

# Stability Assessment and Emergency Control Method Using Trajectory Sensitivities

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**Abstract**—Very stressed power systems often lack ability to withstand certain disturbances, what might result in serious consequences. Therefore the tools mitigating such a situation have a crucial significance. This paper presents a method for real-time emergency control of power systems based on the trajectory sensitivities. Out of the present conditions a nominal trajectory and its sensitivities is pre-calculated. If an incipient instability is detected, necessary stabilizing control measures are determined. A short review of trajectory sensitivities and Model Predictive Control is included. A principal description of the method together with a theoretical foundation and examples with simulation results is provided.

**Index Terms**—trajectory sensitivities, power system dynamic stability, power system protection, power system monitoring, power system control, system protection scheme, predictive control

## I. INTRODUCTION

Extensions of transmission systems have been kept to the minimum in the last decade by various reasons. On the other hand, the electricity demand continuously grows, thus also the amount of transmitted power. Some countries have undergone the liberalization of the electricity market, which introduces significant hourly variations of the generation and transmission pattern. This results in larger stress of transmission systems that are operated much closer to their stability boundaries than ever. This creates a demand for tools allowing very accurate monitoring of power systems stability. This challenging task is a subject of ongoing research activities [3], which have adopted two approaches - preventive and emergency.

Trajectory sensitivities have a potential for both preventive and emergency control. Trajectory sensitivities provide an insight into the behavior of a dynamic system, which would not be otherwise obvious only from its nominal trajectory [1], [2], [4]. An impact of initial conditions and/or parameters on the system trajectory can be analyzed [4]. These capabilities of trajectory sensitivities have been used so far mainly for post-mortem analysis [2]. This paper shows how trajectory

sensitivities could be applied on real-time emergency control. The paper is structured as follows. First a short overview of the trajectory sensitivities concept is provided followed by introduction and explanation of Model Predictive Control basics. The next part describes the proposed method itself on the theoretical basis. The next section presents simulation examples.

## II. TRAJECTORY SENSITIVITY ANALYSIS

Power systems dynamics can be modeled, taking into account their hybrid nature (combination of continuous and discrete dynamics), as follows:

$$\begin{aligned} \dot{\underline{x}} &= \underline{f}(\underline{x}, y) \\ 0 &= \underline{g}^{(0)}(\underline{x}, y) \\ 0 &= \begin{cases} \underline{g}^{(i-)}(\underline{x}, y) & y_{d,i} < 0 \quad i=1, \dots, d \\ \underline{g}^{(i+)}(\underline{x}, y) & y_{d,i} > 0 \end{cases} \\ \underline{x}^+ &= \underline{h}_j(\underline{x}^-, y^-) \quad y_{e,j} = 0 \quad j \in \{1, \dots, e\} \end{aligned} \quad (1)$$

with the vectors

$$\underline{x} = \begin{bmatrix} x \\ z \\ \lambda \end{bmatrix} \quad \underline{f} = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix} \quad \underline{h}_j = \begin{bmatrix} x \\ h_j \\ \lambda \end{bmatrix} \quad (2)$$

where  $x$  are the dynamic states (generators angles, velocities, fluxes),  $z$  represents discrete states (e.g. tap positions),  $\lambda$  parameters (for example line impedances) and  $y$  algebraic states (such as voltages). Switching conditions between different dynamic modes are described using the functions  $h$  and states  $y_d, y_e$ .

Flow of the system can be written:

$$\phi(\underline{x}_0, t) = \begin{bmatrix} \phi_x(\underline{x}_0, t) \\ \phi_y(\underline{x}_0, t) \end{bmatrix} = \begin{bmatrix} \underline{x}(t) \\ \underline{y}(t) \end{bmatrix} \quad (3)$$

Sensitivities of the system flow to the initial conditions and parameters are obtained by a Taylor expansion of above equation (3):

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$$\Delta \underline{x}(t) = \frac{\partial \underline{x}(t)}{\partial \underline{x}_0} \Delta \underline{x}_0 + \text{higher order terms}$$

$$\approx \underline{x}_{\underline{x}_0}(t) \Delta \underline{x}_0$$
(4)

