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DC Optimal Power Flow Including HVDC Grids

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Abstract—Many simulations in power systems analysis contain an AC grid which is solved using linearized power flow methods. This paper extends the known DC optimal power flow method for AC grids for use in combined AC and HVDC grids. Future studies would now incorporate in their simulations combined AC and HVDC grids. This paper describes the additions made to the algorithm and the influence of the soft penalties. The algorithm is tested in the IEEE 14 bus test case with a 5 node DC overlay grid. The accuracy compared to the full power flow model is in a reasonable range. The calculation performance is significantly increased.

Index Terms—HVDC transmission, multi-terminal HVDC, optimal power flow.

I. INTRODUCTION

TRANSMISSION grids have to be reinforced to meet future energy demands. There are proposals to solve this issue with voltage-source converter (VSC) Multi-terminal HVDC (MTDC) grids [1], [2]. The introduction of a HVDC grid instead of new AC lines has several advantages, mainly the reduction of the overall costs, due to the lower losses and the higher controllability [3].

Steady-state power flow can been calculated with different methods. In order to reduce the costs, or losses, in a grid, optimal power flow (OPF) was introduced. The main obstacle for the accurate full OPF calculation is the high computational effort, which is needed to solve this nonlinear, not convex problem. That is why many linearized, so called DC OPF methods, where introduced to speed up the calculations [4], [5]. It should be noted that with DC power flows methods we denote the linearized methods in order to solve the OPF problem. The DC grid is not incorporated in these methods. This may lead to some confusion. Including the HVDC grid in these methods is the contribution of this paper.

The additional benefits of the DC calculation methods are the reliable and unique solutions with relatively simple methods. The injections and the resulting flows are optimized efficiently, particularly in the demanding area of contingency analysis [6]. The accuracy of the known linearized methods is still in a reasonable range for most applications. A detailed study of the errors is done in [7].

Active and reactive power flows can be calculated with the full OPF. The reactive power production has strong effect only on a local basis, therefore the active power flows are of primary interest for studies in topics such as large scale energy markets. The method presented considers only the active power.

The production costs of the generators in the AC grid are usually calculated with a quadratic cost function to calculate social welfare. Linear cost are used for electricity market bidding. Both are possible to use in this paper. One of these cost models is included in most OPF as an objective function and the linearized steady-state flow equations are embedded as constraints. This results in a convex optimization. Additionally the voltage angle differences are penalized with a soft penalty factor. It allows to estimate the size and pattern of the error and improves the numerical stability and convergence [8]. The equations and constraints for the converter stations and a DC grid are added to use the algorithm for combined AC and DC grids.

This paper first gives a brief theoretical background about the linearized flow in the DC grid, followed by the detailed formulation of the combined AC and DC grid OPF. A study case in the IEEE 14 bus test case compares the presented algorithm with a full OPF for combined AC and HVDC grids, which was presented in [9]. Finally conclusions are provided.

II. OPF FORMULATION

The well known DC power flow for AC grid is expanded to MTDC grids. The AC formulation can be found in detail in [8] and is not repeated here.

A. DC calculation of active power flows in the HVDC grid

The main idea is to linearize the quadratic power flow equations for the DC grid. The exact steady-state power flow in a HVDC line from node $k$ to $m$ is represented in (1), where $U_k$ is the voltage at node $k$ in pu. $R_{km}$ is the resistance of the HVDC line between the these nodes.

$$P_{km} = \frac{U_k \cdot (U_k - U_m)}{R_{km}}$$

If it is assumed that the DC voltages are rather close to the nominal voltage, defined as 1 pu, (1) can be approximated.

$$P_{km} \approx \frac{(U_k - U_m)}{R_{km}}$$

The resulting voltages $U_k$ and $U_m$ give only the deviation from the nominal voltage at the reference bus.

With this approximation the linearization is already done. The losses in the DC grid are neglected, similar to the AC grid. The power balance equations for the AC (3) and DC (4) grid are as follows:

$$P_{L,k} = P_{G,k} - \sum_{m}^N P_{km} - P_{T,k} \quad (3)$$

$$P_{L,k} = \sum_{m}^M P_{km} + P_{T,k} \quad (4)$$
\( P_{Lk}, P_{Gk}, \) and \( P_{Tk} \) are the load, generation and the AC/DC terminal power at node \( k \). \( N \) are the total amount of AC branches. \( M \) the similar value for the DC branches.

