

Power Flow, part 2

Variables associated with each node:

- U_k : voltage magnitude
- θ_k : voltage angle
- P_k : net active power (algebraic sum of generation and load)
- Q_k : net reactive power (algebraic sum of generation and load)

Depending on which of the above four variables are known (given) and which ones are unknown (to be calculated), two basic types of buses can be defined:

- PQ bus: P_k and Q_k are specified; U_k and θ_k are calculated
- PU bus: P_k and U_k are specified; Q_k and θ_k are calculated

PQ buses are normally used to represent load buses without voltage control, and PU buses are used to represent generation buses with voltage control in power flow calculations¹. Synchronous compensators² are also treated as PU buses. A third bus is also needed:

- U θ bus: U_k and θ_k are specified; P_k and Q_k are calculated

The U θ bus, also called reference bus or slack bus, has double functions in the basic formulation of the power flow problem:

1. It serves as the voltage angle reference
2. Since the active power losses are unknown in advance, the active power generation of the U θ bus is used to balance generation, load, and losses

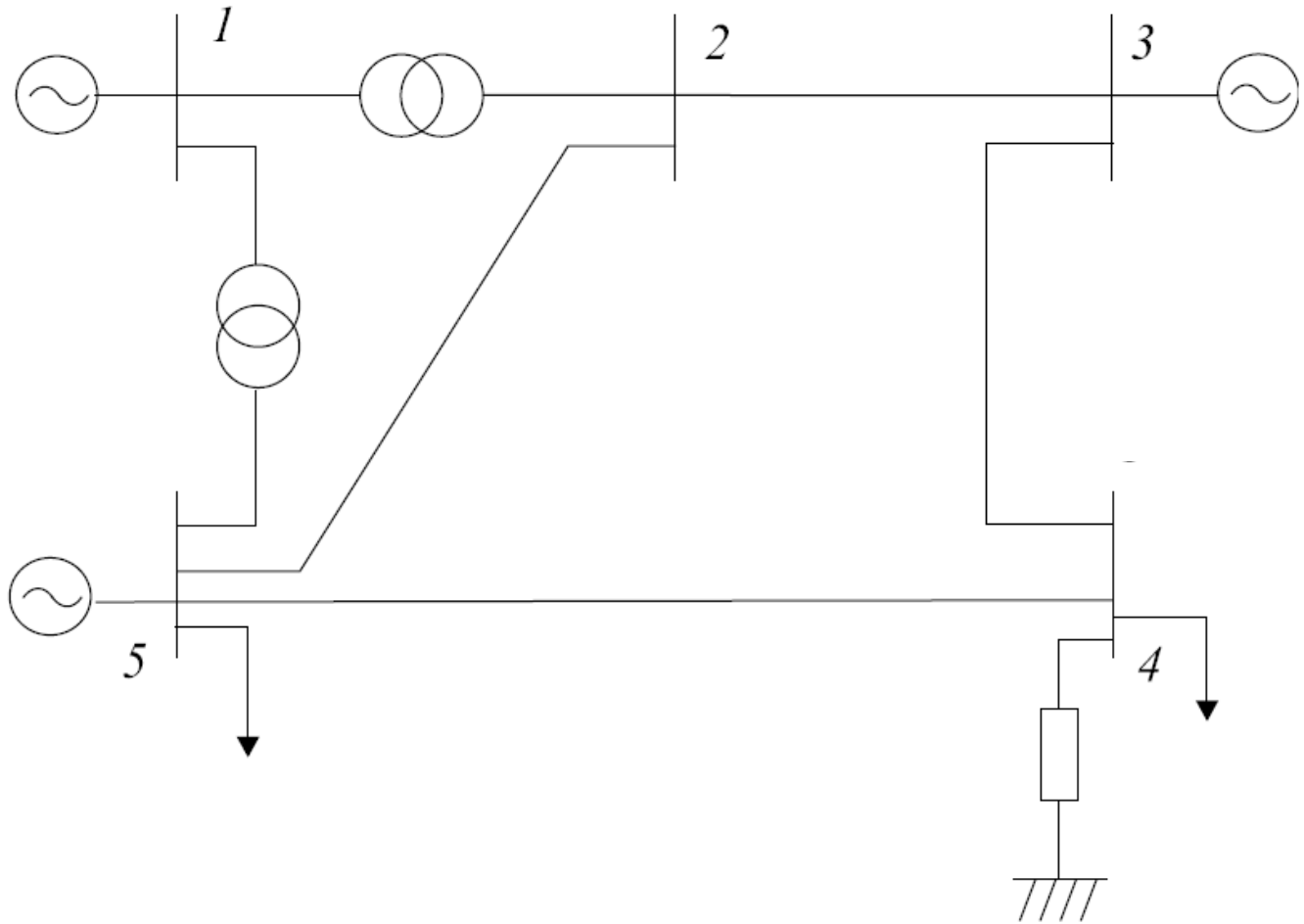


Figure 5.1. 5-bus system

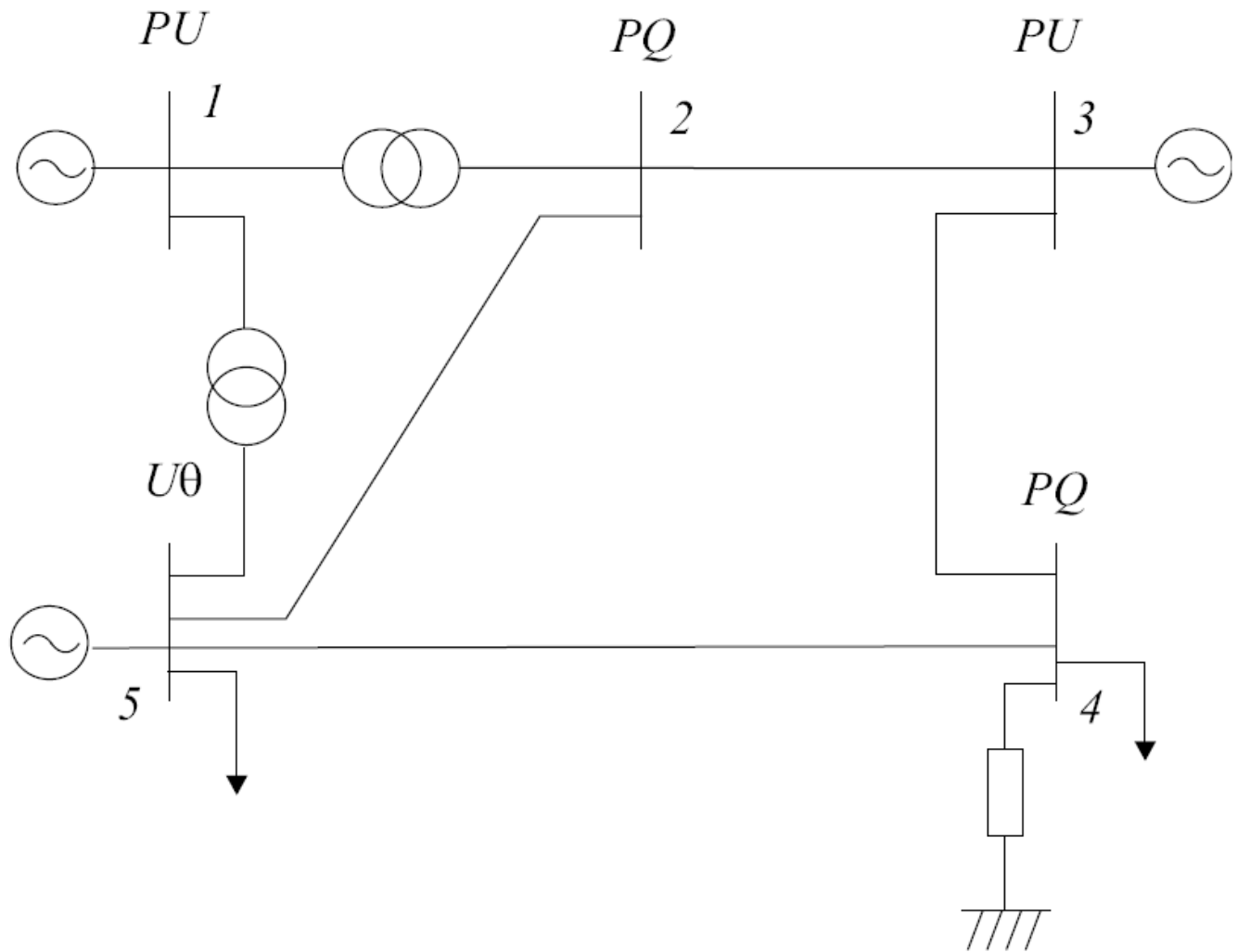


Figure 5.1. 5-bus system

Basic power flow equations

$$P_k = \sum_{m \in \Omega_k} P_{km}(U_k, U_m, \theta_k, \theta_m) \quad (5.1)$$

$$Q_k + Q_k^{sh}(U_k) = \sum_{m \in \Omega_k} Q_{km}(U_k, U_m, \theta_k, \theta_m) \quad (5.2)$$

which also can be written as:

$$S_k = E_k \sum_{m \in K} Y_{km}^* E_m^* \quad k = 1, 2, \dots, N$$

Inequality Constraints

$$Q_k^{min} \leq Q_k \leq Q_k^{max} \quad (5.3)$$

