Solution: Options

http://www.hoadley.net/options/
http://www.eeh.ee.ethz.ch/en/power/power-systems-laboratory/services

1. Option valuation using Black / Scholes

a) Valuate the following call option with the following properties:

- Type: European
- Strike price: 40
- Price of the underlying: 40 (at-the-money)
- Time to expiration: 90 days
- Volatility: 70%
- Interest rate: 0

- Determine the Delta, Gamma, Vega and Theta!
- Determine the change of Delta, Gamma, Vega and Theta as a function of the price of the underlying.
- How does the option price change as a function of the volatility and the time to expiration?

Delta, gamma and theta:

![Option Price Chart](image-url)
Delta, Gamma, Vega and Theta as a function of the price of the underlying:

**Delta:**

![Call Option Delta by Stock Price](image)

**Gamma:**

![Call Option Gamma by Stock Price](image)

**Vega:**

![Call Option Vega by Stock Price](image)

**Theta:**

![Call Option Theta by Stock Price](image)

The option price as a function of volatility:

![Call Option Price & Time Value by Volatility](image)

The option price as a function of the time to expiration:

![Call Option Price & Time Remaining to Expiry](image)
b) Valuate the following call option, which is deep out-of-the-money:

<table>
<thead>
<tr>
<th>Type</th>
<th>European</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strike price</td>
<td>100</td>
</tr>
<tr>
<td>Price of the underlying</td>
<td>40</td>
</tr>
<tr>
<td>Time to expiration</td>
<td>90 days</td>
</tr>
<tr>
<td>Volatility</td>
<td>30 %</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0</td>
</tr>
</tbody>
</table>

- What's the value of the option if the time to expiration is 1 year/10 years?

- What is the probability that the option is in-the-money at expiration for each of the three times to expiration?

90 days to the expiration:

The probability that the option is in-the-money at expiration: 0 %
Time to expiration 1 year:

The probability that the option is in-the-money at expiration: 0.1%

10 years to the expiration:

The probability that the option is in-the-money at expiration: 7.4%
c) Valuate the following put option:

Type: European
Strike price: 40
Price of the underlying: 41
Time to expiration: 90 days
Volatility: 70%
Interest rate: 0

- Determine the Delta, Gamma, Vega and Theta!
- Determine the change of Delta, Gamma, Vega and Theta as a function of the price of the underlying.
- How does the option price change as a function of the volatility and the time to expiration?

Delta, gamma, vega and theta:
Delta, Gamma, Vega and Theta as a function of the price of the underlying:

**Delta:**

![Put Option Delta by Stock Price Graph](image)

**Gamma:**

![Put Option Gamma by Stock Price Graph](image)

**Vega:**

![Put Option Vega by Stock Price Graph](image)

**Theta:**

![Put Option Theta by Stock Price Graph](image)

The option price as a function of volatility:

![Put Option Price & Time Value by Volatility Graph](image)

The option price as a function of the time to expiration:

![Put Option Price & Time Value by Time Remaining to Expiry Graph](image)
2. Binomial trees

a) Valuate the option from exercise 1a) using the binomial model. Use 6 discrete time steps.

b) How does the option price change with an increasing number of time steps? Use 15, 30, 45 and 90 discrete time steps.

c) Compare them with the value from the Black-Scholes formula!

Binomial model:

- 6 discrete time steps: 5.2948
- 15 discrete time steps: 5.6098
- 30 discrete time steps: 5.4732
- 45 discrete time steps: 5.5491
- 90 discrete time steps: 5.5037
3. Complex contract I

a) Solution: - short put (X = 28) plus bull spread (X1, X2) that consists of
   - long call (X1 = 30) and short call (X2 = 32) or
   - long put (X1 = 30) and short put (X2 = 32)

Another possible solution:
   - long fence (X1, X2) plus short call (X=32)
   - long fence consists of a short put (X1 = 28) and long call (X2 = 30)

b) The buyer expects increasing prices; he expects that prices are between 28 - 32; prices from 30 to 28 are hedged, below 28 the prices are not hedged anymore, i.e. a decreasing volatility is expected.

4. Complex contract II

a) - Underlying plus
   - long call with the strike at 30
   - 2 short calls with the strike at 35

b) The buyer pays a premium for the long call option and gets a premium from 2 short call options. The premium of the short call is smaller than the premium of the long call option, but still over 50 % from the premium of the long call. From the premiums, the buyer makes a profit: Profit = 2 short call premiums – 1 long call premium.

The buyer expects slightly increasing prices, and in any case he does not expect falling prices.