Additionally some parameters have to stay within certain limits. The active power generation at each bus is limited between the maximal and minimal generation.

\[
P_{Gk}^{\text{min}} \leq P_{Gk} \leq P_{Gk}^{\text{max}} \tag{5}
\]

\[
P_{Gk}^{\text{min}} \leq P_{Gk} \leq P_{Gk}^{\text{max}} \tag{6}
\]

The power flow through the terminals is also limited. It is assumed that in each direction the same maximum power can be used.

\[
P_{Tk}^{\text{max}} \leq P_{Tk} \leq P_{Tk}^{\text{max}} \tag{7}
\]

\[
P_{km}^{\text{max}} \leq P_{km} \leq P_{km}^{\text{max}} \tag{8}
\]

The limits in the branches, regardless of whether AC or DC, are also fixed. Similar to the converter stations, the flow can either be positive or negative up to the same limit.

Equation 9 is valid for all DC and AC lines.

\[
\sum_{i=1}^{I} [c_{\text{lin}} P_{Gi} + c_{\text{quad}} P_{Gk}^2] + \pi^{\text{AC}} \left[ \sum_{km} (\delta_k - \delta_m)^2 \right] + \pi^{\text{DC}} \left[ \sum_{km} (U_{DCk} - U_{DCm})^2 \right] \tag{11}
\]

The coefficients for the linear costs are \( c_{\text{lin}} \) and \( c_{\text{quad}} \) are the coefficients for the quadratic costs for all \( I \) generators. The AC voltage angle differences and DC voltage differences are considered over all lines in the AC and DC grid.

This objective function is only one possibility, several other options are possible to choose depending on the purpose of the simulation. E.g. if \( c_{\text{quad}} = 0 \) and both \( \pi^{i} = 0 \), the whole algorithm is linear.

\section{Decision variable vector \( x \)}

The vector \( x \) for the combined OPF contains different sets of values. All active power generations \( P_{G} \) are represented in the state vector. Additionally the AC voltage angles \( \delta \) are contained in \( x \). To control the power flow between the AC and the DC grid also the power transfers through the terminals \( P_{T} \) are in the state vector. Finally the voltages in the DC grid \( U_{DC} \) are added to \( x \). To calculate the flows in both grids only the AC voltage angles differences and DC voltage differences are of interest, therefore in each grid a slack bus can be chosen. At these buses the angle \( \delta \), respectively voltage \( U_{DC} \) are defined as reference values and can be removed from \( x \).

\section{Nomenclature}

The symbols used for the formulation of the algorithm are summarized in Table I.

\begin{table}[h]
\centering
\caption{Nomenclature}
\begin{tabular}{|c|c|}
\hline
I & Total amount of generators \\
K & Total amount of AC nodes \\
L & Total amount of terminal stations \\
M & Total amount of DC nodes \\
N & Total amount of AC branches \\
O & Total amount of DC branches \\
Z & \( I \times N + L + M - 2 \) Length of the state vector \( x \)
Y & \( 2(I+N+L+O) \) Amount of inequality constraints \\
\hline
\end{tabular}
\end{table}

\section{Matrix form}

The optimization problem can be formulated in matrix form as follows:

\[
\min \ f(x) = \min \frac{1}{2} x^T G x + a x \tag{12}
\]

with respect to

\[
x = [P_{G1} \ldots P_{G1} \delta_2 \ldots \delta_K]
\]

\[
P_{T1} \ldots P_{TL} U_{DC2} \ldots U_{DCM}]^T \tag{13}
\]

subject to

\[
C_{eq} x = b_{eq} \tag{14}
\]

\[
C_{iq} x \leq b_{iq} \tag{15}
\]

The matrices used are constructed as explained in the following paragraphs. The quadratic costs matrix \( G \) is constructed out of \( QC \) and the reduced version of \( W_{AC} \) and \( W_{DC} \). The generators cost coefficients for the quadratic costs are composed to a diagonal matrix \( QC \) with the size of \( I \times I \).

\[
QC = \text{diag}[2c_{\text{quad} 1} 2c_{\text{quad} 2} \ldots 2c_{\text{quad} I}] \tag{16}
\]

\( W_{DC} \) is the voltage difference weight matrix. The diagonal elements \( W_{ii} \) stand for the amount of DC lines connected to node \( i \). The off diagonal elements \( W_{ij} \) indicate whenever a line from \( i \) to \( j \) with a -1.