Differentiations with respect to the initial conditions and parameters yields:

$$\dot{\underline{x}}_{\underline{x}_0} = \underline{f}_{\underline{x}}(t) \underline{x}_{\underline{x}_0} + \underline{f}_{\underline{y}}(t) \underline{y}_{\underline{x}_0}$$

$$0 = \underline{g}_{\underline{x}}(t) \underline{x}_{\underline{x}_0} + \underline{g}_{\underline{y}}(t) \underline{y}_{\underline{x}_0}$$
(5)

Initial trajectory sensitivities values are then:

$$\underline{x}_{\underline{x}_0}(t_0) = I$$

$$\underline{y}_{\underline{x}_0}(t_0) = -(\underline{g}_{\underline{y}}(t_0))^{-1} \underline{g}_{\underline{x}}(t_0)$$
(6)

Applying the trapezoidal integration method, the numerical expression for computation of the trajectory sensitivities in the time instant  $k+1$  is:

$$\begin{bmatrix} \underline{x}_{\underline{x}_0}^{k+1} \\ \underline{y}_{\underline{x}_0}^{k+1} \end{bmatrix} = \begin{bmatrix} \frac{\eta}{2} \underline{f}_{\underline{x}}^{k+1} - I & \frac{\eta}{2} \underline{f}_{\underline{y}}^{k+1} \\ \underline{g}_{\underline{x}}^{k+1} & \underline{g}_{\underline{y}}^{k+1} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\eta}{2} (\underline{f}_{\underline{x}}^k \underline{x}_{\underline{x}_0}^k + \underline{f}_{\underline{y}}^k \underline{y}_{\underline{x}_0}^k) - \underline{x}_{\underline{x}_0}^k \\ 0 \end{bmatrix}$$
(7)

where  $\eta$  is the integration time step. More details about derivation and possible numerical solution approaches for trajectory sensitivities can be found in [1].

### III. MODEL PREDICTIVE CONTROL

Various definitions of Model Predictive Control (MPC) can be found in literature. However, the main principle is usually the same and can be explained with the help of figure 1 and the Finite Horizon Constrained Linear Programming (LP) formulation (8) with the cost function based on 1/infinity - norm:

$$\min_U \|P(x_{ref}(N) - x(N))\|_p + \sum_{k=0}^{N-1} \|Q(x_{ref}(k) - x(k))\|_p + \|R u(k)\|_p$$

$$s.t. \quad x(k+1) = A x(k) + B u(k)$$

$$E x(k) + L u(k) \leq M$$

$$u_{\min} \leq u(k) \leq u_{\max}$$

$$x_{\min} \leq x(k) \leq x_{\max}$$

$$k = 1, \dots, N$$

$$U = \{u(0), \dots, u(k), \dots, u(N-1)\}$$
(8)

MPC intends to force the controlled system (expressed by the system states  $x$ ) to follow the desired trajectory  $x_{ref}$  by applying an optimal sequence of manipulated control inputs  $u(k)$  in the time instants/samples  $t(k)$  within the specified time horizon  $t(N)$ . Both the system states values as well as control inputs

