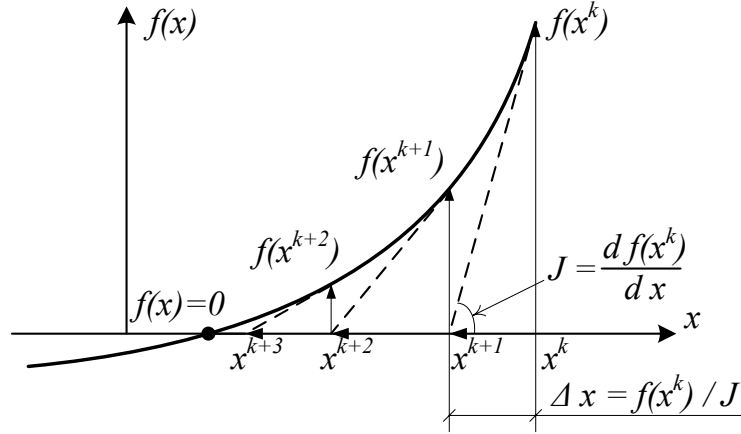


Figure A.9: Newton-Raphon algorithm for single variable function

Thus, (m) angles corresponding to PV-busses are to be determined, as well as ($n-m-1$) voltages and ($n-m-1$) angles corresponding to PQ-busses. The rest of the unknowns such as P_i, Q_i of the slack bus can be computed afterwards, after all the angles and voltages are obtained.

In this case, the function can be formulated as:

$$f(x) = \begin{bmatrix} P(\theta, V) - P_{GL} \\ Q(\theta, V) - Q_{GL} \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = 0, \quad (\text{A.51})$$

where $P(\theta, V)$ and $Q(\theta, V)$ are active and reactive power functions of voltage magnitudes and angles (A.47), P_{GL} and Q_{GL} are the known sums of loads and generation in the busses;

ΔP and ΔQ in the expression above are called power mismatches.

Therefore, the problem becomes:

$$\begin{bmatrix} \Delta P^k \\ \Delta Q^k \end{bmatrix} = -J \begin{bmatrix} \theta^{k+1} - \theta^k \\ (V^{k+1} - V^k) / V^k \end{bmatrix}, \quad (\text{A.52})$$

where J is the Jacobian of the function $f(x)$, superscript k denotes number of iteration, and $k+1$ number of next iteration. The division of $(V^{k+1} - V^k)$ by respective bus voltage magnitude V^k simplifies construction of the Jacobian as will be shown below.

Jacobian matrix of power flow equations is formed by the partial derivatives of the set of functions $f(x)$ by the variables x . In case of power flow equations, it can be written as follows:

$$J = \begin{bmatrix} H & N \\ J & L \end{bmatrix}, \quad (\text{A.53})$$

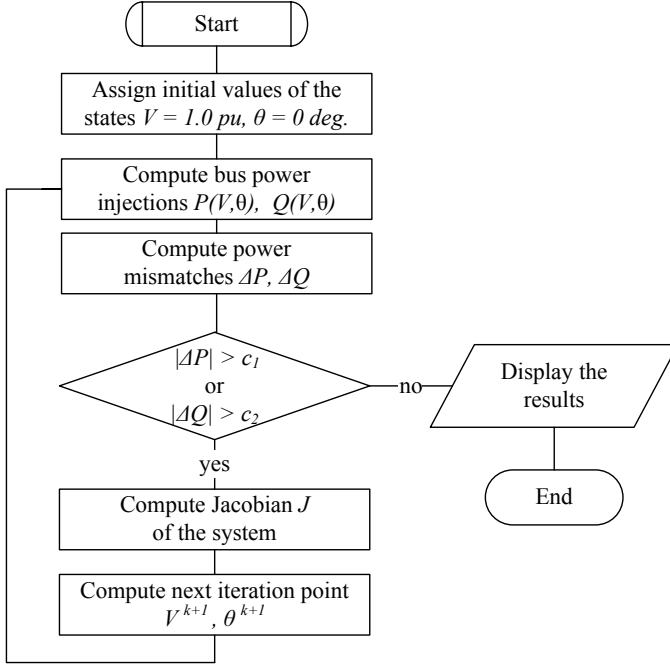


Figure A.10: power flow algorithm

where the diagonal elements taking into account (A.47):