\[
W_{DC} = 2\pi^{\text{DC}}
\]

\[
\begin{bmatrix}
-\sum_{k \neq 1} \mathbb{I}_{k1} & \mathbb{I}_{12} & \ldots & \mathbb{I}_{1M} \\
\mathbb{I}_{21} & -\sum_{k \neq 2} \mathbb{I}_{k2} & \ldots & \mathbb{I}_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbb{I}_{M1} & \mathbb{I}_{M2} & \ldots & -\sum_{k \neq M} \mathbb{I}_{kM}
\end{bmatrix}_{[M \times M]} \tag{17}
\]
The first row of the admittance matrix can be removed due to calculate is limited by the inequality constraints. The first matrix to calculate is the admittance matrix \( B^{AC} \). Here \( B^{AC} \) is defined as follows:

\[
B^{AC} = \begin{cases} \frac{1}{X_{km}[pu]} & \text{if branch } km \text{ or } mk \text{ exists} \\ 0 & \text{otherwise} \end{cases}
\]

where \( X_{km} \) is the line reactance and \( R_{km} = R_{mk} \).

The same matrix for the AC grid is \( B^{DC} \). Here \( B^{DC} \) is defined as follows:

\[
B^{DC} = \begin{cases} \frac{1}{X_{km}[pu]} & \text{if branch } km \text{ or } mk \text{ exists} \\ 0 & \text{otherwise} \end{cases}
\]

The voltage angle difference weight matrix for the AC grid can be constructed similarly. \( W^{AC} \) is then a \( K \times K \) matrix. Node 1 is chosen as the slack bus in the DC grid, similarly AC node 1 is defined as the slack bus in the AC grid. Since only the relative angle of the AC buses and the relative voltage of the DC buses are of interest, the voltage angle at the AC slack bus can be defined as zero, equally the voltage difference at the DC slack bus is zero. Therefore the matrices \( W^{AC} \) and \( W^{DC} \) can be reduced. The first row and column of each of them can be removed to get \( W^{AC} \) and \( W^{DC} \).

The block diagonal matrix \( G \) is then composed out of \( QC \), \( W^{AC} \), and \( W^{DC} \).

\[
G = \text{diag}[QC, W^{AC}, \ldots, W^{DC}](Z \times Z)
\]  

(18)

The zero-columns with width of \( L \) correspond to the power flow through the terminals. The zeros mean that no costs are assigned to the terminal flows.

In vector \( a \) the linear cost coefficients \( c_{in} \) are included. No other linear costs are considered in the proposed objective function.

\[
a = [c_{in1} \ c_{in2} \ldots \ c_{inL} \ 0_{[1 \times (z - i)]}](1 \times Z)
\]

(19)

\[F.\] Constraints

The equality constraints are given by the power balances at each node. The power flow through the lines and terminals is limited by the inequality constraints. The first matrix to calculate is \( H \), it allocates the different generators to the nodes.

\[
H = \begin{bmatrix} \mathbb{H}_{11} & \ldots & \mathbb{H}_{1T} \\ \mathbb{H}_{21} & \ldots & \mathbb{H}_{2T} \\ \vdots & \ddots & \vdots \\ \mathbb{H}_{K1} & \ldots & \mathbb{H}_{KT} \end{bmatrix} \quad [K \times T]
\]

(20)

\[\mathbb{H}_{pq} = \begin{cases} 1 & \text{if generator } q \text{ is connected at node } p \\ 0 & \text{otherwise} \end{cases}\]

The next matrix to calculate is the admittance matrix \( B^{DC} \). The first row of the admittance matrix can be removed due to the fixation of the voltage at bus 1. Therefore

\[
B^{DC} = \begin{bmatrix} -B^{DC}_{21} & \ldots & -B^{DC}_{2K} \\ -B^{DC}_{31} & \ldots & -B^{DC}_{3K} \\ \vdots & \ddots & \vdots \\ -B^{DC}_{M1} & \ldots & -B^{DC}_{MK} \end{bmatrix} \quad [(M-1) \times M]
\]

(21)

\[
B^{km} = \begin{cases} \frac{1}{R_{km}[pu]} & \text{if branch } km \text{ or } mk \text{ exist} \\ 0 & \text{otherwise} \end{cases}
\]
\[ b_{iq} = [P_{L1} \ P_{L2} \ \ldots \ P_{L(K+M)}]_{(K+M) \times 1}^T \]  \hspace{1cm} (26)