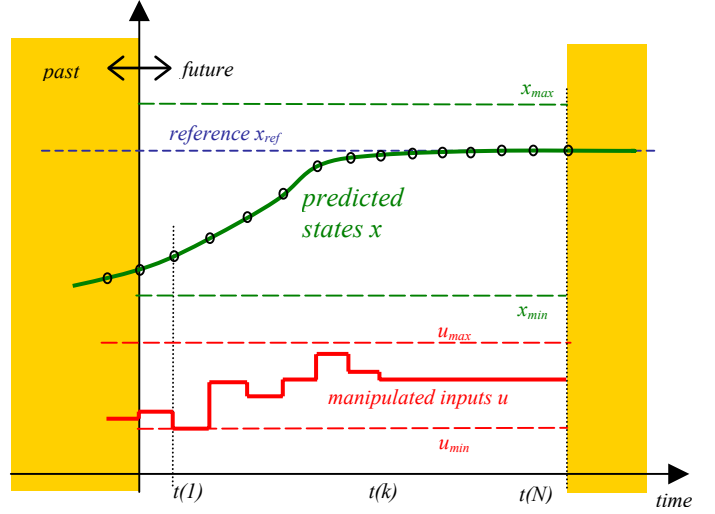
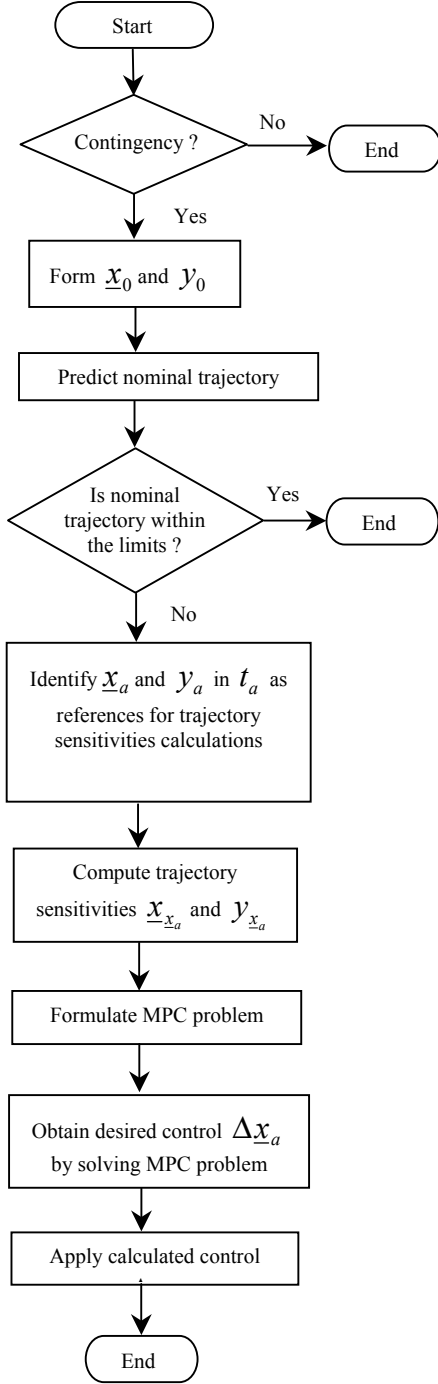


Fig. 1. Model Predictive Control principle.

can be subjected to the inequality or/and equality constraints. To capture the system dynamics and predict the system response to the control inputs, the model of the system is needed; here it is accomplished by introducing the equality constraints. The first term in the above cost function (8) penalizes the deviation of the system trajectory from the reference value in the end of the prediction horizon whereas the second term in all other samples/points. The third term is the penalty serving to keep the employment/engagement of the control to the minimal necessary level. The weight matrices/vectors P and Q express the importance of the close tracking of the reference for various states. The weight matrix R can be used to define the preferred controls. The overall controller performance is tuned using P, Q and R, e.g. accuracy, “aggressiveness” etc. Please note that the magnitude of the elements values of P, Q and R does not matter but their mutual relations/ratios do.

### IV. PROPOSED CONTROL METHOD THEORY

The structure of the method we propose is shown in flowchart 1. A contingency can be quickly detected and the state of the system accurately estimated as described in [6]. The data coming from the state estimator in this case are voltage and current phasors, which are processed to obtain the initial states  $\underline{x}_0$  and  $\underline{y}_0$ . Having them and the model description, the nominal trajectory of the system is predicted according to the equations mentioned in section II. Referring to figure 2, the contingency is detected at time  $t_c$ . To capture the dynamics of the system and especially its transition between different discrete states, the prediction takes the values one step back as the initial state. Since the calculation process together with the execution of a corrective action (if needed) take a certain time, the earliest time for applying a control will be  $t_a$  (we assume that this time delay is known). Therefore the checking if the post-fault voltage is within the acceptable band makes sense only from this time. The tolerance band can be wider in the beginning to allow for



Flowchart. 1. Flowchart outlining the principle of the proposed method.

larger excursions of the voltage. However, the voltage has to recover to the normal operation range before a local under-voltage relay protection would act at time  $t_{invs}$ . If the predicted nominal voltage is not within this specified range within the specified time horizon, trajectory sensitivities calculations start. Because we execute an action at time  $t_a$ , the trajectory sensitivities are computed with respect to the values expected at that time (available from nominal trajectory calculation):  $\underline{x}_a$  and  $\underline{y}_a$ . In contrast to the traditional MPC, where a sequence of control inputs is determined and only the first of

