Abstract—In this paper, we propose a novel hierarchical control algorithm to enable simultaneous participation of aggregations of Thermostatically Controlled Loads (TCLs) in power system Load Frequency Control (LFC) and active distribution network management in order to increase the integration of Renewable Energy Sources (RES). The algorithm assumes a two-way communication infrastructure and consists of two phases: day-ahead scheduling and real-time operation. In the scheduling phase, the optimal load dispatch is determined by considering demand and RES predictions and solving a robust multi-period AC Optimal Power Flow (AC-OPF). In real-time, a two-step procedure is applied to control the load aggregation to desired set-points that guarantee LFC provision, maximize the absorption of RES power and satisfy Distribution Network (DN) constraints. The effectiveness of the algorithm is illustrated by considering a benchmark Medium Voltage (MV) DN with large shares of photovoltaic (PV) generation and a controllable aggregation of residential Electric Water Heaters (EWHs). The results show that the algorithm properly exploits the demand-side flexibility and guarantees the provision of significant LFC reserves. At the same time, it reduces the curtailed PV energy due to DN stresses and minimizes adverse effects on user comfort.

I. INTRODUCTION

A. Motivation

Increased penetration of Renewable Energy Sources (RES) in the grid poses new challenges to traditional power system operation and planning. Due to the fluctuating nature of RES, balancing production and demand is not an easy task. Imbalances cause system-wide frequency deviations but also voltage deviations at the Distribution Network (DN). Traditionally, frequency deviations are managed by employing active power reserves from contracted power plants, whereas voltage deviations are handled by reactive power compensation. Nowadays, there is a rising interest in offering ancillary services, namely frequency and voltage regulation, by demand-side resources.

Multi-tasking with controllable loads, i.e., the concurrent provision of two or more services to the grid, will leverage the full potential of the demand-side for power system control tasks. However, investigating the existing potential for each service separately may lead to inaccurate conclusions due to the possibly conflicting objectives. For instance, consider the case where a load aggregation situated at a Medium Voltage (MV) network with large photovoltaic (PV) shares participates in Load Frequency Control (LFC). In cases when down regulation is requested for a prolonged period of time, the reduced power consumption of the aggregation in combination with a large PV production may lead to overvoltages in certain network locations. In this case, a more sophisticated reserve allocation algorithm that takes into account the DN topology could minimize the adverse effects.

This is exactly the motivation of our work: the development of an algorithm that enables multi-tasking with Thermostatically Controlled Loads (TCLs), namely frequency and voltage regulation, and provides satisfying performance with respect to both goals.

B. Related Work

There is a lot of work reported on how load aggregations can provide voltage or frequency control, sometimes in parallel to battery storage. In [1], the authors propose an approach based on multi-period AC Optimal Power Flow (AC-OPF) to reduce peak demand by controlling heating loads and Electric Vehicles (EVs). A discussion of centralized and decentralized schemes for frequency control with Demand Response (DR) is provided in [2]. Simulation results of power trajectory tracking for frequency regulation are presented in [3] and [4] using refrigerators and Electric Water Heaters (EWHs) as controllable loads, respectively. The integration of certain DR reserves in a stochastic DC Optimal Power Flow (DC-OPF) is studied in [5]. Reference [6] additionally considers DR reserve uncertainty in the DC-OPF problem.

Multi-tasking with EVs has been investigated in the literature. In [7], a co-optimization of smart-charging and frequency regulation for an aggregation of EVs is proposed. It consists of a day-ahead multi-period DC-OPF for scheduling and a decentralized approach for the real-time dispatch of the regulation signal. The combined provision of frequency and voltage regulation by EV aggregations is explicitly addressed in [8], but without including the network constraints.