Loads can be connected directly to the DC grid, if no such connections exist, \( P_{L_i} = 0 \) for all \( i > K \). The inequality constraints are represented by the \( C_{iq} \) matrix. \( I_r \) and \( I_L \) are identity matrices with the size of \( I \) and \( L \) respectively.

\[
C_{iq} = \begin{bmatrix}
I_r & 0 & 0 & 0 \\
-I_r & 0 & 0 & 0 \\
0 & A_{AC}^T & 0 & 0 \\
0 & -A_{AC}^T & 0 & 0 \\
0 & 0 & I_L & 0 \\
0 & 0 & -I_L & 0 \\
0 & 0 & 0 & A_{DC}^T \\
0 & 0 & 0 & -A_{DC}^T \\
\end{bmatrix}_{[Y \times Z]} \hspace{1cm} (27)
\]

The first two rows correspond to the generators limits, followed by two rows for the AC line limits. The transfer capacities of the terminal are limited in row 5 and 6. The last two rows limit the flow through the DC lines. The vector \( b_{iq} \) consists of 8 other vectors. There \( b_{Ug} \) and \( b_{g} \) stand for the upper and lower limits of the generators. \( b_{br} \) and \( b_{DCbr} \) represent the line limits for the AC and DC lines, respectively. The active power through each terminal is limited by the values in \( b_{ter} \).

All this vectors together result in \( b_{iq} \).

\[
b_{iq} = [b_{Ug}^{up} - b_{Ug}^{lo} \ b_{br} - b_{br} \ b_{ter} - b_{ter} \ b_{DCbr} - b_{DCbr}]_{[Y \times 1]}^T \hspace{1cm} (33)
\]

The power injection to the DC grid is in all simulations node 1, as shown in Fig. 2. This is reasonable since the cheapest and highest generation capacity is located at this node. All other terminals take power out of the DC grid. The most load is connected to node 3, 4 and 9 are next in the loading. To reduce the losses it makes sense to control the DC grid in such a way that the output is as close as possible to the highest loads. For the full OPF case, node 3 takes the highest power out of the DC grid. In the linearized OPF it is depending on the penalty terms, where the weighted values fits best compared to the full OPF.

The flows in the DC lines are shown in Fig. 3. Except the small flow on line 13-25, all branch flows have the same direction for all simulations. Only two lines are loaded close to their limit of 50 MW for the full OPF. In the linearized method the line loadings are far below this level, except the simulations with the weighted penalty terms. For the other two linearized simulations the utilization of the DC grid is much smaller than for the full OPF. Since losses are considered in the full OPF, its preferable to transfer the power over the DC lines instead of using the AC lines. Therefore an opposite situation is observable in the AC grid.

Fig. 4 shows the AC lines loadings, which are in general more loaded in the linearized OPF. All major flows in the AC lines are similar for the full and the linearized OPF. Again the simulation with weighted penalty factors fits best to the full OPF.

In the simulation without penalty terms the flow are not penalized, therefore its hard to decide if the power should flow through the AC or DC grid. If similar values for the penalty terms are used, the flows are penalized, but not in a similar amount. The voltage difference are usually bigger than the voltage angle differences. Therefore the AC grid is preferred choice in this simulation. With the weighted soft penalty factors all flows are penalized in the same amount.
The main advantage is the fast calculation performance. The full OPF for the combined AC and DC grid, with its nonlinear constraints and the quadratic objective function, requires 2.57 seconds to find a solution within the tolerances. The problem solved on the same office pc (Q9950 2.83GHz, 8GB RAM) with the linearized OPF takes 14.09 milliseconds, this is over 180 times faster. This could be a major advantage especially for all simulations which use multiple runs such as Monte Carlo simulations. If several ten thousand repetitions are needed, for example to calculate varying renewable infeed. It is expected that the time ratio for bigger systems could even increase, due to the simplicity of the algorithm.

IV. Conclusion

The method presented in this paper allows the inclusion of multi-terminal HVDC grids in several different studies. Especially the topic of energy markets and fluctuating infeed could profit and it is possible to include the future grid structure. It can be simply implemented in most existing simulations. The calculation performance gained compared to the full OPF justifies the simplifications, whereas the simulation results are never exact, but still in a reasonable range. Especially if the weighted penalty factors are used. A future expansion of this algorithm will include some security assessments, allowing an OPF which fulfills the N-1 security constraints.

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REFERENCES


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