$$U_k^{min} \leq U_k \leq U_k^{max} \quad (5.4)$$

Number of equations = Number of unknowns? ?

Solution of Power Flow Equations

1. Consider the formulation

$$S_k = E_k \sum_{m \in K} Y_{km}^* E_m^* \quad k = 1, 2, \dots, N$$

This non-linear complex system of equations can be solved by Gauss (- Seidel) iteration.

Gauss Iteration

Gauss iteration:

$x = h(x)$ is a (non linear) equation

Iterative scheme:

$$x^{\nu+1} = h(x^{\nu}), \quad \nu = 0, 1, \dots$$

If the sequence x^0, x^1, \dots converges to x^*

then x^* is a solution to the equation $x = h(x)$

i.e. $x^* = h(x^*)$

Gauss-Seidel Iteration 1

$$S_k = E_k \sum_{m \in K} Y_{km}^* E_m^* \quad k = 1, 2, \dots, N$$

$$E_k = \frac{1}{Y_{kk}} \left[\frac{S_k^*}{E_k^*} - \sum_{m \in \Omega_k} Y_{km} E_m \right] \quad k = 1, 2, \dots, N$$

$$E_2 = h_2(E_1, E_2, \dots, E_N)$$

$$E_3 = h_3(E_1, E_2, \dots, E_N)$$

⋮

$$E_N = h_N(E_1, E_2, \dots, E_N)$$

Gauss-Seidel Iteration 2

Gauss Iteration

$$x_2^{\nu+1} = h_2(x_1, x_2^{\nu}, \dots, x_N^{\nu})$$

$$x_3^{\nu+1} = h_3(x_1, x_2^{\nu}, \dots, x_N^{\nu})$$

\vdots

$$x_N^{\nu+1} = h_N(x_1, x_2^{\nu}, \dots, x_N^{\nu})$$

$$x_2^{\nu+1} = h_2(x_1, x_2^{\nu}, \dots, x_N^{\nu})$$

$$x_3^{\nu+1} = h_3(x_1, x_2^{\nu+1}, \dots, x_N^{\nu})$$

\vdots

$$x_N^{\nu+1} = h_N(x_1, x_2^{\nu+1}, \dots, x_{N-1}^{\nu+1}, x_N^{\nu})$$

Gauss-Seidel Iteration

Gauss-Seidel Iteration 3

1. Start values ?

To be discussed

2. Convergence properties?

When it converges, the convergence is linear.

Sometimes no convergence, even if solution exists!

Generally hard to determine convergence properties.

Better solution technique is needed:

Newton - Raphson

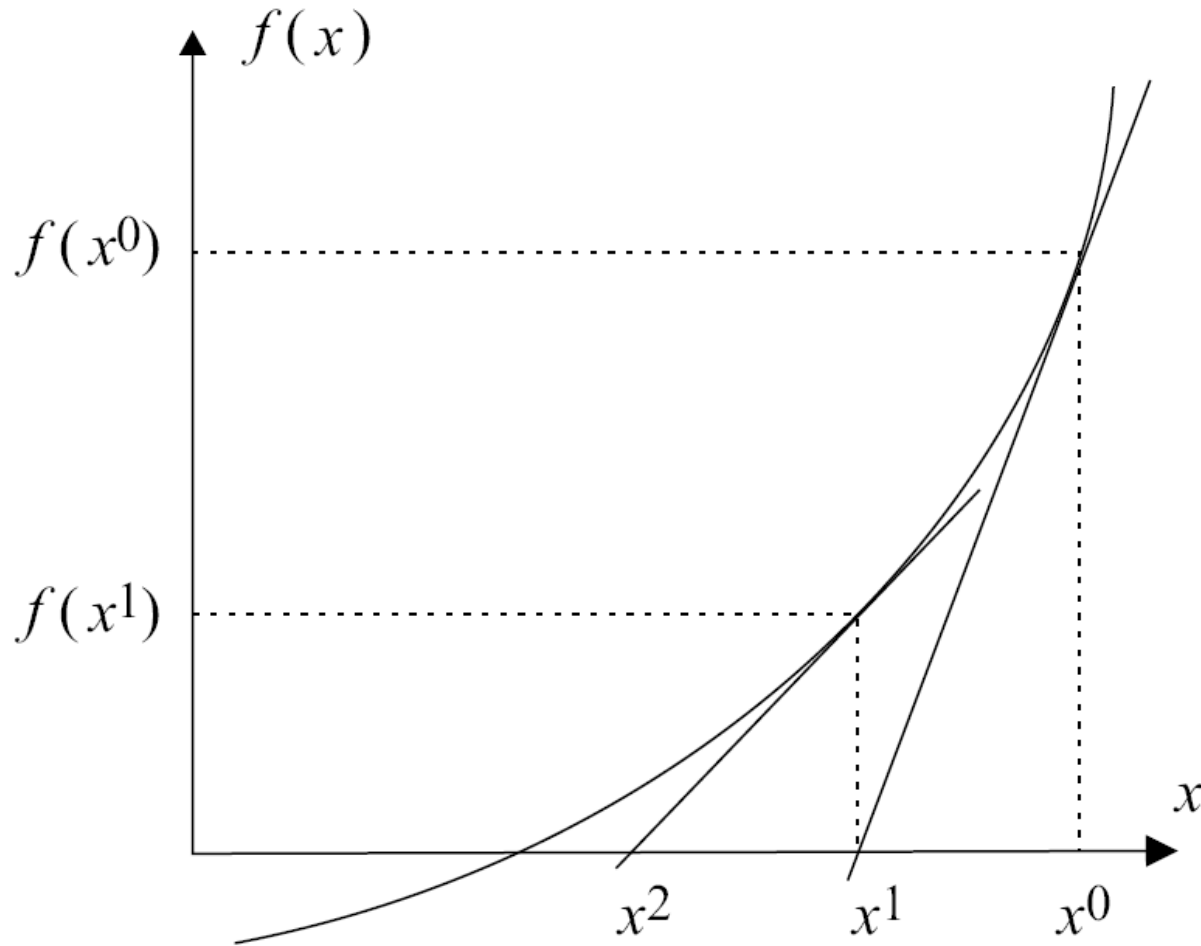


Figure 6.1. Newton-Raphson method in unidimensional case

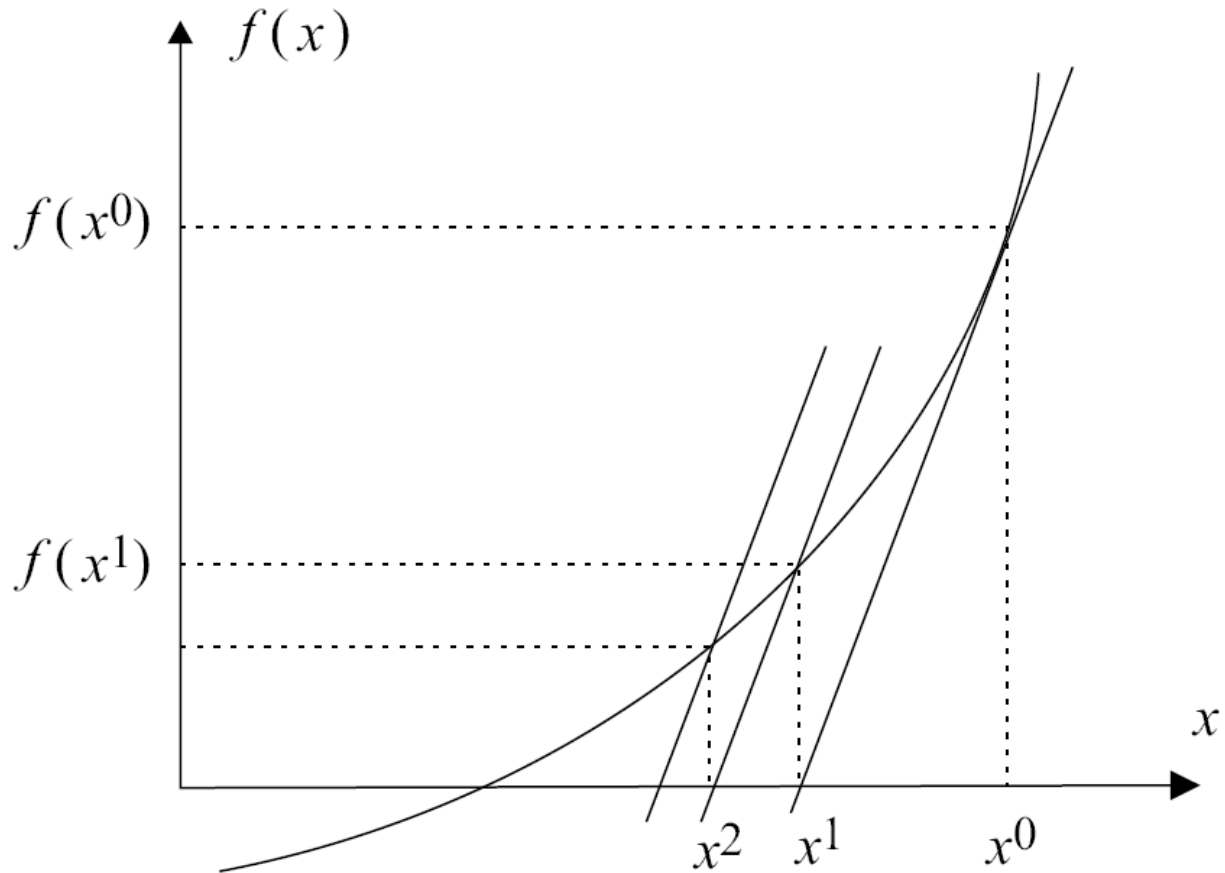


Figure 6.2. Dishonest Newton-Raphson method in unidimensional case

Newton-Raphson Algorithm

$$\mathbf{f}(\mathbf{x}) = \mathbf{0}$$

1. Set $\nu = 0$ and choose an appropriate starting value \mathbf{x}^0 ;
2. Compute $\mathbf{f}(\mathbf{x}^\nu)$;
3. Test convergence:
If $|f_i(\mathbf{x}^\nu)| \leq \varepsilon$ for $i = 1, 2, \dots, n$, then \mathbf{x}^ν is the solution
Otherwise go to 4;
4. Compute the Jacobian matrix $\mathbf{J}(\mathbf{x}^\nu)$;
5. Update the solution

$$\Delta \mathbf{x}^\nu = -\mathbf{J}^{-1}(\mathbf{x}^\nu) \mathbf{f}(\mathbf{x}^\nu) \tag{6.28}$$

$$\mathbf{x}^{\nu+1} = \mathbf{x}^\nu + \Delta \mathbf{x}^\nu$$

6. Update iteration counter $\nu + 1 \rightarrow \nu$ and go to step 2.

Power Flow Equations

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} \Delta \mathbf{P}(\mathbf{x}) \\ \Delta \mathbf{Q}(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \mathbf{P}(\mathbf{x}) - \mathbf{P}^{(s)} \\ \mathbf{Q}(\mathbf{x}) - \mathbf{Q}^{(s)} \end{pmatrix} \quad (6.30)$$

with

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} P_2(\mathbf{x}) - P_2^{(s)} \\ \vdots \\ P_m(\mathbf{x}) - P_m^{(s)} \\ \text{---} \\ Q_2(\mathbf{x}) - Q_2^{(s)} \\ \vdots \\ Q_n(\mathbf{x}) - Q_n^{(s)} \end{pmatrix} \quad (6.31)$$

