Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Operation, Monitoring and Control Technology of Power Systems

Course 227-0528-00

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Contents

- Power Systems Modeling
- Power Flow Computations
- Voltage Stability Assessment

Components Modeling

Component	Quantity	Expression
Load		
$..P = const$	$P = P_0$	$P_l = P_0$
	$Q = Q_0$	$Q_l = Q_0$
$..I = const$	$P(V)$	$P_l = \Re(\underline{I}_0^* \underline{V})$
	$Q(V)$	$Q_l = \Im(\underline{I}_0^* \underline{V})$
$..Z = const$	$P(V)$	$P_l = rV^2/(r^2 + x^2)$
	$Q(V)$	$Q_l = xV^2/(r^2 + x^2)$
Generator	P	P_g
	Q	Q_g according to maximal heating
Line	flow, P_{ij}	$P_{ij} = V_i^2(g_{ij}^{sh} + g_{ij}) - V_i V_j (g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij})$
	flow, Q_{ij}	$Q_{ij} = -V_i^2(b_{ij}^{sh} + b_{ij}) - V_i V_j (g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij})$
	losses, P_w	$P_w = 3rI^2 = r[P_{ij}^2 + (Q_{ij} + b_{ij}^{sh} V_i^2)^2]/V_i^2$
	losses, Q_w	$Q_w = 3xI^2 = x[P_{ij}^2 + (Q_{ij} + b_{ij}^{sh} V_i^2)^2]/V_i^2$
Transformer	flow, P_{ij}	$P_{ij} = a^2 V_i^2 g_{ij} - aV_i V_j (g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij})$
	flow, Q_{ij}	$Q_{ij} = -a^2 V_i^2 b_{ij} - aV_i V_j (g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij})$
Shunt	P_{sh}	losses are neglected
	Q_{sh}	$Q = V_i^2 b_i^{sh}$

Per-unit System

Quantity	Notation	Value
Base Voltage	V_b	phase-to-phase voltage, kV
Base Power	S_b	three-phase power, MVA
Base Current	$I_b = S_b/(V_b\sqrt{3})$	current/phase, kA
Base Impedance	$Z_b = V_b^2/S_b$	impedance, Ohm

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Power Flow

- Exploration how a change of some parameters or variables would affect the system state (i.e. modify the flow pattern and voltages)
- Parameters and variables change:
  - Different topology (e.g. line outage)
  - Load increase (compensated by generation increase)
  - Generation outage (compensated by generation increase)
- From mathematical point of view it is just solving a set of nonlinear equations => iterative solution techniques, e.g: Newton – Raphson etc.

Power Flow

- General form: $$0 = g(x,u)$$

- Power Flow (in each bus *i*):

$$0 = P_{Gi} - P_{Di} - V_i \sum_{k=1}^{N} V_k \left[G_{ik} \cos(\delta_{ik}) + B_{ik} \sin(\delta_{ik}) \right]$$

$$0 = Q_{Gi} - Q_{Di} - V_i \sum_{k=1}^{N} V_k \left[G_{ik} \sin(\delta_{ik}) - B_{ik} \cos(\delta_{ik}) \right]$$

  - States *x*: δ, ***V***
    - => having states x and the system topology, we can compute all flows
  - Controls *u*:
    - Slack bus: δ, ***V***
    - PV bus: ***V***, ***P***
    - PQ bus: ***P***, ***Q***

Power Flow – Iteration Procedure

- Set *j = 0*
- Choose initial values of voltages and phase angles
- Construct Y_{bus} matrix
- Compute bus generation/consumption:

$$P_{GDi} = P_{Gi} - P_{Di}$$

$$Q_{GDi} = Q_{Gi} - Q_{Di}$$

1. Update iteration number *j = j + 1*
2. Calculate injected powers:

$$P_i = V_i \sum_{k=1}^{N} V_k \left[G_{ik} \cos(\delta_{ik}) + B_{ik} \sin(\delta_{ik}) \right]$$

$$Q_i = V_i \sum_{k=1}^{N} V_k \left[G_{ik} \sin(\delta_{ik}) - B_{ik} \cos(\delta_{ik}) \right]$$

