ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

eeh power systems laboratory

Small-Signal Stability and Power System Stabilizer

Dynamics and Control of Electric Power Systems
(Prof. Dr. Göran Andersson)

Stephan Koch

Contents

- Review: Closed-Loop Stability
- Third-Order Model of the Synchronous Machine
- Heffron-Phillips Model
- Dynamic Analysis of the Heffron-Phillips Model
  - Split between damping and synchronizing torque
  - SMIB with classical generator model
  - SMIB including field circuit dynamics
  - SMIB including excitation system
- Power System Stabilizer
  - Block diagram
  - Effect on system dynamics

Review: Closed-Loop Stability

State space formulation of dynamical system

- Autonomous dynamical system with initial condition:

$$\dot{x} = Ax, \quad x(t=0) = x_0$$

- Rate of change of each state is a linear combination of all states:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2$$

- Transformation to diagonal form in order to derive solution easily:

$$\dot{z}_1 = \lambda_1 z_1$$

$$z_1 = z_1(0) \cdot e^{\lambda_1 t}$$

Review: Closed-Loop Stability

State space formulation of dynamical system

- Our aim is to transform the equation to the “easy“ form:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \Leftrightarrow \dot{z} = \Lambda \cdot z$$

- Linear coordinate transformation:

$$\begin{aligned} x &= \Phi \cdot z \\ \dot{x} &= \Phi \cdot \dot{z} \end{aligned} \quad \longrightarrow \quad \begin{aligned} \Phi \cdot \dot{z} &= A \cdot \Phi \cdot z \\ \dot{z} &= \underbrace{\Phi^{-1} \cdot A \cdot \Phi}_{\Lambda} \cdot z \end{aligned} \quad \longrightarrow \quad \dot{z} = \Lambda \cdot z$$

- This is equivalent to:

$$\Phi = [\phi_1, \phi_2 .....\phi_n]$$

$$\Lambda = diag(\lambda_1, \lambda_2 ....., \lambda_n) \qquad \lambda_i ....eigenvalues$$

$$\phi_i \cdot \lambda_i = A \cdot \phi_i \Rightarrow (A - \lambda_i I) \cdot \phi_i = 0 \qquad \phi_i ....right\ eigenvectors$$

$$\det(A - \lambda_i I) = 0$$

Review: Closed-Loop Stability

Eigenvalues, stability, oscillation frequency and damping ratio

- Let λ_1 be a real eigenvalue of matrix A. Then holds:

  $\lambda_1 < 0:$ The corresponding mode is **stable** (decaying exponential).

  $\lambda_1 > 0:$ The corresponding mode is **unstable** (growing exponential).

  $\lambda_1 = 0:$ The corresponding mode has **integrating** characteristics.

- Let $\lambda_{1,2} = \sigma \pm j\omega$ be a complex conjugate pair of eigenvalues of A. Then:

  $\mathrm{Re}\,\lambda_{2,3} < 0$: The corresponding mode is **stable** (decaying oscillation).

  $\mathrm{Re}\,\lambda_{2,3} > 0$: The corresponding mode is **unstable** (growing oscillation).

  $\mathrm{Re}\,\lambda_{2,3} = 0$: The corresponding mode is **critically stable** (undamped osc.).

  The following dynamic properties can be established:

  - Oscillation frequency: $f = \dfrac{\omega}{2\pi}$
  - Damping ratio: $\zeta = \dfrac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$

Third-Order Model of the Synchronous Machine

- Voltage deviation in d- and q-axis:

$$\begin{pmatrix} \Delta u_d \\ \Delta u_q \end{pmatrix} = \begin{pmatrix} 0 \\ \Delta e'_q \end{pmatrix} + \begin{pmatrix} -r_d & x_q \\ -x'_d & -r_q \end{pmatrix} \begin{pmatrix} \Delta i_d \\ \Delta i_q \end{pmatrix} \qquad (5.88)$$

with $\Delta e'_q = -\dfrac{\Delta e_F + (x_d - x'_d)\Delta i_d}{1 + sT'_{do}}$ (5.87)

- Linearized swing equation:

$$\Delta\omega = \frac{1}{2Hs + K_D}(\Delta T_m - \Delta T_e)$$

$$\Delta\delta = \frac{2\pi f_0}{s}\Delta\omega$$

Heffron-Phillips Model

Purpose:

- Simplified representation of synchronous machine, suitable for stability studies:

  "Small Signal Stability" → linearized model

Basis:

- Third-order Model of synchronous machine

Starting point for derivation:

- Single-Machine Infinite-Bus (SMIB) System
- Linearized generator swing equation:

$$\Delta\omega = \frac{1}{2Hs + K_D}(\Delta T_m - \Delta T_e)$$

$$\Delta\delta = \frac{2\pi f_0}{s}\Delta\omega$$

Heffron-Phillips Model

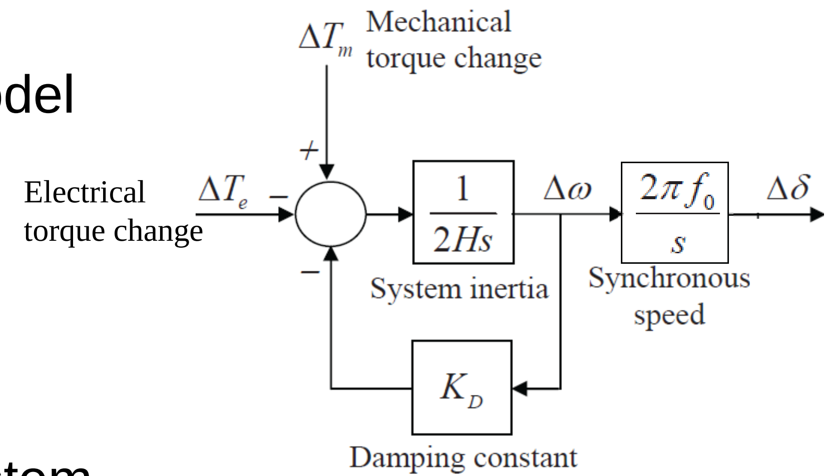
Purpose:

- Simplified representation of synchronous machine, suitable for stability studies:

  "Small Signal Stability" → linearized model

Basis:

- Third-order Model of synchronous machine



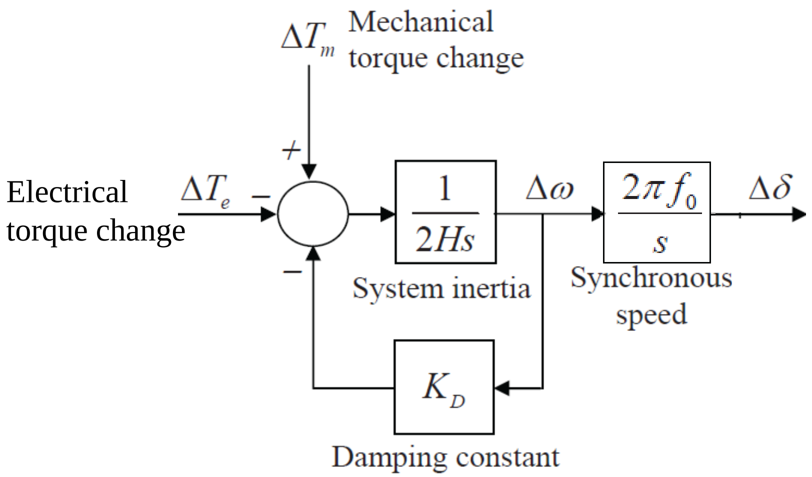
Starting point for derivation:

- Single-Machine Infinite-Bus (SMIB) System
- Linearized generator swing equation:

$$\Delta\omega = \frac{1}{2Hs + K_D}(\Delta T_m - \Delta T_e)$$

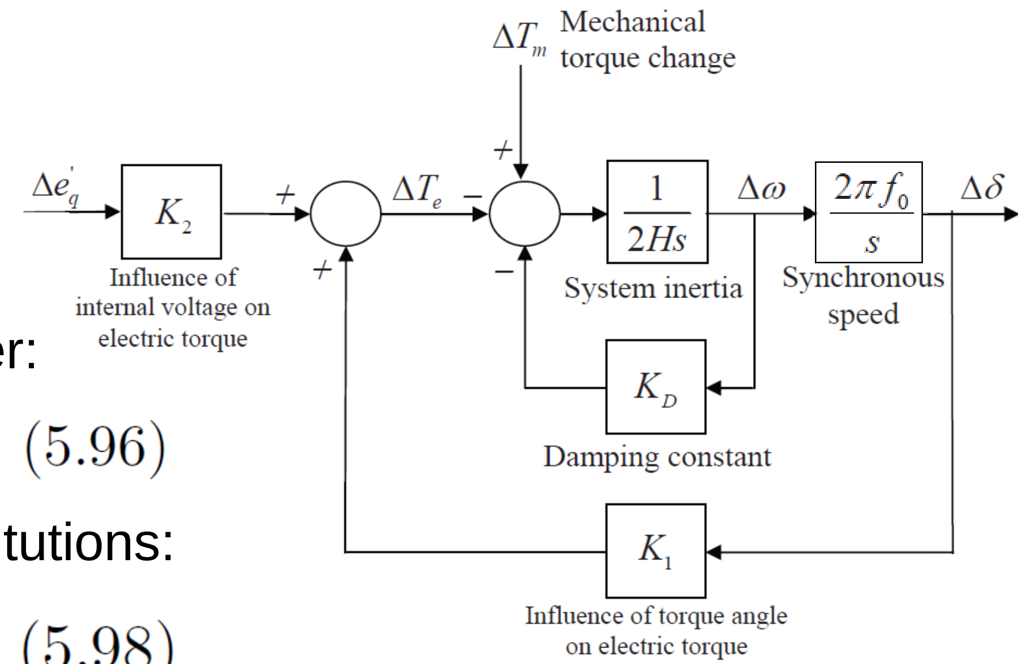
$$\Delta\delta = \frac{2\pi f_0}{s}\Delta\omega$$

Heffron-Phillips Model



Heffron-Phillips Model

… including the composition of the electric torque:



Approximation of torque with power:

$$T_e \approx P_e = i_d u_d + i_q u_q \qquad (5.96)$$

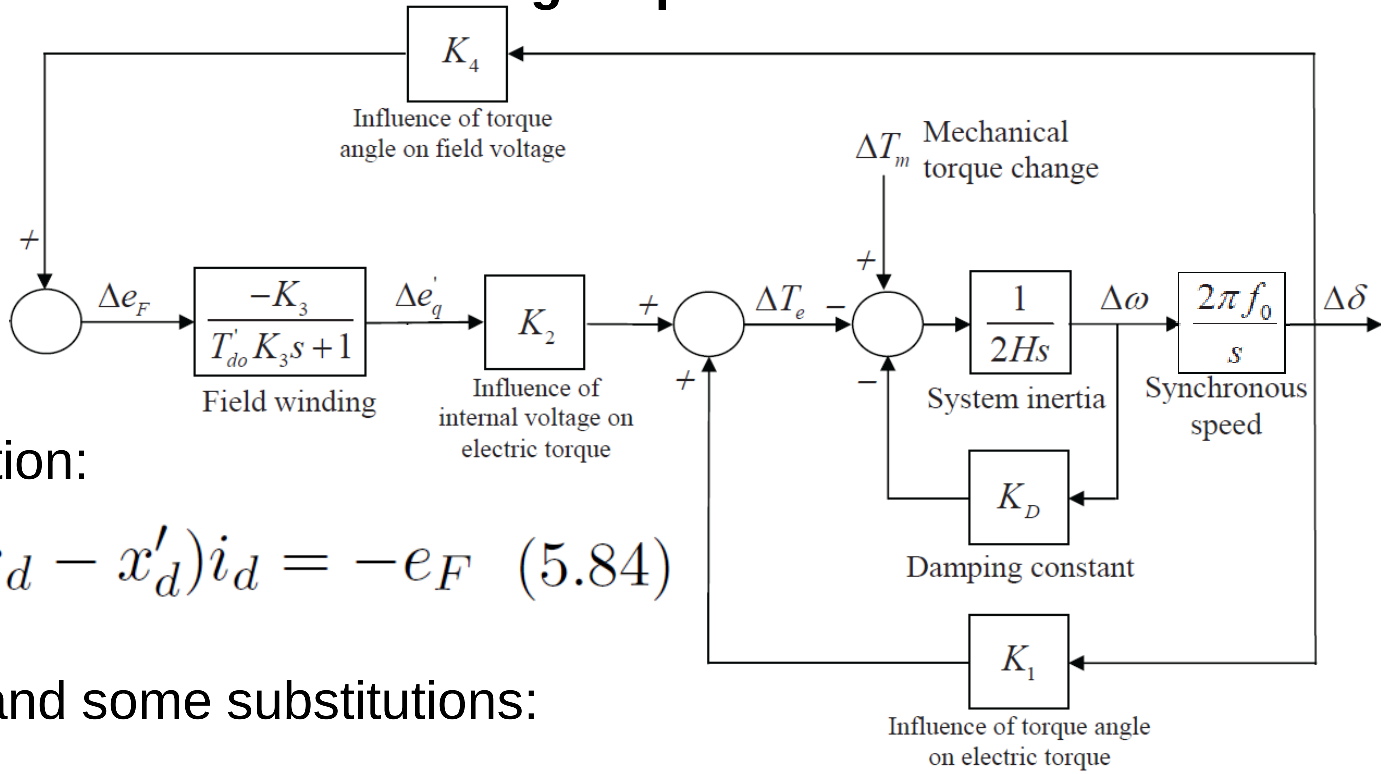
After linearization and some substitutions:

$$\Delta T_e = K_1 \Delta\delta + K_2 \Delta e_q' \qquad (5.98)$$

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0 \\ i_{q0} \end{bmatrix} + \begin{bmatrix} F_d & F_q \\ Y_d & Y_q \end{bmatrix} \begin{bmatrix} (x_q - x_d') i_{q0} \\ e_{q0}' + (x_q - x_d') i_{d0} \end{bmatrix} \qquad (5.99)$$

Heffron-Phillips Model

… including the effect of the field voltage equation:



Field voltage equation:

$$T'_{do}\dot{e}'_q + e'_q + (x_d - x'_d)i_d = -e_F \quad (5.84)$$

After linearization and some substitutions:

$$(1 + s\,T'_{do}K_3)\Delta e'_q = -K_3(\Delta e_F + K_4\Delta\delta) \quad (5.101)$$

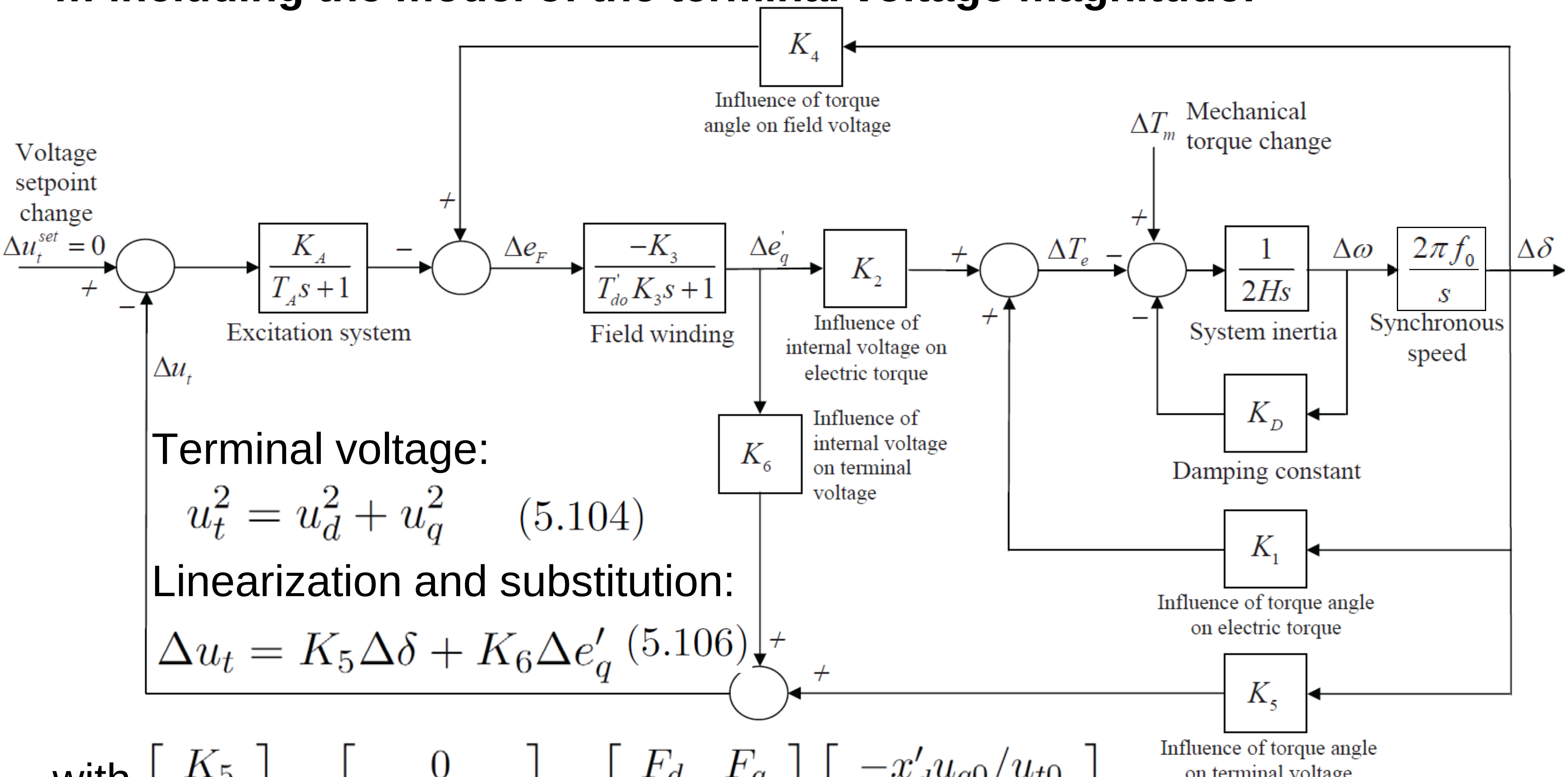
with:

$$K_3 = 1/(1 + (x_d - x'_d)Y_d) \quad (5.102)$$

$$K_4 = (x_d - x'_d)F_d \quad (5.103)$$

Heffron-Phillips Model

… including the model of the terminal voltage magnitude:



Terminal voltage:

$$u_t^2 = u_d^2 + u_q^2 \qquad (5.104)$$

Linearization and substitution:

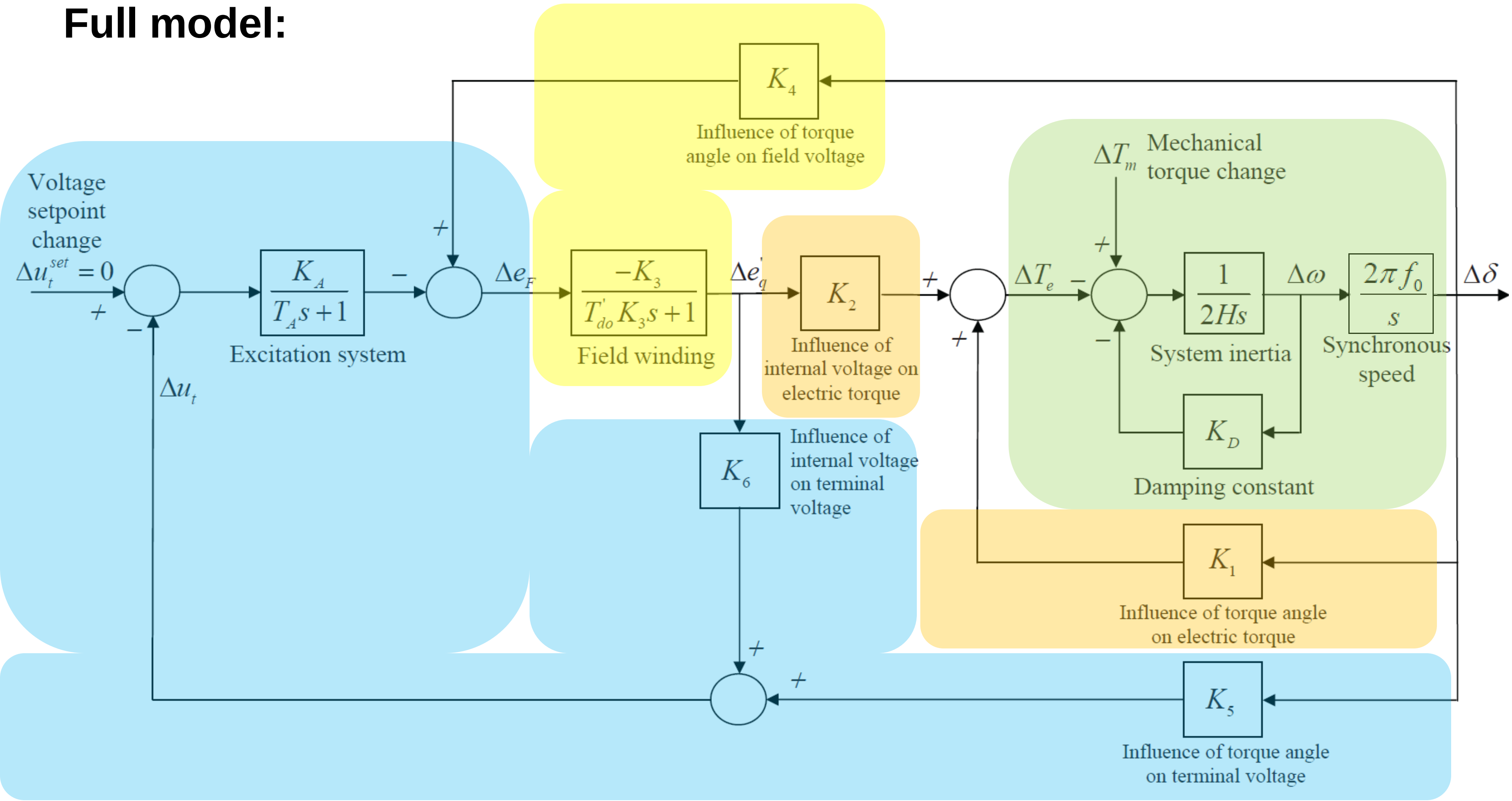
$$\Delta u_t = K_5 \Delta\delta + K_6 \Delta e'_q \quad (5.106)$$

with

$$\begin{bmatrix} K_5 \\ K_6 \end{bmatrix} = \begin{bmatrix} 0 \\ u_{q0}/u_{t0} \end{bmatrix} + \begin{bmatrix} F_d & F_q \\ Y_d & Y_q \end{bmatrix} \begin{bmatrix} -x'_d u_{q0}/u_{t0} \\ x_q u_{d0}/u_{t0} \end{bmatrix} \quad (5.107)$$