$$\mathbf{x} = \begin{pmatrix} \theta \\ \mathbf{U} \end{pmatrix} \quad (6.29)$$

$$\mathbf{J}(\mathbf{x}^\nu) \begin{pmatrix} \Delta\theta^\nu \\ \Delta\mathbf{U}^\nu \end{pmatrix} + \begin{pmatrix} \Delta\mathbf{P}(\mathbf{x}^\nu) \\ \Delta\mathbf{Q}(\mathbf{x}^\nu) \end{pmatrix} = 0 \quad (6.33)$$

$$\mathbf{J} = \begin{pmatrix} \frac{\partial \mathbf{P}}{\partial \theta} & \frac{\partial \mathbf{P}}{\partial \mathbf{U}} \\ \frac{\partial \mathbf{Q}}{\partial \theta} & \frac{\partial \mathbf{Q}}{\partial \mathbf{U}} \end{pmatrix} \quad (6.36)$$

$$\mathbf{J}(\mathbf{x}^\nu) \begin{pmatrix} \Delta\theta^\nu \\ \Delta\mathbf{U}^\nu \end{pmatrix} + \begin{pmatrix} \Delta\mathbf{P}(\mathbf{x}^\nu) \\ \Delta\mathbf{Q}(\mathbf{x}^\nu) \end{pmatrix} = 0$$

When solving for the updates in the equation above, the inverse of the Jacobian, \mathbf{J} , is **not** explicitly computed.

Instead the updates, $\Delta\theta$ and $\Delta\mathbf{U}$, are calculated by use of Gauss elimination, (LU-factorization) or similar methods.

Line losses neglected: $r = 0$
Valid for high voltage systems

$$P_{km} = \frac{U_k U_m \sin \theta_{km}}{x_{km}} \quad (6.37)$$

$$Q_{km} = \frac{U_k^2 - U_k U_m \cos \theta_{km}}{x_{km}} \quad (6.38)$$

$$\frac{\partial P_{km}}{\partial \theta_k} = \frac{U_k U_m \cos \theta_{km}}{x_{km}} \quad \frac{\partial P_{km}}{\partial U_k} = \frac{U_m \sin \theta_{km}}{x_{km}} \quad (6.39)$$

$$\frac{\partial Q_{km}}{\partial \theta_k} = \frac{U_k U_m \sin \theta_{km}}{x_{km}} \quad \frac{\partial Q_{km}}{\partial U_k} = \frac{2U_k - U_m \cos \theta_{km}}{x_{km}} \quad (6.40)$$

When $\theta_{km} = 0$, perfect decoupling conditions are observed, i.e.

$$\frac{\partial P_{km}}{\partial \theta_k} = \frac{U_k U_m}{x_{km}} \quad \frac{\partial P_{km}}{\partial U_k} = 0 \quad (6.41)$$

$$\frac{\partial Q_{km}}{\partial \theta_k} = 0 \quad \frac{\partial Q_{km}}{\partial U_k} = \frac{2U_k - U_m}{x_{km}} \quad (6.42)$$

Example 6.1. A 750 kV transmission line has 0.0175 p.u. series reactance (the series reactance and the shunt admittance are ignored in this example). The terminal bus voltage magnitudes are 0.984 and 0.962 p.u. and the angle difference is 10° . Calculate the sensitivities of the active and reactive power flows with respect to voltage magnitude and phase angle.

Solution The four sensitivities are calculated by using eqs. (6.39) and (6.40):

$$\frac{\partial P_{km}}{\partial \theta_k} = \frac{U_k U_m \cos \theta_{km}}{x_{km}} = \frac{0.984 \cdot 0.962 \cos 10^\circ}{0.0175} = 54.1$$

$$\frac{\partial P_{km}}{\partial U_k} = \frac{U_m \sin \theta_{km}}{x_{km}} = \frac{0.962 \sin 10^\circ}{0.0175} = 9.5$$

$$\frac{\partial Q_{km}}{\partial \theta_k} = \frac{U_k U_m \sin \theta_{km}}{x_{km}} = \frac{0.984 \cdot 0.962 \sin 10^\circ}{0.0175} = 9.4$$

$$\frac{\partial Q_{km}}{\partial U_k} = \frac{2U_k - U_m \cos \theta_{km}}{x_{km}} = \frac{2 \cdot 0.984 - 0.962 \cos 10^\circ}{0.0175} = 58.3$$