Power Flow – Iteration Procedure

3. Compute node power balance/deviation:

$$\Delta P_i = P_{GDi} - P_i$$
$$\Delta Q_i = Q_{GDi} - Q_i$$

4. Construct Jacobian matrix

5. Compute

$$\begin{bmatrix} \Delta\delta \\ \Delta V / V \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

6. Update voltage angle and magnitude values

$$\delta^{j+1} = \delta^{j} + \Delta\delta^{j+1}$$
$$V^{j+1} = V^{j}\left(1 + \Delta V^{j+1} / V^{j}\right)$$

7. Stop if result has converged: || ΔV^{j+1} || < *tolerance*, otherwise go to 1

Power Flow – Jacobian Matrix

$$\begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_N} & \frac{\partial P_1}{\partial V_1} & \cdots & \frac{\partial P_1}{\partial V_N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_N}{\partial \delta_N} & \cdots & \frac{\partial P_N}{\partial \delta_N} & \frac{\partial P_N}{\partial V_1} & \cdots & \frac{\partial P_N}{\partial V_N} \\ \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_N} & \frac{\partial Q_1}{\partial V_1} & \cdots & \frac{\partial Q_1}{\partial V_N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_N}{\partial \delta_1} & \cdots & \frac{\partial Q_N}{\partial \delta_N} & \frac{\partial Q_N}{\partial V_1} & \cdots & \frac{\partial Q_N}{\partial V_N} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \vdots \\ \vdots \\ \Delta\delta_N \\ \Delta V_1 \\ \vdots \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} \Delta P_1 \\ \vdots \\ \vdots \\ \Delta P_N \\ \Delta Q_1 \\ \vdots \\ \vdots \\ \Delta Q_N \end{bmatrix}$$

- Off-diagonal elements:

$$\frac{\partial P_i}{\partial \delta_k} = V_i V_k \left(G_{ik} \sin(\delta_{ik}) - B_{ik} \cos(\delta_{ik})\right) \qquad V_k \frac{\partial P_i}{\partial V_k} = V_i V_k \left(G_{ik} \cos(\delta_{ik}) + B_{ik} \sin(\delta_{ik})\right)$$

$$\frac{\partial Q_i}{\partial \delta_k} = -V_i V_k \left(G_{ik} \cos(\delta_{ik}) + B_{ik} \sin(\delta_{ik})\right) \qquad V_k \frac{\partial Q_i}{\partial V_k} = V_i V_k \left(G_{ik} \sin(\delta_{ik}) - B_{ik} \cos(\delta_{ik})\right)$$

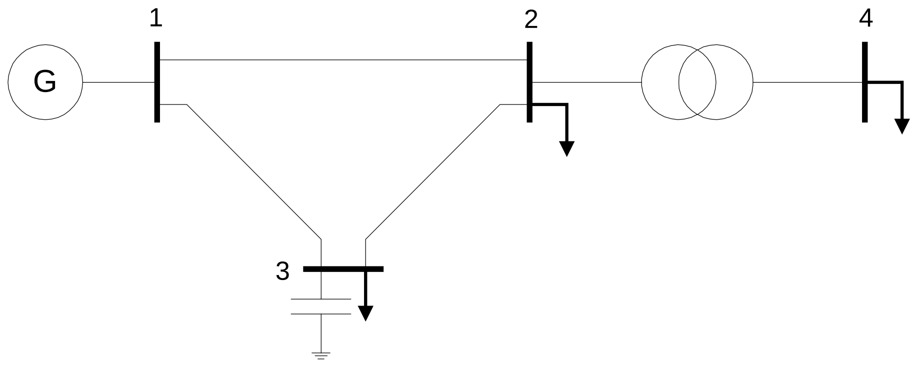
- Diagonal elements:

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii} V_i^2 \qquad \frac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii} V_i^2 \qquad V_i \frac{\partial P_i}{\partial V_i} = P_i + G_{ii} V_i^2 \qquad V_k \frac{\partial Q_i}{\partial V_i} = Q_i - B_{ii} V_i^2$$