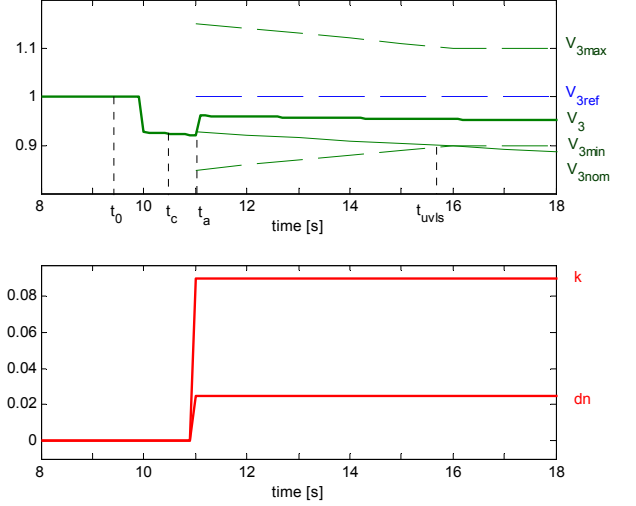


Fig. 2. The principle of TS MPC (Trajectory Sensitivities based Model Predictive Control). The dashed lines represent the boundaries, i.e. maximal and minimal acceptable voltage levels.  $k$  is load shedding factor and  $dn$  change of the tap position.

them is applied, we have adopted a more protection-oriented approach – to compute only one set of control inputs, which remains same for the whole prediction horizon. The modified version of MPC employing linear programming is then derived as follows. Since the impact of a parameter and an initial state change is expressed with help of trajectory sensitivities, a new trajectory is:

$$\begin{bmatrix} \underline{x} \\ \underline{y} \end{bmatrix} = \begin{bmatrix} \underline{x}_{nom} \\ \underline{y}_{nom} \end{bmatrix} + \begin{bmatrix} \underline{x}_{\underline{x}_a} \\ \underline{y}_{\underline{x}_a} \end{bmatrix} \Delta \underline{x}_a \quad (9)$$

Since  $\Delta \underline{x}_a$  represents our control inputs, the objective function of MPC is:

$$\min \left\{ \left\| \begin{bmatrix} \underline{x}_{ref} \\ \underline{y}_{ref} \end{bmatrix} - \left( \begin{bmatrix} \underline{x}_{nom} \\ \underline{y}_{nom} \end{bmatrix} + \begin{bmatrix} \underline{x}_{\underline{x}_a} \\ \underline{y}_{\underline{x}_a} \end{bmatrix} \Delta \underline{x}_a \right) \right\| + \|R \Delta \underline{x}_a\|_1 \right\} \quad (10)$$

where the sensitivity vector contains only the relevant entries (corresponding to the manipulated control inputs) for the whole prediction horizon (i.e. for all sample times starting at  $t_a$ ). The constraints on the system states (here voltages) are:

$$\begin{bmatrix} \underline{x}_{\underline{x}_a} \\ \underline{y}_{\underline{x}_a} \end{bmatrix} \Delta \underline{x}_a \leq \begin{bmatrix} \underline{x}_{boundary} \\ \underline{y}_{boundary} \end{bmatrix} - \begin{bmatrix} \underline{x}_{nom} \\ \underline{y}_{nom} \end{bmatrix} \quad (11)$$

and on the control variables:

$$\Delta \underline{x}_{a \min} \leq \Delta \underline{x}_a \leq \Delta \underline{x}_{a \max} \quad (12)$$

In the above equations we assumed the possibility for control inputs to vary continuously within the specified constraints.