However, the combined provision of frequency and voltage regulation by TCL aggregations is not studied yet. Addressing the two problems in a separated manner may lead to globally suboptimal or even infeasible solutions. Therefore, a unified and systematic approach is needed.
C. Contribution

To the best of our knowledge, this is the first time that an hierarchical algorithm that enables multi-tasking with TCLs is proposed. The algorithm consists of a day-ahead scheduling phase, which determines the optimal load dispatch solving a multi-period AC-OPF, and a real-time operation phase to dynamically allocate set-points to the available resources. The set-points satisfy particular requirements for LFC provision, RES integration and user comfort. Although the underlying optimization problems are demanding, the algorithm is designed in a computationally efficient way. Moreover, the algorithm is robust against uncertainties in demand and RES predictions, as well as model mismatches, and guarantees successful reserve provision exploiting the LFC statistical properties, which are derived from historical data.

The remainder of the paper is organized as follows: in Section II the communication setup and the considered models are introduced. In Section III the control algorithm is detailed, while in Section IV our case study is explained. Section V shows the simulation results, which are then discussed in Section VI. Last, Section VII concludes the work.

II. PROBLEM DESCRIPTION

A. Communication Setup

In this paper, domestic EWHs are considered as the controllable loads; however, the algorithm is generic and could be applied also to other types of TCLs. It is assumed that a pool of EWHs managed by an aggregator participates in the LFC market. The same aggregator is responsible for dispatching the EWHs in a way that maximizes RES penetration.

Two-way real-time communication is available between the aggregator and each EWH through direct wired or wireless communication links. These links can be used to send control signals and receive information about EWH temperatures and ON/OFF states. Although other approaches based on signal broadcasting exist, as in [3], [4], [7], [9], their flexibility here is limited since they cannot easily handle the requirement of spatial allocation of control actions. Note that the requirement for real-time load measurements can be relaxed using estimates, provided that a reliable state estimation method is available, which is outside the scope of this work.

The communication is assumed perfect, i.e., there are neither bandwidth constraints nor communication delays. Even in the presence of communication delays, the proposed algorithm can be effective in practice since TSOs usually allow a sufficient response time for the response to the LFC signal. For example, swissgrid, the Swiss TSO, allows a response time of 20 seconds [10].

B. Electric Water Heater Modeling

An EWH is modeled as a hybrid system with hysteresis control based on a deadband. The temperature evolution is described by the following linear time-varying model, which was proposed in [11]:

\[ T_{k+1} = a_k T_k + b_k u_k + e_k \]  

where \( a_k = \exp(-\Delta t/RC) \), \( b_k = \eta R (1 - a_k) \), \( e_k = (G R T_{\text{out}} + B R T_{\text{in}})(1 - a_k) \), \( R = 1/(G + B) \), \( B = \rho D U C \), \( G = A_1 U \), \( C = \rho m \),

where \( T_k \) is the water temperature, \( u_k \in \{0, 1\} \) is the ON/OFF state, \( \Delta t \) is the simulation time step, \( \eta \) is the EWH efficiency, \( T_{\text{out}} \) is the ambient temperature, \( T_{\text{in}} \) is the incoming water temperature, \( \rho \) is the water density, \( D \) is the water draw at time step \( k \), \( \rho \) is the specific heat of water, \( A_1 \) is the total EWH surface, \( U \) is the inverse of the tank thermal resistance, and \( m \) is the mass of the contained water.

C. EWH Population Modeling

Two aggregated EWH models are considered in this work. The first one is obtained by simply stacking together individual models, one for each EWH. If we denote by \( n_{ap} \) the total number of EWHs and we apply (1)-(4) for each one of them, we result in the following aggregated model:

\[ T_{k+1} = A_k T_k + B_k u_k + E_k \]  

where \( A_k = \text{diag}([a_k^1 \ldots a_k^{n_{ap}}]) \), \( B_k = \text{diag}([b_k^1 \ldots b_k^{n_{ap}}]) \), \( E_k = \text{diag}([e_k^1 \ldots e_k^{n_{ap}}]) \), \( T_k = [T_k^1 \ldots T_k^{n_{ap}}]^T \), \( U_k = [u_k^1 \ldots u_k^{n_{ap}}]^T \),

where diag denotes the operator that transforms a vector to a diagonal matrix with this vector in the diagonal.