Example – Test System

- A. Abur and A. G. Exposito, *“Power System State Estimation,”* Marcel Dekker, 2004.
- Reference voltage in bus 1 is 1.00 pu
- Capacitor susceptance is 0.5 pu

Bus	Pg [pu]	Pl [pu]	Ql [pu]
1	2.00		
2		0.50	0.30
3		1.20	0.80
4		0.25	0.10

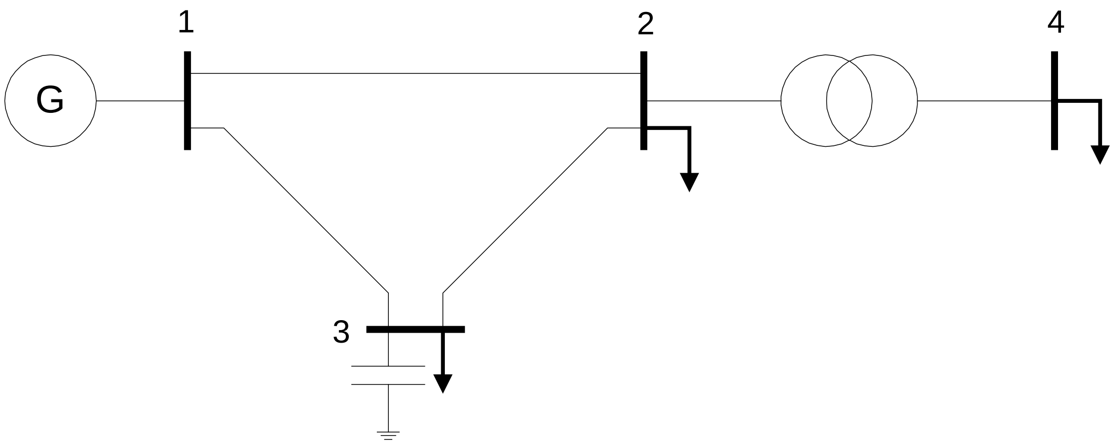


From	To	R [pu]	X [pu]	B_s [pu]	Tap
1	2	0.02	0.06	0.20	
1	3	0.02	0.06	0.25	
2	3	0.05	0.10	0.00	
2	4	0.00	0.08	0.00	0.98

Example – Power Flow Problem

- Slack/Swing bus: 1
- PV bus:
- PQ buses: 2, 3, 4

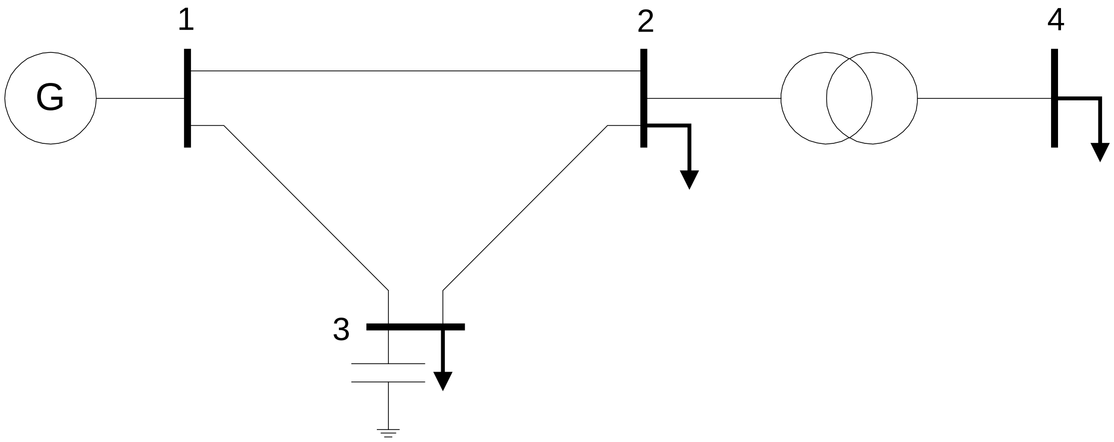
Bus	δ [rad]	V [pu]
1	0	1.0000
2	?	?
3	?	?
4	?	?