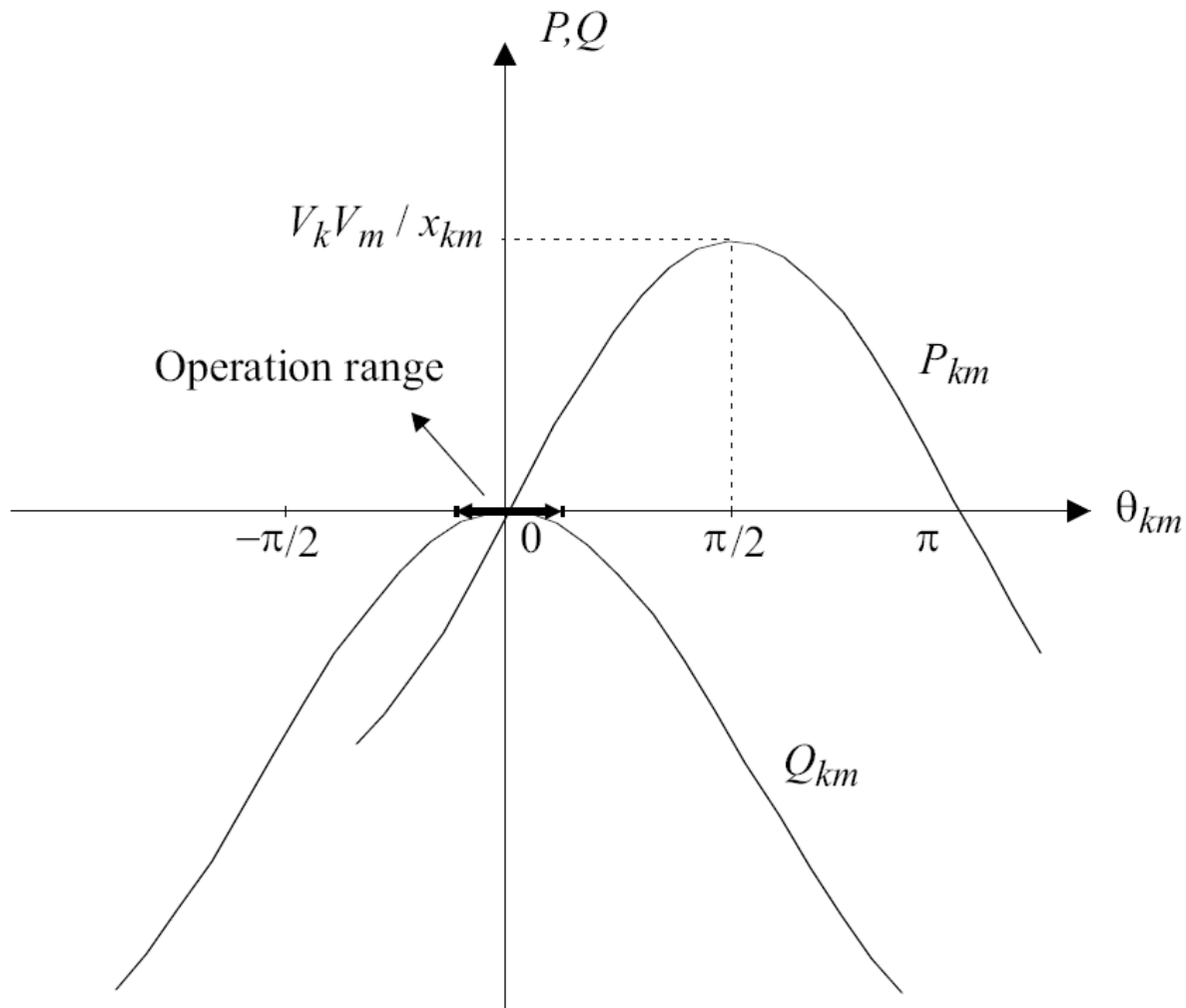


Figure 6.3. $P - \theta$ and $Q - \theta$ curves for a line with a series resistance and a shunt admittance of zero and considering terminal voltages $U_k = U_m = 1.0$ p.u.

De-coupled Power Flow

$$\mathbf{J}_{DEC} = \begin{pmatrix} \frac{\partial \mathbf{P}}{\partial \theta} & 0 \\ 0 & \frac{\partial \mathbf{Q}}{\partial \mathbf{U}} \end{pmatrix} \quad (6.43)$$

$$\frac{\partial \mathbf{P}}{\partial \theta} \Delta \theta^\nu + \Delta \mathbf{P}(\theta^\nu, \mathbf{U}^\nu) = 0 \quad (6.44)$$

$$\frac{\partial \mathbf{Q}}{\partial \mathbf{U}} \Delta \mathbf{U}^\nu + \Delta \mathbf{Q}(\theta^{\nu+1}, \mathbf{U}^\nu) = 0 \quad (6.45)$$

Convergence

If the de-coupled Newton – Raphson converges to a solution, it is the correct solution.

(No approximations in the computation of mismatches, only in the Jacobian.)

Convergence properties

- Gauss – Seidel has **linear** convergence:

$$|\varepsilon_{v+1}| \leq k \cdot |\varepsilon_v| ; 0 \leq k < 1$$

ε_v is the local error, i.e. $\varepsilon_v = x^* - x^v$,

where x^* is the (true) solution.

Newton – Raphson has **quadratic** convergence:

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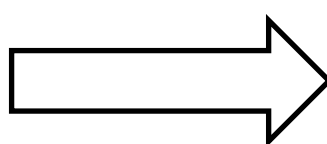
Since,

$$\frac{f(x^\nu)}{f'(x^\nu)} + \varepsilon_\nu = \frac{f(x^\nu)}{f'(x^\nu)} + x^* - x^\nu = x^* - x^{\nu+1} = \varepsilon_{\nu+1} \quad (6.19)$$

the following relationship between ε_ν and $\varepsilon_{\nu+1}$ results:

$$\frac{\varepsilon_{\nu+1}}{\varepsilon_\nu^2} = -\frac{1}{2} \frac{f''(\bar{x})}{f'(x^\nu)} \quad (6.20)$$

In the vicinity of the root, i.e. as $x^\nu \rightarrow x^*$, $\bar{x} \rightarrow x^*$, and we thus have


$$\frac{|\varepsilon_{\nu+1}|}{\varepsilon_\nu^2} = \frac{1}{2} \frac{|f''(x^*)|}{|f'(x^*)|} \quad (6.21)$$

Initial solution, x_0 ?

As initial solution, if no better solution is known, quite often the so called “flat start” is selected:

$$\theta_i = 0 \quad (\text{angle of slack bus})$$

$$U_i = U_1 = \quad \text{voltage magnitude of slack bus}$$

Initial solution, x_0 ?

If the power flow to be calculated is a perturbation of an already known power flow solution, e.g. outage of a line ($N - 1$ calculation) or a change in generator dispatch, the known power flow solution can be used as a initial solution.

Combination of Gauss - Seidel and Newton - Raphson

1. Initial solution (flat start)
2. (Gauss – Seidel iterations)
3. Newton – Raphson iterations possibly with last Gauss –Seidel iteration as initial solution.

Jacobian Matrix

For realistic power systems the Jacobian, J , is large but sparse.

Special techniques for storage and factorization of sparse matrices exist.

Optimal ordering of buses possible.

(Lecture by Dr. Demiray next week)

Linearized Power Flow (1) (DC Power Flow)

An approximative power flow solution can be obtained by linearizing the power flow equations. Assumptions:

1. Angle differences between buses are small
2. Line series resistances are neglected
3. All voltage magnitudes in the system are equal, e.g. 1 p.u.

Linearized Power Flow (2)

This means that the active power flows on line is given by:

$$P_{km} = -P_{mk} = -U_k U_m b_{km} \sin \theta_{km} \quad (6.49)$$

and since

$$U_k \approx U_m \approx 1 \text{ p.u.} \quad (6.52)$$

$$\sin \theta_{km} \approx \theta_{km} \quad (6.53)$$

$$b_{km} = -1/x_{km} \quad (6.54)$$

Linearized Power Flow (3)

$$P_{km} = \theta_{km}/x_{km} = \frac{\theta_k - \theta_m}{x_{km}} \quad (6.55)$$

which can be interpreted as

- P_{km} is the dc current;
- θ_k and θ_m are the dc voltages at the resistor terminals;
- x_{km} is the resistance.