If the model (5)-(8), with \( n_{ap} \) on the order of a few hundreds or thousands, was included directly in a day-ahead multi-period AC-OPF this would lead to a large scale nonlinear optimization problem with binary variables, which is practically intractable even for relatively small networks. To render the optimization problem computationally attractive, a simpler aggregated EWH model is needed.

The EWHs are grouped according to the MV/LV substation where they belong, i.e., the aggregated EWH is the sum of all the EWHs that belong to substation \( i \). Each group is represented by a parent EWH with a tank volume and an aggregated water draw equal to the sum of the volumes and the water draws of the children EWHs, respectively. The deadband limits of the parent EWH are set using the State of Charge (SOC) concept \( SOC_k^i = (T_k^i - T_{\text{min}}^i)/(T_{\text{max}}^i - T_{\text{min}}^i) \), where \( T_k^i \) is its water temperature [4]. Let us define the input for each parent EWH as the total power consumption of the group and denote it by \( P_{CL,k}^i \). By defining \( \text{SOC}_k := [SOC_k^1 \ldots SOC_k^{n_{ap}}]^T \) and \( P_{CL,k} := [P_{CL,k}^1 \ldots P_{CL,k}^{n_{ap}}]^T \), where \( n_{ib} \) is the number of buses of the MV network, the aggregated EWH dynamics can be written in the compact form:

\[ \text{SOC}_{k+1} = A_k \text{SOC}_k + B_k P_{CL,k} + E_k \]  

where \( A_k \), \( B_k \) and \( E_k \) are appropriate matrices calculated applying (2)-(4) for the parent EWHs. Model (9) was compared against model (5)-(8) to assess its accuracy in terms...
of predicting the aggregated SOC of EWH groups at each substation. The Mean Absolute Percentage Error (MAPE) between the two models is around 9.7% on average.

III. CONTROL ALGORITHM

A. Day-ahead Scheduling Phase

At the end of each day the aggregator predicts the demand, RES production and water draws and builds aggregated models for each MV/LV substation, as in (9). The aggregator also knows the LFC control band of the EWH aggregation, which is defined as a percentage $\alpha$% of the hourly scheduled power consumption\(^1\). All this information is used in a multi-period AC-OPF to determine the optimal EWH dispatch, i.e. the aggregated hourly EWH consumption at each MV/LV substation that minimizes electricity costs by maximizing local RES energy absorption, while leaving enough space for provision of LFC reserves. The optimization problem can be formulated as follows:

\[
\begin{align*}
\min_{P_{G,t}, P_{CL,t}, V_t, \delta_t} & \sum_{t=1}^{N_t} c^T_{P,G} P_{G,t} + c^T_{Q,G} Q_{G,t} \\
\text{s.t.} & \quad P_{G,t} - P_{CL,t} - P_{i,n,t} = P_{\text{inj},t} \quad \forall \ t \\
& \quad Q_{G,t} - Q_{\text{inj},t} = 0 \quad \forall \ t \\
& \quad P_{G,t}^{\min} \leq P_{G,t} \leq P_{G,t}^{\max} \quad \forall \ t \\
& \quad Q_{G,t}^{\min} \leq Q_{G,t} \leq Q_{G,t}^{\max} \quad \forall \ t \\
& \quad V^{\min} \leq V_t \leq V^{\max} \quad \forall \ t \\
& \quad \delta_t(i) = 0 \quad \forall \ t, \quad \forall i \in I_{\text{ref}} \\
& \quad \text{SOC}_{t+1} = \bar{A}_t \text{SOC}_t + \bar{B}_t P_{CL,t} + \bar{E}_t \quad \forall \ t \\
& \quad 0 \leq P_{CL,t} \leq P_{CL,\text{max}} \quad \forall \ t \\
& \quad \text{SOC}_0 = \text{SOC}_{N_t} \\
& \quad \alpha (1^T P_{CL,t}) \geq \beta \frac{P_{\text{inj},t}}{P_{\text{max},t}} \quad \forall \ t \\
& \quad \text{SOC}_{t+1} + \alpha E_{\text{wc}}^u B_w (1^T P_{CL,t}) \leq \text{SOC}_{\text{max}} \quad \forall \ t \\
& \quad \text{SOC}_{t+1} - \alpha E_{\text{wc}}^d B_w (1^T P_{CL,t}) \geq \text{SOC}_{\text{min}} \quad \forall \ t \\
& \quad (1 + \alpha) (1^T P_{CL,t}) \leq P_{\text{max},t} \quad \forall \ t,
\end{align*}
\]