Example – Power Flow Results

Bus	δ [rad]	V [pu]
1	0	1.0000
2	-0.0482	0.9629
3	-0.0624	0.9597
4	-0.0691	0.9742

From	To	P_{ft} [pu]	Q_{ft} [pu]	P_{tf} [pu]	Q_{tf} [pu]
1	2	0.8864	0.2406	-0.8684	-0.3793
1	3	1.1092	0.2083	-1.0824	-0.3679
2	3	0.1184	-0.0269	-0.1176	0.0284
2	4	0.2500	-0.1353	-0.2500	0.1421



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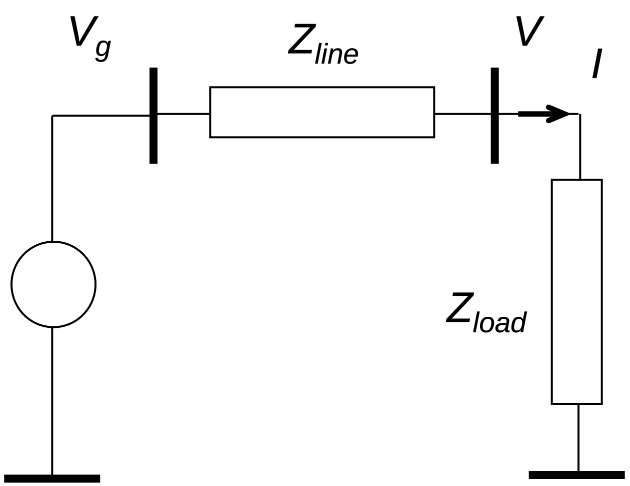
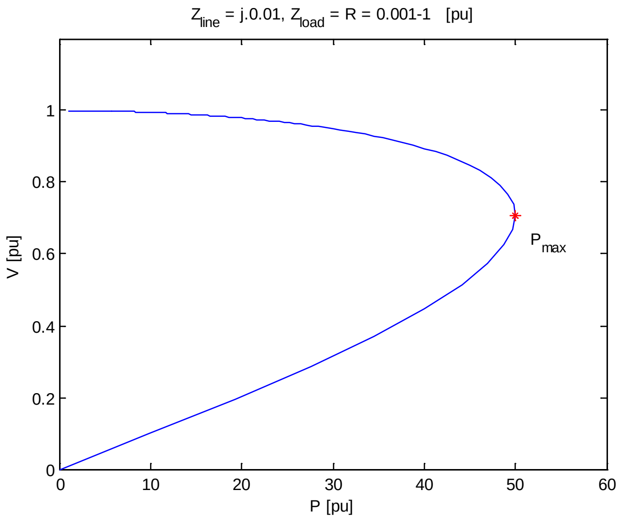
Voltage Instability – Principle

$$V = \left|\overline{V}\right| = \left|\overline{I}.\overline{Z}_{load}\right| = \left|\frac{\overline{V}_g.\overline{Z}_{load}}{\overline{Z}_{line} + \overline{Z}_{load}}\right|$$

$$P = real\left\{\overline{Z}_{load}.\left|I\right|^2\right\} = real\left\{\overline{Z}_{load}.\left|\frac{\overline{V}_g}{\overline{Z}_{line} + \overline{Z}_{load}}\right|^2\right\}$$

- PV ("nose") curves used for graphical interpretation
- increase of the loading (decrease of the impedance) causes the nonlinear voltage drop
- maximal possible power transfer to the load P_{max}
- impedance load => ok
- constant power => collapse may occur !





Voltage Instability – Principle

$$V = \left|\overline{V}\right| = \left|\overline{I}.\overline{Z}_{load}\right| = \left|\frac{\overline{V}_g.\overline{Z}_{load}}{\overline{Z}_{line} + \overline{Z}_{load}}\right|$$