However, this is not always the case in reality. Many available control inputs in power systems are of discrete nature, e.g. tap changers can move only in steps, load is connected through feeders in certain discrete amounts etc. Therefore the inclusion of this feature (discrete controls) in our model/control algorithm is needed. Mixed Logic Dynamic (MLD) concept has been proposed in [7] for control of hybrid systems. Although MLD is primarily intended to cover the hybrid behavior of the system itself (but in our case we captured this already in the trajectory sensitivities), the ideas can be applied here as well. The following formula is used for control variables:

$$\Delta \underline{x}_a = C \cdot \delta \quad (13)$$

where  $C$  is the diagonal matrix containing values of available controls, for the details see Appendix. Each diagonal element is actually a row vector corresponding to the discrete values of one particular control, e.g. all possible tap positions of one tap changing transformer.  $\delta$  is a column vector of auxiliary binary variables (they can be either 1 or 0) consisting of sub-vectors corresponding to the  $C$  elements. Thus the sought result of the optimization procedure becomes vector  $\delta$ , where all elements will be zero except the ones filled with one, pointing at the needed control input. To guarantee that only one control will be chosen per control object (e.g. only one out of the possible tap positions can be used), new constraints have to be added:

$$D \cdot \delta \leq E \quad (14)$$

where  $D$  is the matrix of the same structure as  $C$  but all elements in the sub-vectors equal to one.  $E$  is a column vector with all elements equal to one. The inclusion of equality constraints is necessary when there is a tight connection/relation between some controls (e.g. load shedding of active and reactive power is physically coupled):

$$F \cdot \delta = G \quad (15)$$

The optimization problem containing a norm in its cost function is conventionally rewritten into the easier solvable form by introducing slack variables. However, this is not convenient in our case since it introduces as many new variables as the number of samples within the prediction horizon times the number of controlled variables, so the problem scale becomes for larger systems enormous. Therefore we propose another solution with a slight modification of the cost function and adding new inequality constraints. The first term in the cost function (10) is basically the sum of distances of the actual trajectory from the reference one, which is expressed by 1-norm; sum of absolute values. We propose its replacement by plain sum. To prevent it from going negative, we formulate additional constraints that this sum must be equal or greater than zero. So the Trajectory Sensitivities based Model Predictive Control problem can be summarized in the compact form for discrete control inputs:

$$\min \left\{ \begin{bmatrix} Q & R \end{bmatrix} \begin{bmatrix} - \begin{bmatrix} \underline{x}_{x_0} \\ y_{x_0} \\ C \end{bmatrix} \cdot \begin{bmatrix} \underline{x}_{ref} - \underline{x}_{nom} \\ y_{ref} - y_{nom} \\ 0 \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \varepsilon \end{bmatrix} \right\} \quad (16)$$

$$\text{s.t.} \quad \begin{bmatrix} \underline{x}_{x_a} \cdot C & 0 \\ y_{x_a} \cdot C & 0 \\ -\underline{x}_{x_a} \cdot C & 0 \\ -y_{x_a} \cdot C & 0 \\ D & 0 \\ -Q \cdot \begin{bmatrix} \underline{x}_{x_a} \\ y_{x_a} \end{bmatrix} & Q \cdot \begin{bmatrix} \underline{x}_{ref} - \underline{x}_{nom} \\ y_{ref} - y_{nom} \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \varepsilon \end{bmatrix} \leq \begin{bmatrix} \underline{x}_{max} - \underline{x}_{nom} \\ y_{max} - y_{nom} \\ -\underline{x}_{min} + \underline{x}_{nom} \\ -y_{min} + y_{nom} \\ E \\ 0 \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} F & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \delta \\ \varepsilon \end{bmatrix} = \begin{bmatrix} G \\ 1 \end{bmatrix} \quad (18)$$

$$\delta_i \in \{0,1\}$$

Note that the result of the vector multiplication  $R \cdot C$  is a vector with constant entries, therefore it is simple to determine its absolute value before the employment of the optimization solver. Auxiliary variable  $\varepsilon$  is introduced only for reasons to meet the standard input requirements for available optimization solvers.

We have solved the above formulated optimization problem by use of the Matlab function *miqp* [8], which employs Branch and Bound optimization technique.