$$\begin{aligned}
 H_{ii} &= \frac{\partial P_i}{\partial \theta_i} = \sum_{j=1}^N V_i V_j (-G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij}) - B_{ii} V_i^2 = -Q_i - B_{ii} V_i^2 \\
 N_{ii} &= V_i \frac{\partial P_i}{\partial V_i} = \sum_{j=1}^N V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) + G_{ii} V_i = P_i + G_{ii} V_i^2 \\
 J_{ii} &= \frac{\partial Q_i}{\partial \theta_i} = \sum_{j=1}^N V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) - G_{ii} V_i^2 = P_i - G_{ii} V_i^2 \\
 L_{ii} &= V_i \frac{\partial Q_i}{\partial V_i} = \sum_{j=1}^N V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) - B_{ii} V_i = Q_i - B_{ii} V_i^2 \quad (\text{A.54})
 \end{aligned}$$

while the off-diagonal elements:

$$\begin{aligned}
 H_{ij} &= \frac{\partial P_i}{\partial \theta_j} = V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \\
 N_{ij} &= V_j \frac{\partial P_i}{\partial V_j} = V_i V_j (G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij}) \\
 J_{ij} &= \frac{\partial Q_i}{\partial \theta_j} = -V_i V_j (G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij}) \\
 L_{ij} &= V_j \frac{\partial Q_i}{\partial V_j} = V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}). \quad (\text{A.55})
 \end{aligned}$$

Similarly as for the power mismatches in (A.52), the Jacobian elements H, N are formed for the PV- and PQ-busses and J, L elements consider only PQ-busses.

Constrained power flow

A set of inequality constraints may describe for example reactive power production limits at the power plant:

$$Q_i^{min} \leq Q_i \leq Q_i^{max}. \quad (\text{A.56})$$

Therefore, in addition to power balance equations (A.47), it may necessary to consider some of the constraints, such as reactive power production capacity, directly in the power flow solution procedure: if the reactive power production limit is reached at PV-bus, the voltage cannot be regulated anymore and becomes unknown parameter, while the reactive power production is determined. Thus PV-bus turns into PQ-bus.

For the other types of constraints (thermal limitation, voltage limits) it is sufficient to perform a violation check after the iterative solution procedure is finished.

Convergence criteria and tests

The decision about the convergence and the sufficient accuracy of the solution requires certain criteria. The usual criteria for power flow problem are bus power mismatches to be sufficiently small:

$$\begin{aligned} |\Delta P_i| &\leq c_1, \quad \forall \text{ PQ- and PV-busses,} \\ |\Delta Q_i| &\leq c_2, \quad \forall \text{ PQ-busses,} \end{aligned}$$

where c_1 and c_2 are small empirical constants. Usually, $c_1 = c_2$ and the particular value differs depending on the problem and the system. In a large system $c = 1$ MW(MVAr) shall give reasonable accuracy, while for the smaller systems $c = 0.1$ MW(MVAr) may be a better choice. A variant of the criteria above is:

$$\sum_i \Delta P_i^2 + \sum_i \Delta Q_i^2 \leq c_3. \quad (\text{A.57})$$

The power flow problem for the physically unstable operating conditions *usually* diverges. However, this is not always the case. To verify whether obtained solution is physically feasible, certain simple checks may be introduced in the program, such as transmission angle and voltage drop across each of the line. Another example of the useful check is for the voltage magnitude being within arbitrary limits 0.5-1.5 p.u. It is unlikely that a meaningful solution would be outside of this range.

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