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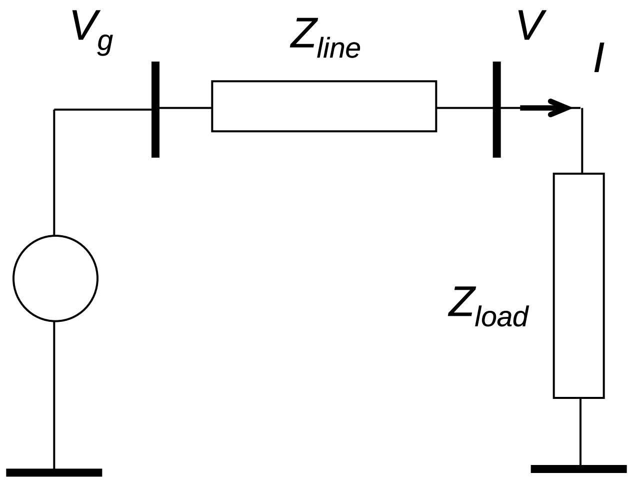
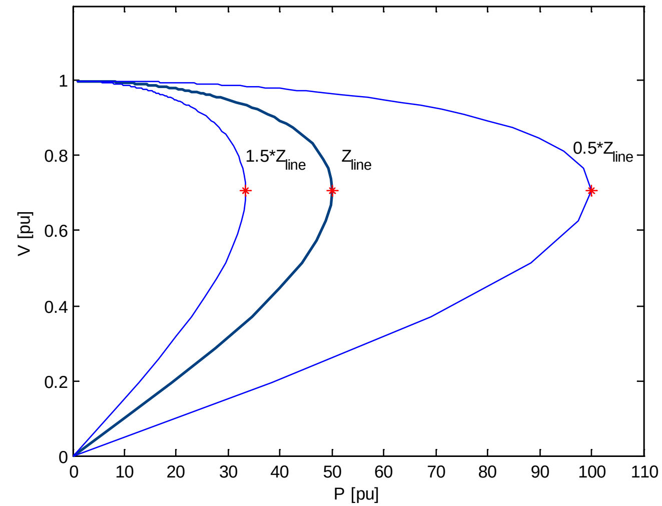
- losses of the reactive power in the lines are decisive => impact of the line impedance on the maximum loadability (P_{max}) is large
- voltage collapse much more probable in the systems with the long electrical distances (reactances of the lines)