## V. EXAMPLES – SIMULATION RESULTS

The example of a small power system has been used to demonstrate a functionality of trajectory sensitivities in [1]. This power system exhibits important features of hybrid systems, i.e. combination of algebraic states  $y$  (voltages) and discrete states  $z$  (tap position of the transformer). The tripping of one of the parallel lines leads to the drastic drop of the vol-

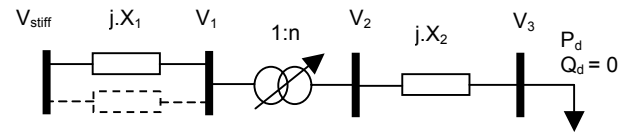


Fig. 3. Single line diagram of the example from [1]. Reactance  $X_1$  changes its value due to the outage of one of the parallel lines at time 10 seconds.

tage. Exponential load recovery models the dynamics of the load. In this system two types of control inputs are available: load shedding and tap changing. In case of changing the tap position, we assume the possibility to control it directly, i.e. we can move the tap more than one-step at the time and we can do that immediately (in reality practical limitations would delay the actual action), without waiting for tap-changer time constant if the voltage has recovered. The result of the

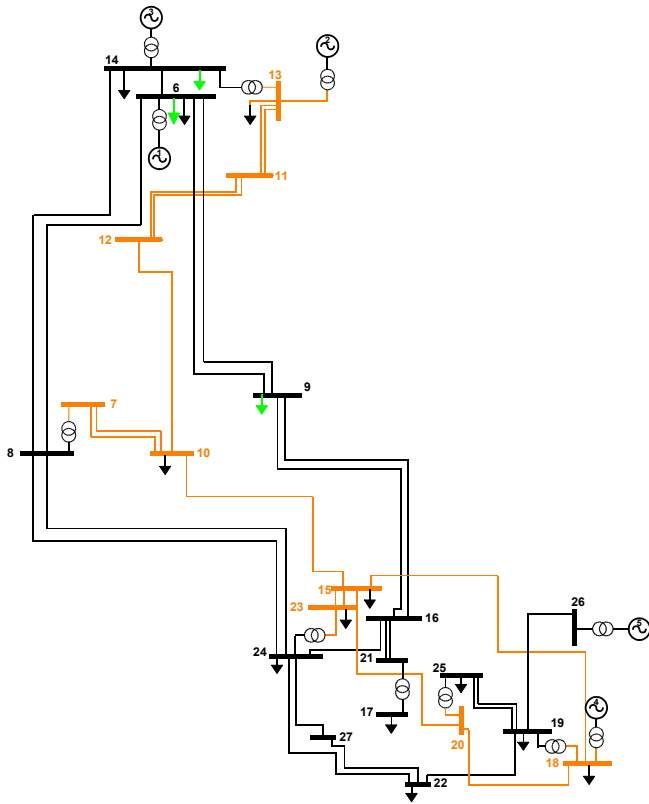


Fig. 4. Single line diagram of the part of a real system. 400 kV part of the system is drawn in darker and 130 kV part in lighter color.

proposed approach application is shown in figure 2. The voltage safely settles down after a little amount of load shedding and moving the tap position carried out immediately in time  $t_a$ . The second example shown in figure 4 represents a part of a real system. The generation is concentrated in the northern part of the system whereas the load areas in the south. A heavy flow of the power is from the north to the south, therefore the system is very sensitive to the outages of the lines interconnecting these two areas. The main problem is voltage instability only based on the topology change (contingency driven), i.e. tripping of the two or three lines, without any load increase. The reduced version we have used has 27 nodes and 5 generators. The data of this system can be obtained from the authors upon request. Since the southern part is prone to the voltage instability, we focus on voltages of nodes 15 – 27 as our controlled variables. We have decided to use following possible controls:

- tap changer of the transformer between nodes 21 and 17, the largest load is connected here
- load shedding of all available loads
- change of the voltage reference point setting of the generators voltage regulators

The last mentioned type of control allows utilizing of unused reactive power generation capacities (if they are available, i.e. the generators are not operated on their limits) and thus keeping the system voltage profile on the acceptable level. Since the load shedding should be used only as a last measure,