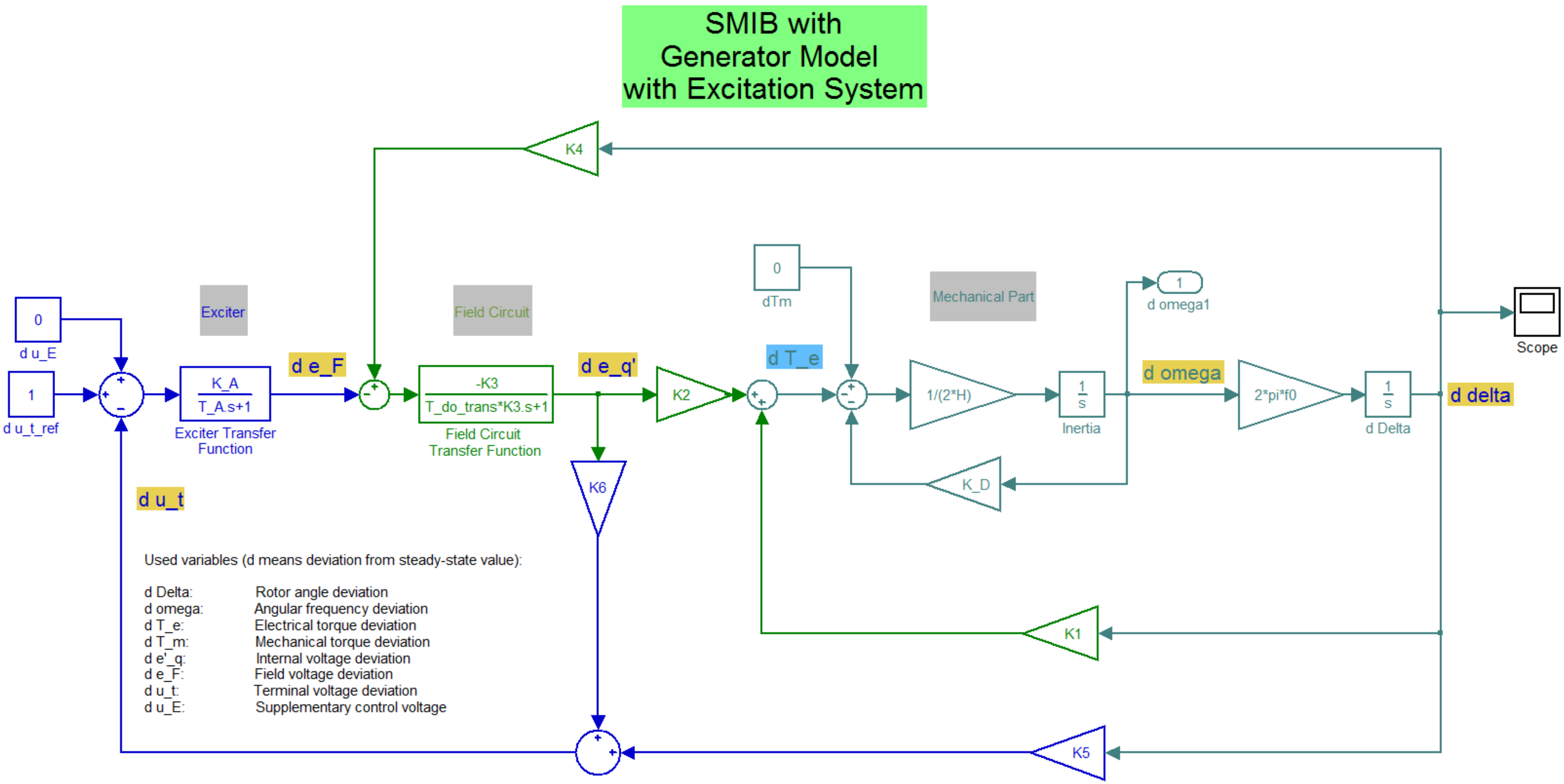
Heffron-Phillips Model

Full model:



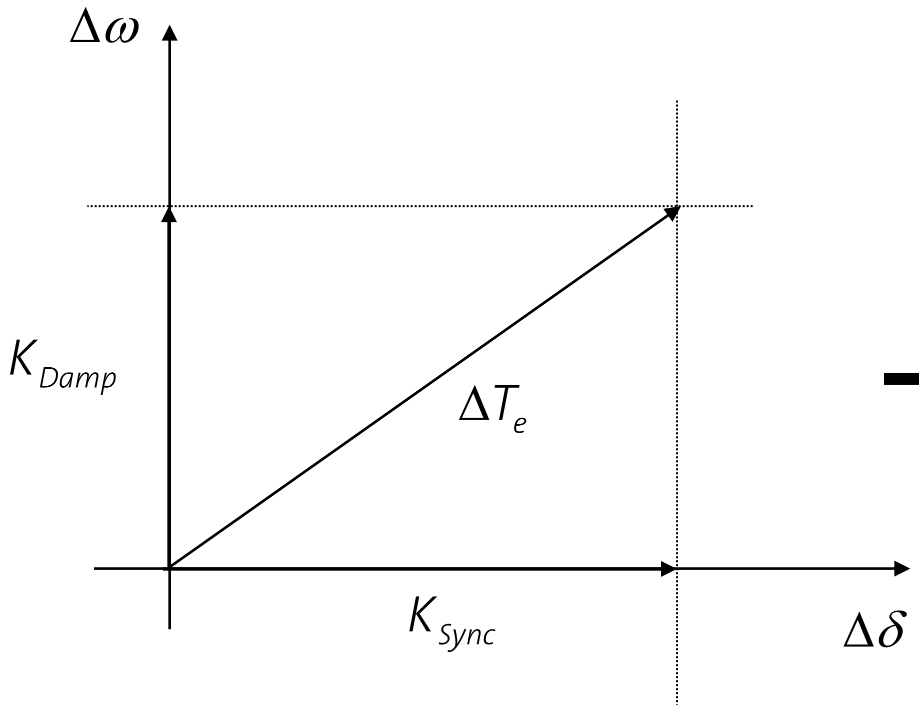
Heffron-Phillips Model

Simulink implementation



Dynamic Analysis of the Heffron-Phillips Model

Splitting between synchronizing and damping torque

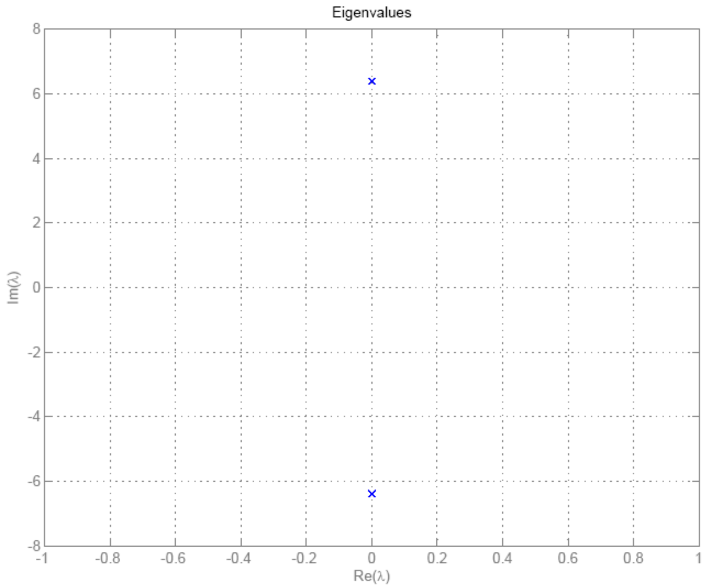


→ Exercise 3!

$$\Delta T_e = K_{Sync} \cdot \Delta\delta + K_{Damp} \cdot \Delta\omega$$

Dynamic Analysis of the Heffron-Phillips Model

SMIB with classical generator model (mechanical damping torque $K_D = 0$)



Eigenvalues on imaginary axis
→ system is critically stable

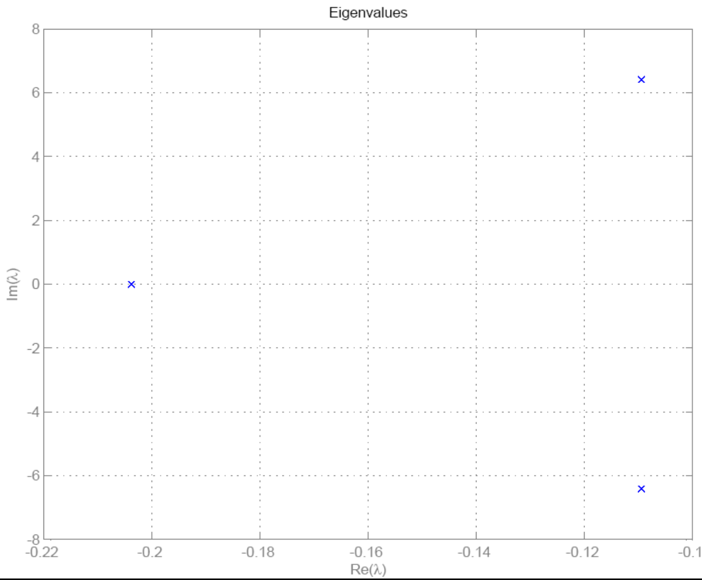
Eigenvalues	Real	Imaginary	Damping Ratio	f [Hz]
$\lambda_{1,2}$	0	$\pm$ 6.385	-	1.016

Synchronizing and damping torque coefficients

s	K_{sync}	K_{damp}
$\lambda_{1,2}$	0.757	0

Dynamic Analysis of the Heffron-Phillips Model

SMIB including field circuit dynamics



Eigenvalues moved to the left because field circuit adds damping torque

Eigenvalues	Real	Imaginary	Damping Ratio	*f* [Hz]
$\lambda_{1,2}$	– 0.109	$\pm$ 6.411	0.0170	1.020
λ_3	– 0.204	0	1.0	

Synchronizing and damping torque coefficients due to field circuit

s	K_{sync}	K_{damp}
$\lambda_{1,2}$	– 0.0008	1.5333
λ_3	– 0.7651	0

Dynamic Analysis of the Heffron-Phillips Model

SMIB including excitation system

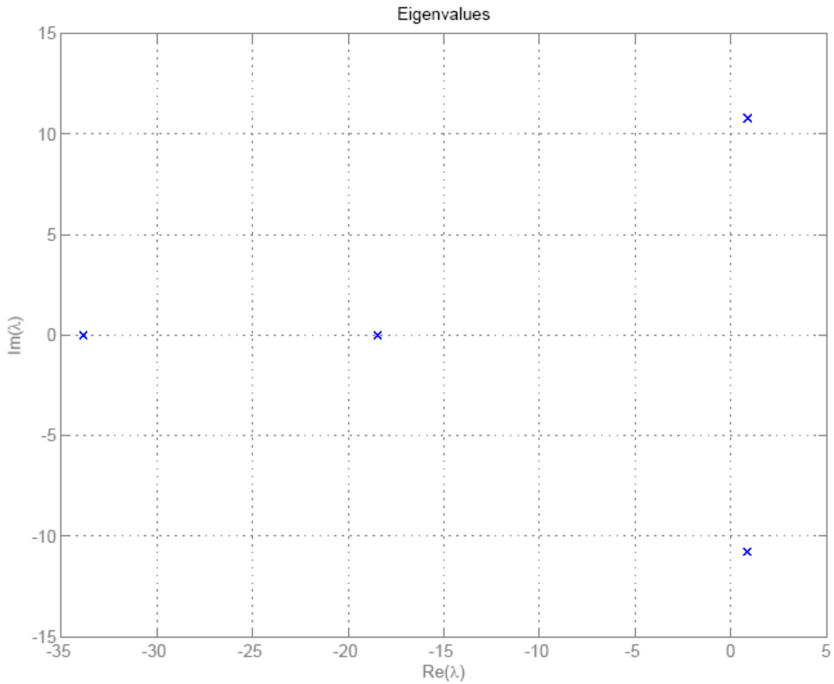
Eigenvalues	Real	Imaginary	Damping Ratio	*f* [Hz]
$\lambda_{1,2}$	0.8837	$\pm$10.7864	– 0.0816	1.7167
λ_3	– 33.8342	0	1.0	0
λ_4	–18.4567	0	1.0	0