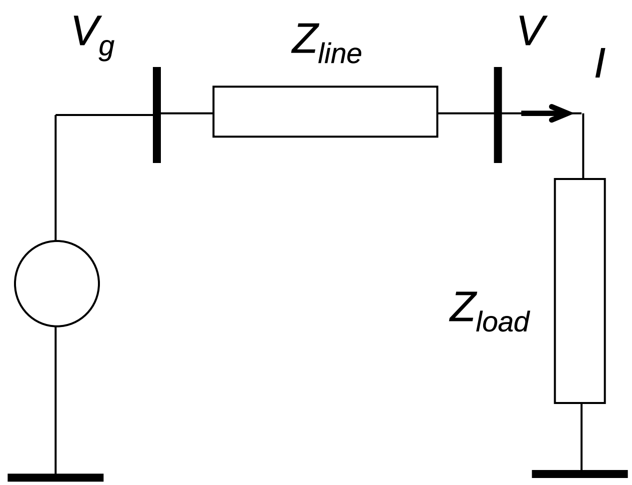




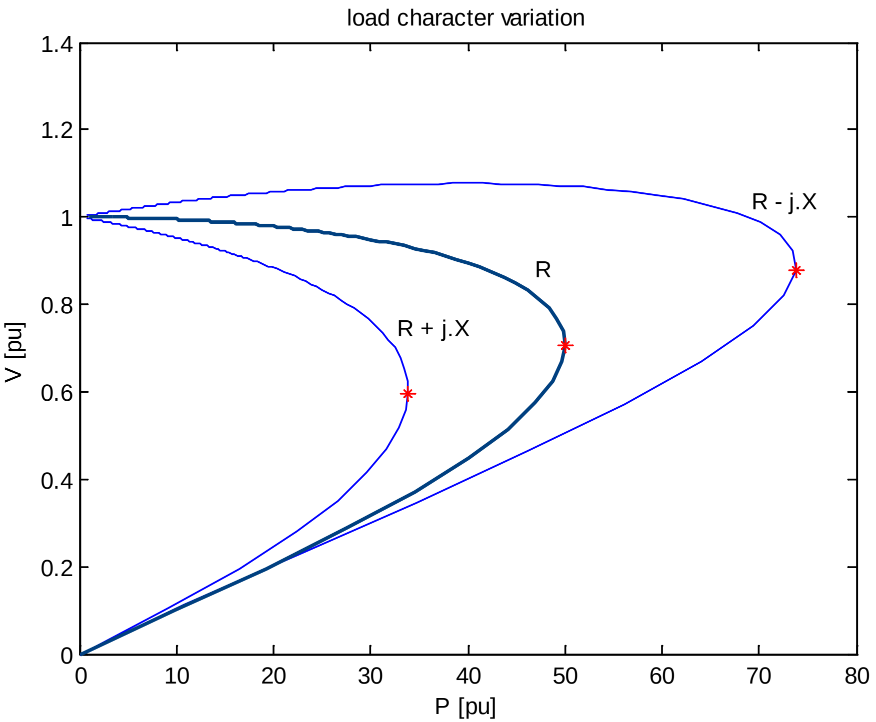
Voltage Instability – Principle

$$V = \left|\overline{V}\right| = \left|\overline{I}.\overline{Z}_{load}\right| = \left|\frac{\overline{V}_g.\overline{Z}_{load}}{\overline{Z}_{line}+\overline{Z}_{load}}\right|$$

$$P = real\left\{\overline{Z}_{load}.\left|I\right|^2\right\} = real\left\{\overline{Z}_{load}.\left|\frac{\overline{V}_g}{\overline{Z}_{line}+\overline{Z}_{load}}\right|^2\right\}$$