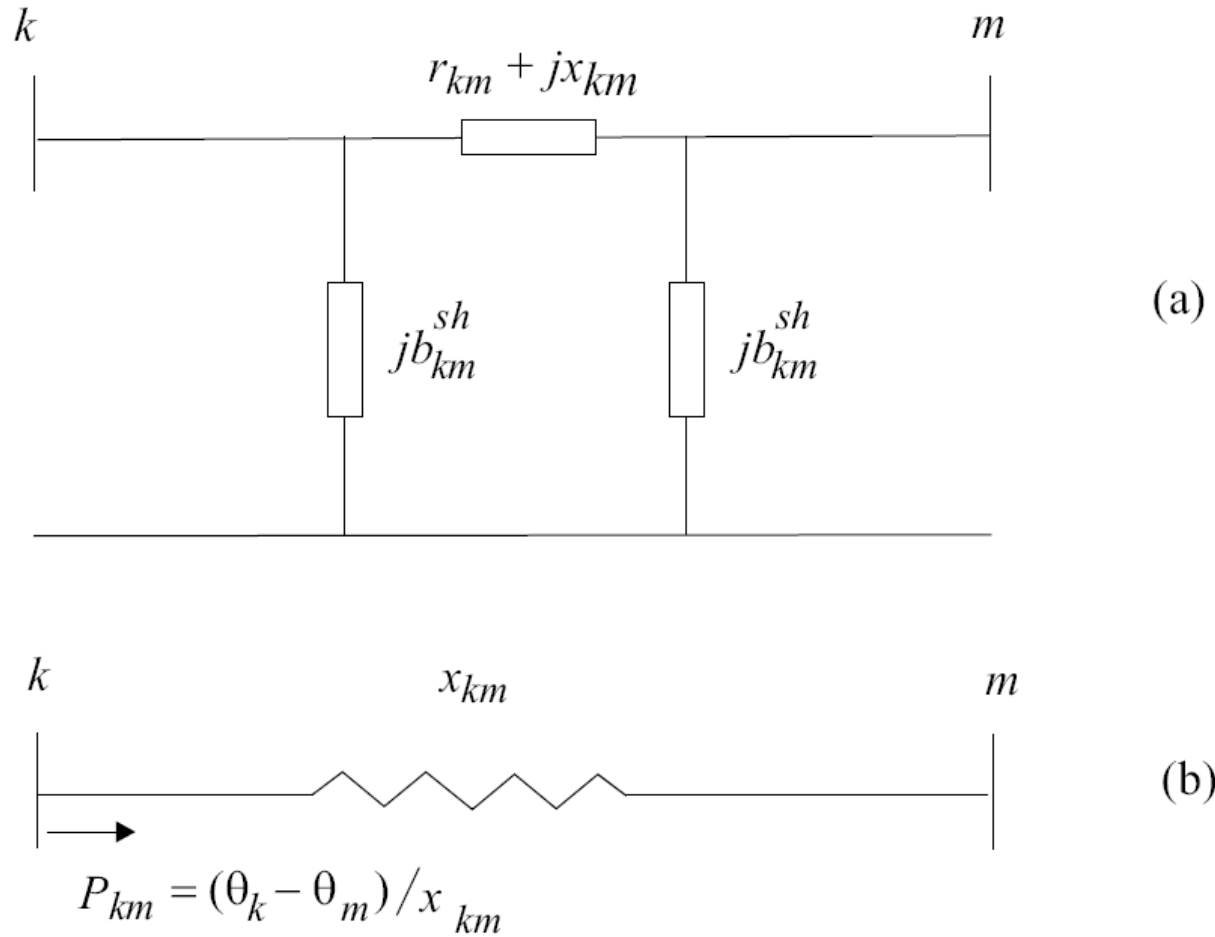


Figure 6.4. Transmission line. (a) Equivalent π -model. (b) DC power flow model

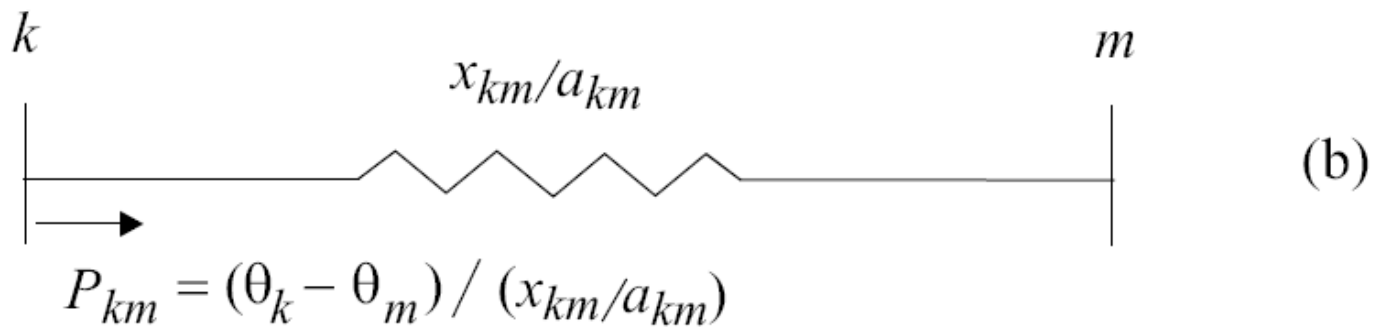
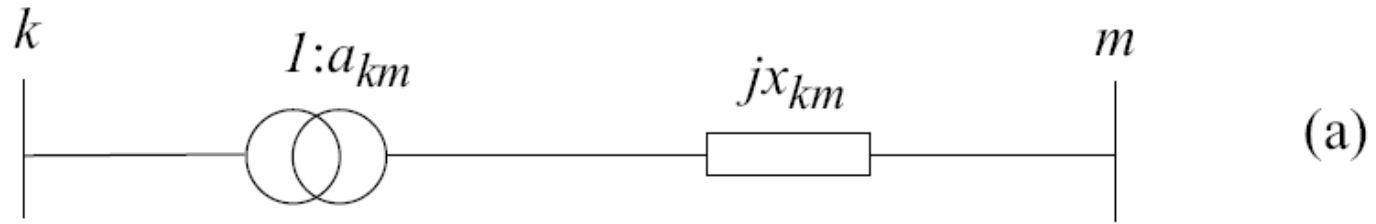


Figure 6.5. In-phase transformer. (a) Transformer comprising ideal transformer and series reactance. (b) DC power flow model

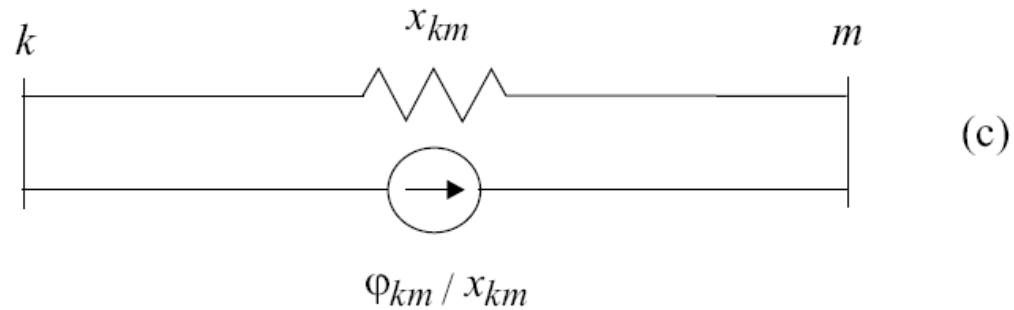
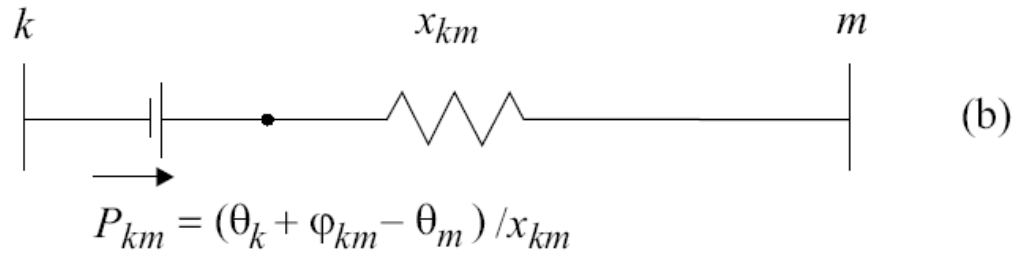
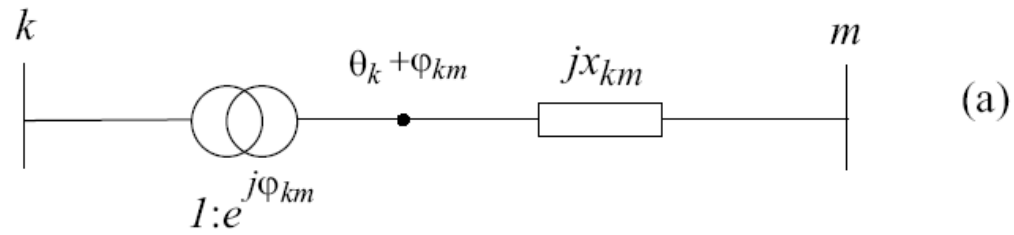