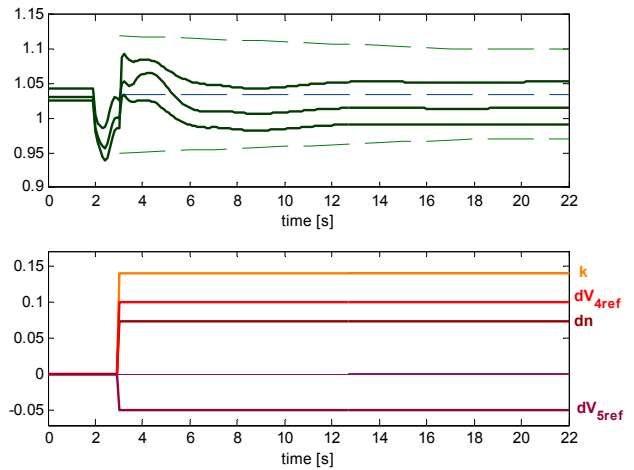


Fig. 5. Voltage magnitudes in the nodes 17, 18 and 22. The lower and upper boundaries are dashed as well as the reference value. Load shedding constant  $k$  (ratio between the shed and initial value of power) corresponds to shedding in the nodes 10, 14, 15, 17, 18, 22, 23 and 25. Weights  $Q$  on tracking the reference value have value 20. Penalty  $R$  on the tap changer action is 0.02, on the generator voltage reference point change 0.5 and on the load shedding 1. The lower part of the figure shows controls, particularly from the top to bottom:  $k$ ,  $dV_{4ref}$ ,  $dn$  and  $dV_{5ref}$ .

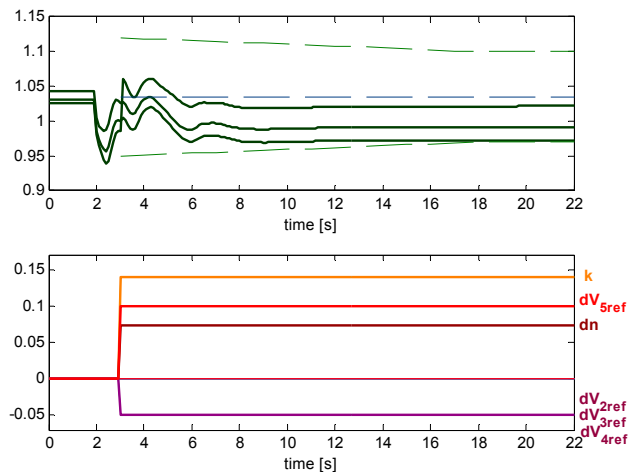


Fig. 6. Voltage magnitudes in the nodes 17, 18 and 22. Load shedding constant  $k$  corresponds to shedding in the nodes 22 and 23. Weights  $Q$  on tracking the reference value have value 0.5. Penalty  $R$  on the tap changer action is 0.02, on the generator voltage reference point change 0.5 and on the load shedding 1. The lower part of the figure shows controls, particularly from the top to bottom:  $k$ ,  $dV_{5ref}$ ,  $dn$ ,  $dV_{2ref}$ ,  $dV_{3ref}$  and  $dV_{4ref}$ .

if absolutely necessary, the penalties for employed controls (elements of  $R$ ) have been set accordingly. The most desired control to be used is tap-changing, then setting of the generators reference points and finally load shedding. Note, that the penalties can vary within each category. The simulated contingency is the tripping of the both lines from node 8 to node 24. This excites the oscillatory voltage drops under the allowed level in several locations/nodes. However, employing the proposed control as shown in figures 5 and 6 safely stabilizes the situation. The role of the weights  $Q$  and  $R$  is clearly visible. In figure 5 the emphasis is given on the accurate control, which results into heavy engagement of the

control mechanisms, especially load shedding. But in emergency situations in power systems the focus is more on being within the acceptable operation range, not to achieve certain exact (optimal) voltages. More important is to employ as little expensive controls (such as load shedding) as possible. This philosophy is followed in according setting of weights yielding figure 6. If this is the main interest, the formulations (16) – (18) can be significantly simplified as in [9].

## VI. CONCLUSIONS AND FUTURE WORK

A new approach for the emergency control of the voltage in power systems has been described and validated on the simulation examples. It has a predictive feature; the system can be stabilized in the early stage after a disturbance by employing only necessary and cheap control. This offers a rather significant advantage over the conventional methods, e.g. under-voltage load shedding relays.