where $N_t$ is the number of lines in the network, $N_h$ is the number of steps for the day-ahead AC-OPF (typically $N_h = 24$), $I_{\text{ref}}$ is the set of all slack buses of the system, $P_{\text{inj},t}$ is the aggregated EWH power rating at substation $i$, and $P_{\text{max},t}$ is the aggregated EWH power rating in the network. In (10)-(24), $P_{G,t} \in R^{N_h}$ and $Q_{G,t} \in R^{N_h}$ are the generation active and reactive power vectors, respectively; $P_{CL,t} \in R^{N_h}$ and $Q_{CL,t} \in R^{N_h}$ are the uncontrollable active and reactive power demand vectors, respectively; $P_{i,n,t} \in R^{N_h}$ and $Q_{i,n,t} \in R^{N_h}$ are the active and reactive power injection vectors, respectively; $V_t \in R^{N_h}$ and $\delta_t \in R^{N_h}$ are the voltage magnitudes and angles, respectively; $S_t \in R^{N_h}$ is the power flow vector; $c_{P,G,t} \in R^{N_h}$ and $c_{Q,G,t} \in R^{N_h}$ are the cost vectors for active and reactive power, respectively. Note that $P_{\text{min},t}$, $P_{\text{max},t}$, $Q_{\text{min},t}$ and $Q_{\text{max},t}$ are determined by the generator capabilities and $V^{\max}$ is determined by the line properties.

Last, we assume $V^{\min} = 0.9 \cdot 1^T$ p.u., $V^{\max} = 1.1 \cdot 1^T$ p.u., and we define $P_{\text{max},t} := [P_{\text{min},t}, \ldots, P_{\text{min},t}]^T$, $w := P_{\text{max},t} / P_{\text{max},t}$.

There are three types of constraints in the previous problem. Constraints (11)-(17) are the standard AC-OPF constraints. Constraints (18)-(20) correspond to the controllable aggregation of EWHs: (18) models the aggregate thermal dynamics, (19) sets limits to the aggregated power, and (20) requires that the SOC between two consecutive days is the same\(^2\). Constraints (21)-(24) guarantee secure LFC provision: (21) requires that a minimum amount of reserves, defined as $\beta \%$ of $P_{\text{inj},t}$, is procured throughout the whole day\(^3\); (22)-(23) ensure that the EWH virtual storage will not be full (depleted) even in the worst-case scenario of prolonged up (down) regulation; and (24) guarantees that the aggregator can increase the aggregated EWH power by the contracted capacity. To ensure that the aggregated EWH power can be decreased by the contracted capacity, it suffices to select $\alpha \leq 1$. The values of $\text{SOC}_{\text{max}}$ and $\text{SOC}_{\text{min}}$ in (22) and (23) depend on the desired level of robustness against uncertainties introduced by water draw predictions and aggregated model mismatches. Note that problem (10)-(24) is a multi-period AC-OPF since constraints (18), (22) and (23) link different time periods.

We calculated the worst-case scenarios for prolonged up and down regulation using LFC data from the Swiss control area for 2009. The intervals during which the LFC signal did not change sign for up to an hour were identified and the corresponding energy requirements were calculated. Based on this analysis, the worst-case amounts of energy that can be requested within an hour during up and down regulation are equal to $E_{\text{wc}} = 96.2\%$ and $E_{\text{wc}} = 97.8\%$ of the control band, respectively.