Figure 6.6. Phase-shifting transformer. (a) Phase-shifting transformer model (b) Thévenin dc power flow model. (c) Norton dc power flow model

Linearized Power Flow Equations (1)

In the linearized power flow formulation only active power flows are considered.

The active power flow injection at bus k :

$$P_k = \sum_{m \in \Omega_k} x_{km}^{-1} \theta_{km} = \left(\sum_{m \in \Omega_k} x_{km}^{-1} \right) \theta_k + \sum_{m \in \Omega_k} (-x_{km}^{-1} \theta_m) \quad (6.63)$$

(In (6.61) it is assumed that only lines constitute the connections. Otherwise it must be modified.)

Linearized Power Flow Equations (2)

The linearized power flow equations in matrix form:

$$\mathbf{P} = \mathbf{B}'\theta \quad (6.64)$$

where

- \mathbf{P} is the vector of the net injections P_k
- \mathbf{B}' is the nodal admittance matrix with the following elements:

$$B'_{km} = -x_{km}^{-1}$$

$$B'_{kk} = \sum_{m \in \Omega_k} x_{km}^{-1}$$

- θ is the vector of voltage angles θ_k

Linearized Power Flow Equations (3)

Problem:

The matrix B' is singular (Why?)

Solution:

Remove the equation for the slack node. (Angle reference.)

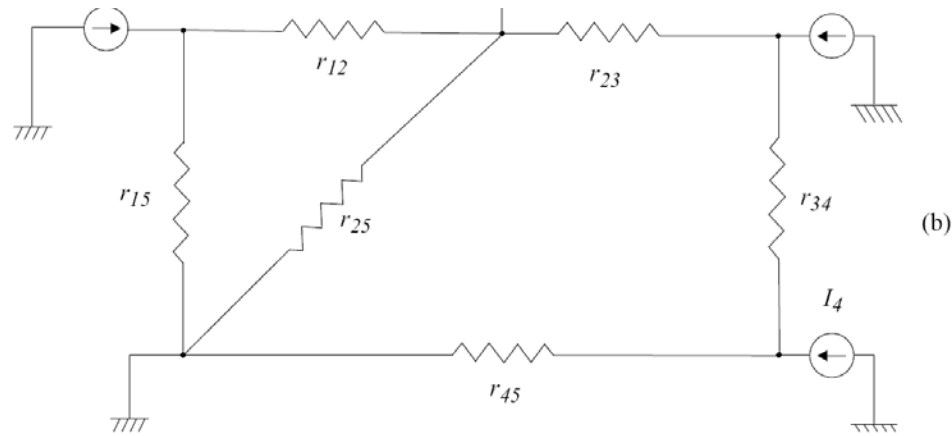
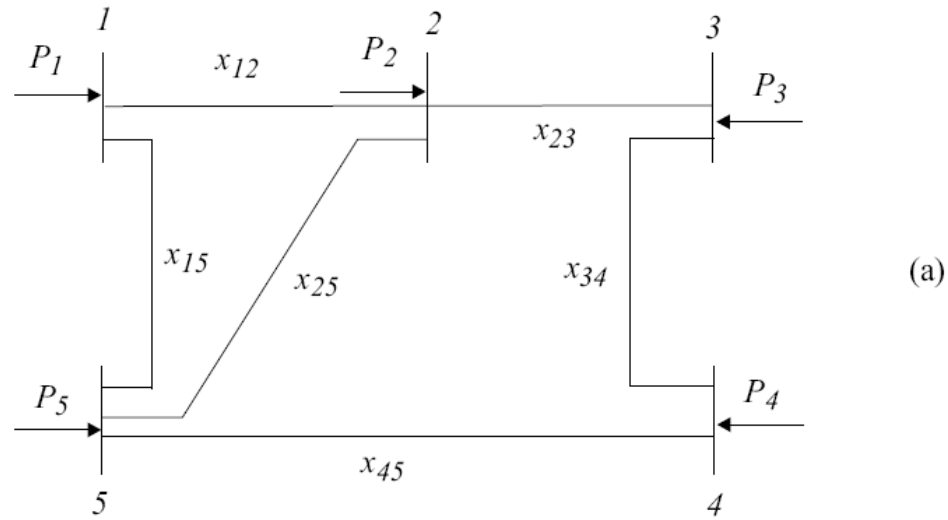
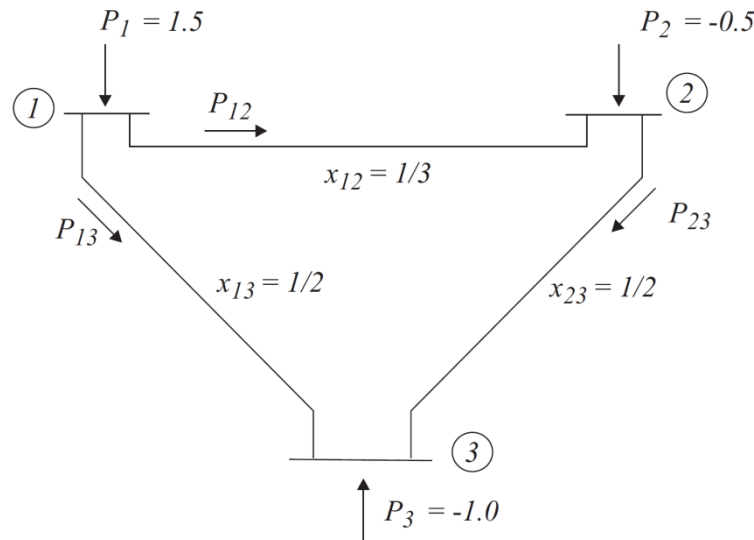


Figure 6.8. 6-bus network. (a) power network. (b) dc power flow model.