Use of trajectory sensitivities has significantly helped to reduce the complexity of the problem allowing using a simple Model Predictive Control even for a larger power system. This paper contains only the initial ideas and their simple application on one field – the voltage emergency control but the main principle of the proposed approach-combination of trajectory sensitivities and simple Model Predictive Control can be seen as a general framework that could be adapted to other types of the control problems in power systems (frequency instability etc.) as well.

We made one simplification acceptable for this application – we computed only one control input set. However, it may be of interest for other implementations/applications to execute several subsequent controls, and therefore it will be a subject of future research as well as other aspects.

## VII. ACKNOWLEDGMENT

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## IX. BIOGRAPHIES

**Marek Zima** (S'02) received his BSc degree from Slovak University of Technology in Bratislava, Slovakia in 1999 and MSc degree from Royal Institute of Technology (KTH) in Stockholm, Sweden in 2001, both in Electric Power Engineering. Since 2001 he has been with ABB Switzerland where he is involved in the research and development of wide area platform for monitoring, protection and control. In 2002 he joined Power Systems Laboratory of Swiss Federal Institute of Technology (ETH) in Zürich, Switzerland where he is pursuing his PhD Degree in area of System Protection Schemes. His research interests are stability issues, analysis and control of power systems.



**Göran Andersson** (M'86, SM'91, F'97) was born in Malmö, Sweden. He obtained his M.S. and Ph.D. degree from the University of Lund in 1975 and 1980, respectively. In 1980 he joined ASEA:s, now ABB, HVDC division in Ludvika, Sweden, and in 1986 he was appointed full professor in electric power systems at the Royal Institute of Technology (KTH), Stockholm, Sweden. Since 2000 he is full professor in electric power systems at the Swiss Federal Institute of Technology (ETH), Zürich, where he heads the powers systems laboratory. His research interests are in power system analysis and control, in particular power systems dynamics and issues involving HVDC and other power electronics based equipment. He is a member of the Royal Swedish Academy of Engineering Sciences and Royal Swedish Academy of Sciences and a Fellow of IEEE.



## X. APPENDIX

To explain the structure of matrices C, D and F let us consider shedding of the loads connected in the node 22 in the second example. Let us assume that some of these loads have the following values:

$$S_{22-1} = 0.05 + j.0.02$$

$$S_{22-2} = 0.12 + j.0.04$$

So the possible values of the loads to be shed are:

$$C_1 = [0.05 \quad 0.12 \quad 0.17]$$

$$C_2 = [0.02 \quad 0.04 \quad 0.06]$$

The binary variables  $\delta$  can be assigned to the each possible value so the load shedding value in node 22 can be then expressed:

$$\Delta P_{22} = C_1 \cdot \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} \quad \Delta Q_{22} = C_2 \cdot \begin{bmatrix} \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix}$$

The rule that only one or none value can be selected is then written:

$$\begin{aligned} \delta_1 + \delta_2 + \delta_3 &\leq 1 \\ \delta_4 + \delta_5 + \delta_6 &\leq 1 \end{aligned}$$

Since there is a physical coupling between the active and reactive power the following equations must hold:

$$\begin{aligned} \delta_1 - \delta_4 &= 0 \\ \delta_2 - \delta_5 &= 0 \\ \delta_3 - \delta_6 &= 0 \end{aligned}$$

Therefore the corresponding sub-vectors and sub-matrices are:

$$\begin{aligned} D_1 &= [1 \quad 1 \quad 1] \\ D_2 &= [1 \quad 1 \quad 1] \\ F_1 &= I \\ F_2 &= -I \end{aligned}$$

And their position within the C, D and F is (N denotes number of available controls):

$$C = \begin{bmatrix} C_1 & 0 & \dots & 0 \\ 0 & C_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_N \end{bmatrix}$$

$$D = \begin{bmatrix} D_1 & 0 & \dots & 0 \\ 0 & D_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & D_N \end{bmatrix}$$

$$F = \begin{bmatrix} F_1 & F_2 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & F_{N-1} & F_N \end{bmatrix}$$

The vectors derived from  $x$  and  $y$  comprise of the column sub-vectors that correspond to the subsequent time instances (here N denotes last sample within the prediction horizon).

$$y_{ref} = \begin{bmatrix} y_{ref}(1) \\ \vdots \\ y_{ref}(N) \end{bmatrix}$$