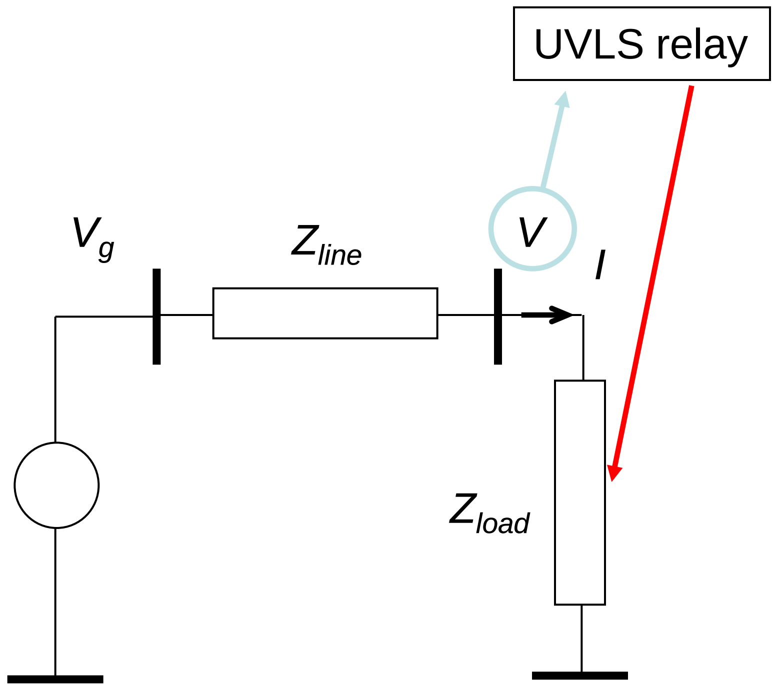


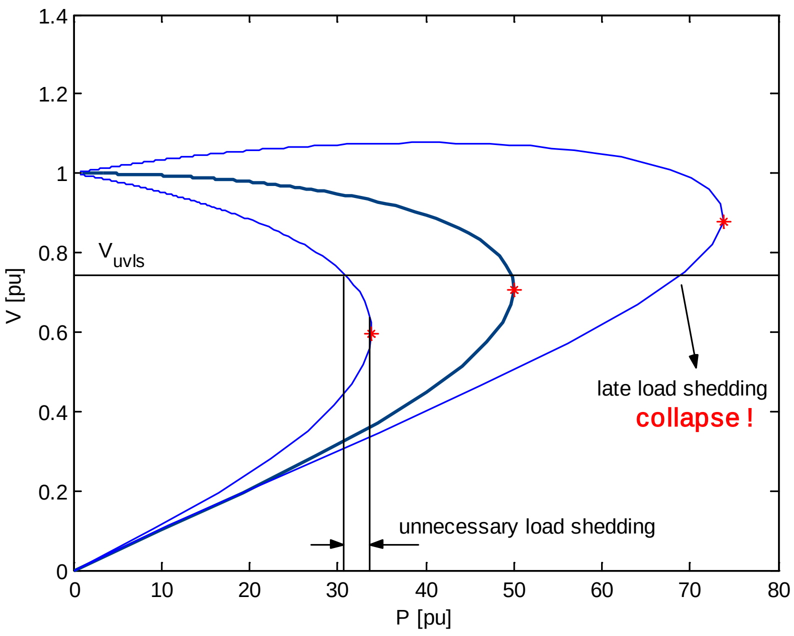
- capacitive load => higher loadability
- inductive load => lower loadability



Voltage Instability – Local Protection

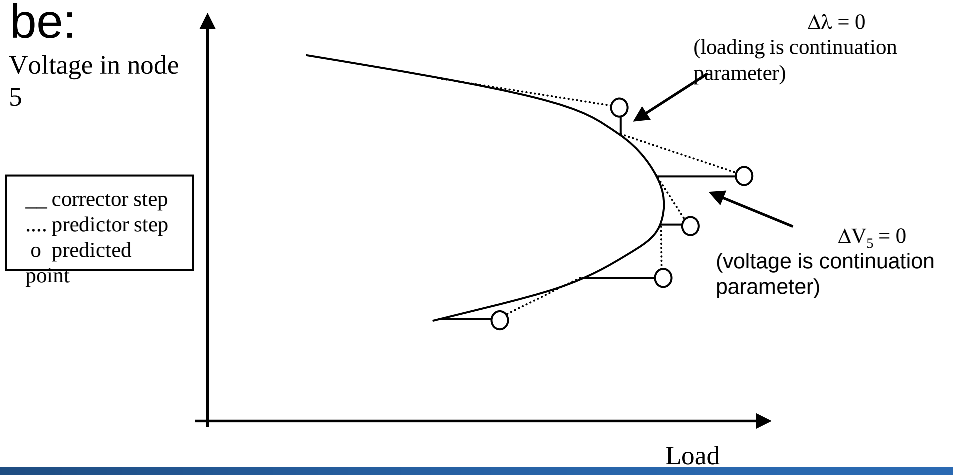
- Under Voltage Load Shedding (UVLS) relay
- disconnection of the part of the load when the voltage in the node drops under the trigger value





Voltage Instability – Continuation Power Flow

- Goal is to avoid numerical problems when approaching instability point in computation of loadability curves (i.e. PV – curves)
- PV – curve tracking is achieved via computation of points in two stages:
  - Predictor
  - Corrector
- Numerical difficulties are avoided by switching **continuation parameter** when moving from point to point
- Continuation parameter can be:
  - Loading factor λ
  - Voltage magnitude



Voltage Instability – Continuation Power Flow – Corrector

- Introduction of the loading factor λ:

$$P_{GDi} = P_{Gio} + \lambda.k_{Gi}.P_{Gio} - P_{Dio} - \lambda.k_{Li}.S_{dbase}.\cos\psi_i$$

$$Q_{GDi} = Q_{Gio} - Q_{Dio} - \lambda.k_{Li}.S_{dbase}.\sin\psi_i$$

- Traditional Power Flow formulation is augmented by the additional equation and variable:

$$\begin{bmatrix} J_1 & J_2 & J_5 \\ J_3 & J_4 & J_6 \\ & J_7 & \end{bmatrix} . \begin{bmatrix} \Delta\delta \\ \Delta V \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} \Delta P \\ \Delta Q \\ 0 \end{bmatrix}$$

$$J_5 : \frac{\partial P_i}{\partial \lambda} = k_{Gi}.P_{Gio} - k_{Li}.S_{dbase}.\cos\psi_i$$

$$J_6 : \frac{\partial Q_i}{\partial \lambda} = -k_{Li}.S_{dbase}.\sin\psi_i$$