Synchronizing and damping torque coefficients due to exciter

s	K_{sync}	K_{damp}
$\lambda_{1,2}$	0.2731	-10.6038
λ_3	– 19.8103	0
λ_4	– 7.0126	0

Dynamic Analysis of the Heffron-Phillips Model

SMIB including excitation system



Eigenvalues moved to the right by the excitation system
→ System is unstable!

Power System Stabilizer

- **Purpose:**

  provide additional damping torque component in order to prevent the system from becoming unstable
- **Approach:**

  insert feedback between angular frequency and voltage setpoint
- **Block diagram:**

Gain:
Tuning parameter for damping torque increase

Washout filter:
Suppress effect of low-frequency speed changes

Phase compensation:
Provide phase-lead characteristic to compensate for lag between exciter input and el. torque

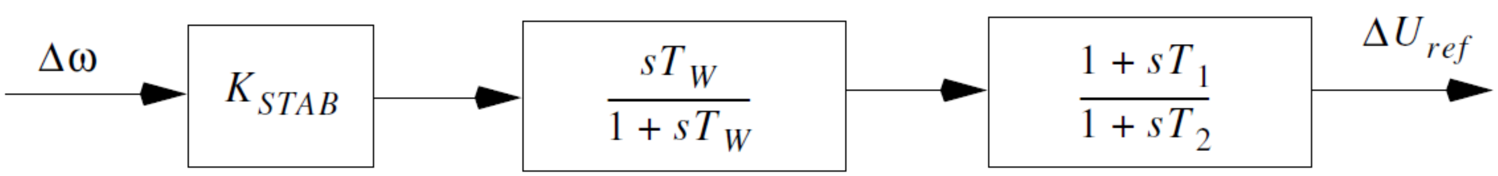
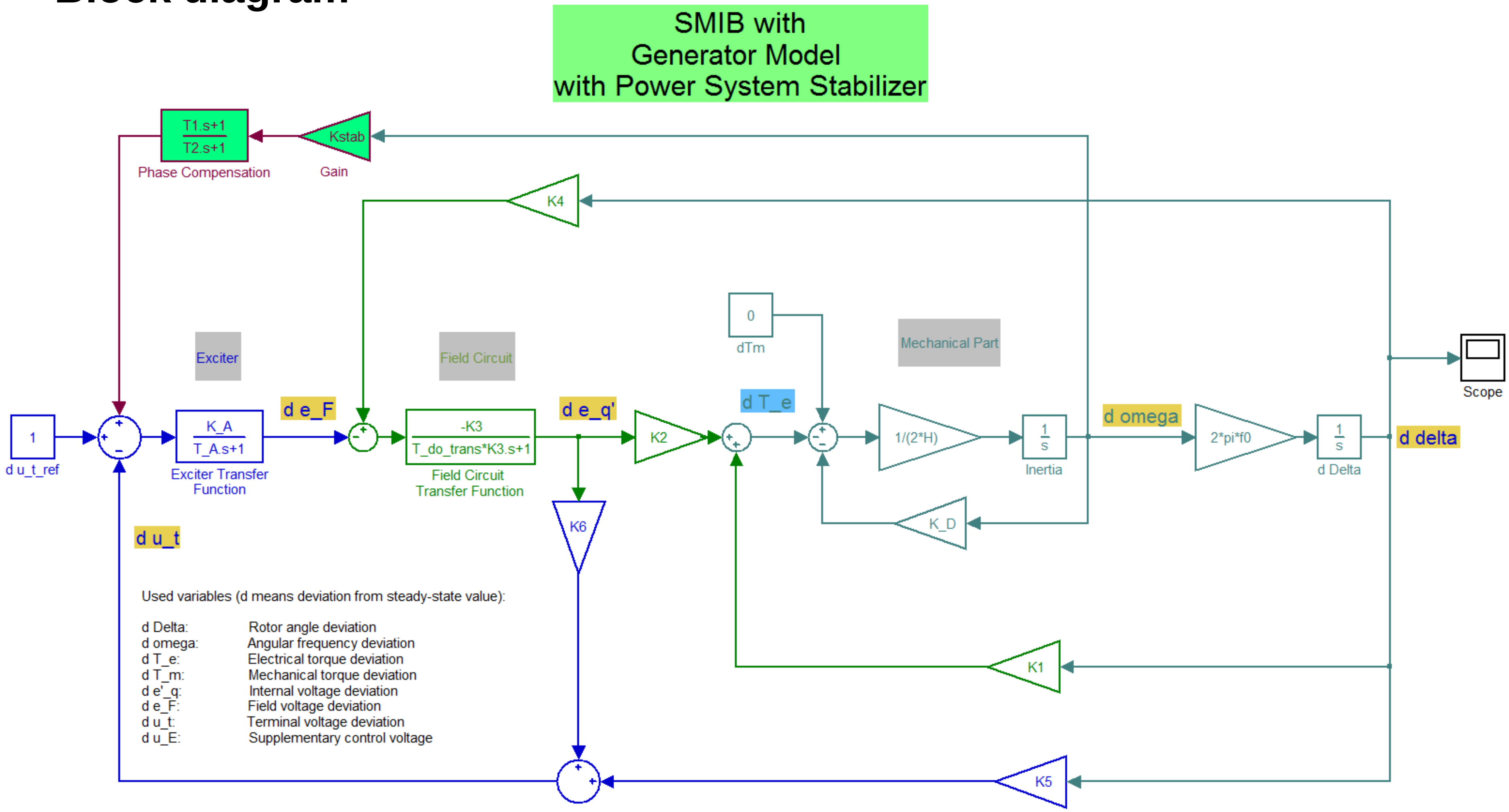


Figure 7.2. Block Diagram of a simple PSS.