Example 6.2. Consider the network given in Figure 6.7 in which the reference angle is $\theta_1 = 0$. Use the dc power flow method to calculate the power flows in the lines.



Solution In this case, the elements of the matrix \mathbf{B}' are calculated as

$$B_{22} = x_{21}^{-1} + x_{23}^{-1} = (1/3)^{-1} + (1/2)^{-1} = 5$$

$$B_{23} = -x_{23}^{-1} = -(1/2)^{-1} = -2$$

$$B_{32} = -x_{32}^{-1} = -(1/2)^{-1} = -2$$

$$B_{33} = x_{31}^{-1} + x_{32}^{-1} = (1/2)^{-1} + (1/2)^{-1} = 4$$

and thus

$$\mathbf{B}' = \begin{pmatrix} 5 & -2 \\ -2 & 4 \end{pmatrix}$$

and

$$(\mathbf{B}')^{-1} = \begin{pmatrix} 1/4 & 1/8 \\ 1/8 & 5/16 \end{pmatrix}$$

The nodal voltage angles (in radians) can now easily be calculated

$$\begin{aligned} \theta &= \begin{pmatrix} \theta_2 \\ \theta_3 \end{pmatrix} = (\mathbf{B}')^{-1} \mathbf{P} \\ &= \begin{pmatrix} 1/4 & 1/8 \\ 1/8 & 5/16 \end{pmatrix} \begin{pmatrix} -0.5 \\ -1.0 \end{pmatrix} = \begin{pmatrix} -0.250 \\ -0.375 \end{pmatrix} \end{aligned}$$

The power flows in the transmission lines are according to the dc power flow model

$$P_{12} = x_{12}^{-1} \theta_{12} = 3 \cdot 0.25 = 0.75 \text{ p.u.}$$

$$P_{13} = x_{13}^{-1} \theta_{13} = 2 \cdot 0.375 = 0.75 \text{ p.u.}$$

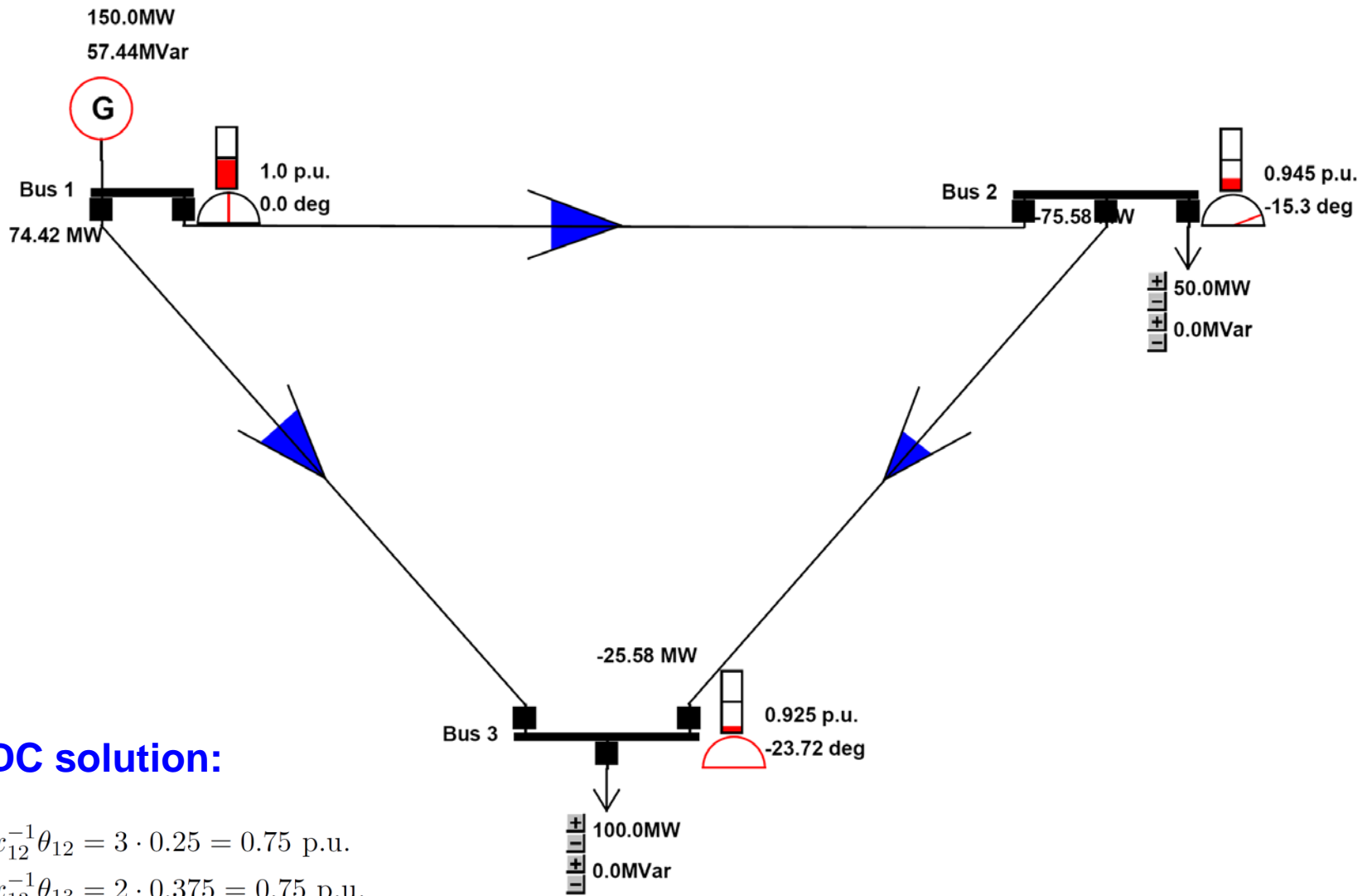
$$P_{23} = x_{23}^{-1} \theta_{23} = 2 \cdot 0.125 = 0.25 \text{ p.u.}$$

Comparison with “exact” solution

DC solution : $\theta_2 = -14.3^\circ$; $\theta_3 = -21.5^\circ$

Exact solution : $\theta_2 = -15.3^\circ$; $\theta_3 = -23.7^\circ$

Exact solution



DC solution:

$$P_{12} = x_{12}^{-1} \theta_{12} = 3 \cdot 0.25 = 0.75 \text{ p.u.}$$

$$P_{13} = x_{13}^{-1} \theta_{13} = 2 \cdot 0.375 = 0.75 \text{ p.u.}$$

$$P_{23} = x_{23}^{-1} \theta_{23} = 2 \cdot 0.125 = 0.25 \text{ p.u.}$$

Extensions

Inclusion of phase shifting transformers:

$$\mathbf{P} = \mathbf{B}'\theta - \mathbf{P}_{\text{pst}} \quad (6.65)$$

Calculation of losses?

What about reactive power?