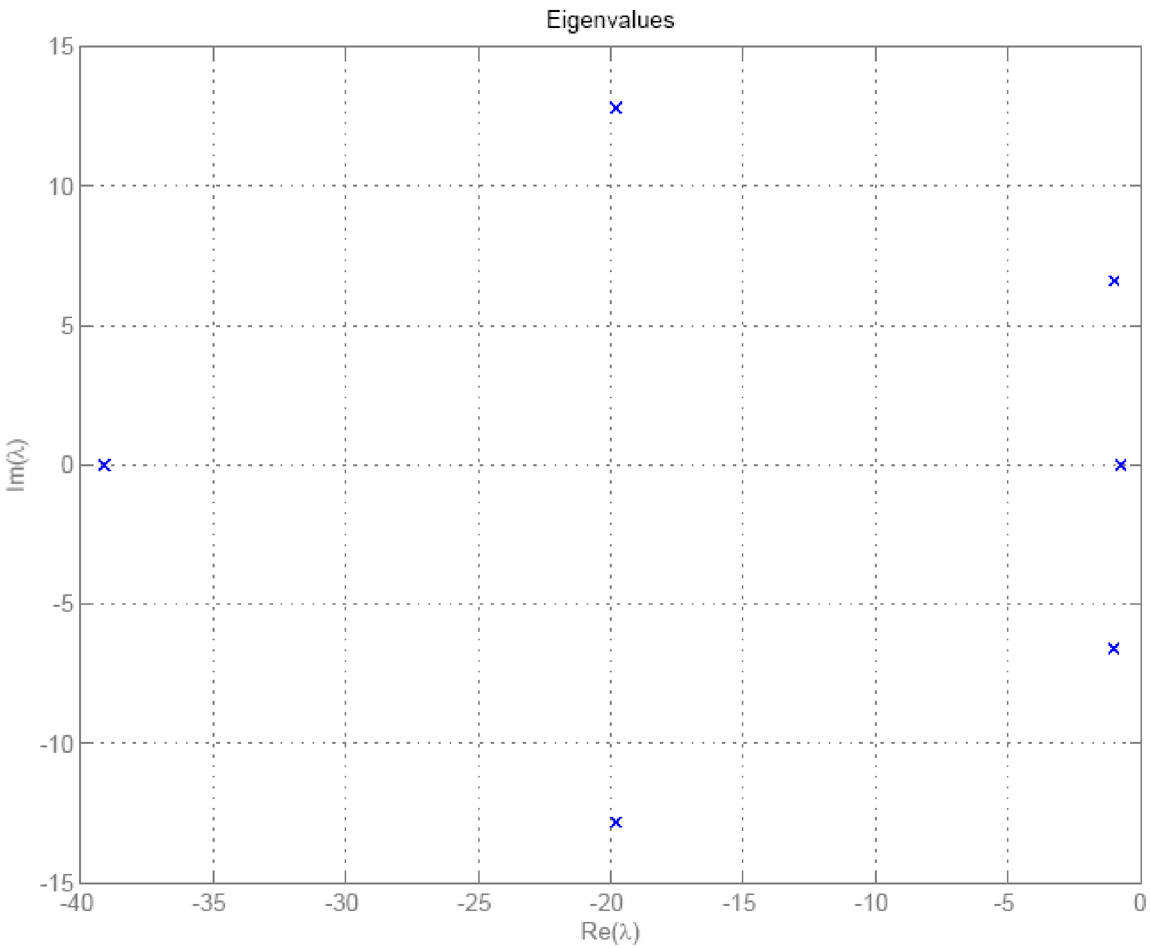
Power System Stabilizer

Block diagram



Power System Stabilizer

Effect on the system dynamics



Power System Stabilizer

Effect on the system dynamics

Eigenvalues	Real	Imaginary	Damping Ratio	*f* [Hz]
$\lambda_{1,2}$	– 1.0052	$\pm$ 6.6071	0.1504	1.0516
$\lambda_{3,4}$	– 19.7970	$\pm$12.8213	0.8394	2.0406
λ_5	– 39.0969	0	-	-
λ_6	– 0.7388	0	-	-

Synchronizing and damping torque coefficients due to exciter

s	K_{sync}	K_{damp}
$\lambda_{1,2}$	0.21	– 8.69
$\lambda_{3,4}$	– 1.27	– 13.00
λ_5	1.16	0
λ_6	0.30	0

Synchronizing and damping torque coefficients due to PSS

s	K_{sync}	K_{damp}
$\lambda_{1,2}$	– 0.145	22.761
$\lambda_{3,4}$	10.838	290.163
λ_5	– 30.306	0
λ_6	–1.072	0

Coming up …

Exercise 3: Power System Stabilizer

- Contents:
  Stability analysis of Heffron-Phillips Model, PSS design and testing
- Date and time:
  Tuesday, 03 May 2010, 10:00 – 12:00
  ETZ D 61.1 (Computer Room)
- Handouts will be sent around one week in advance.
  Please prepare the exercise at home, timing is tight!
- Attendance is compulsory for the “Testat“.
  Please notify us in case you cannot attend → substitute task.

Thank you for your attention!