The outcome of the day-ahead AC-OPF is a set $\{P_{\text{CL},t}, P_{CL,t}, V_t, \delta_t\}$, $\forall t \in [1, N_t]$, containing the optimal values for each bus and each time step. $P_{\text{CL},t}$ is the base part of the target profile for the next day at each time step $t \in [1, N_t]$, where $N_t = N_h (3600 / \Delta t)$. The variable part of this profile is determined during real-time operation by the incoming LFC signal, $P_{\text{LFC},t}$, and the control band of the EWH aggregation, $P_{cb,t} = \alpha (1^T P_{CL,t})$. Therefore, for an arbitrary LFC signal the target power profile is:

\[
P_{\text{fp},t} = 1^T P_{CL,t}^* + P_{\text{LFC},t} P_{cb,t} = (1 + \alpha P_{\text{LFC},t}) (1^T P_{CL,t}^*) \quad \forall t \in [1, N_t].
\]

B. Real-time Operation Phase

During real-time operation, the aggregator receives measurements of the current RES production and demand, the temperatures of EWHs, as well as $P_{\text{LFC},t}$, and he calculates the desired set-point applying (25). Next, the aggregator needs to allocate this set-point to the participating EWHs. The

\(^1\)An ancillary services market where hourly reserve bids are allowed is implicitly assumed to facilitate demand-side participation.

\(^2\)In general, it suffices that $\text{SOC}_{\text{inj},t}$ and $\text{SOC}_{\text{N_t}}$ are given, i.e. it is not necessary that they are equal.

\(^3\)Both $\alpha$ and $\beta$ are design parameters to be selected by the aggregator.
allocation must satisfy the following requirements: (a) the contracted LFC reserves must be provided with nearly 100% reliability, (b) the adverse effects on DN operation should be minimal, and (c) user comfort should be respected.

Ideally, a single-period AC-OPF for the whole MV network incorporating the dynamics of individual EWHs as in (5) would provide the optimal allocation; however, the corresponding optimization problem would be intractable. To guarantee implementability, we devised a two-step optimization-based allocation method as an approximate scheme for the original problem. Requirements (a) and (b) are tackled in the first step by solving a simplified version of the single-period AC-OPF, while requirement (c) is addressed in the second step by solving a number of tractable Mixed Integer Linear Programs (MILPs). The two-step allocation method, which is solved at every time step of the LFC signal, i.e. typically every 4 seconds, is explained in the following.

1) Single-period AC-OPF: First, the aggregator divides the EWHs into subsets based on their SOC using the approach proposed in [4]. In particular, the aggregator identifies the subset of EWHs at each MV/LV substation that are outside their deadbands, which will be automatically toggled by their internal controllers. He also identifies the subset of EWHs at each MV/LV substation that are within their deadbands and thus available for external control actions, which is denoted by $S_{CL,i} = \{ j \in S_{\text{EWH}} | 0 \leq \text{SOC}_j(t) \leq 1 \}$ for substation $i$.

Second, the aggregator determines the aggregated EWH power at each substation that minimizes electricity costs at the current time step by solving the following single-period AC-OPF problem:

$$\begin{align*}
\min_{P_{V,t}, P_{CL,i}, V, \delta_t} & \quad c_{P,t}^T P_{G,t} + c_{Q,t}^T Q_{G,t} \\
\text{s.t.} & \quad \text{constraints (11)-(17)} \\
& \quad P_{C,t} = (1 + \alpha P_{\text{LFC},t})(1^T P_{CL,t}) \\
& \quad 0 \leq P_{CL,i} \leq P_{\text{max}}, \quad P_{\text{max}}(i) = \sum_{j \in S_{CL,i}} P_{j}^d, \\
& \quad c_{P,t} \text{ and } c_{Q,t} \text{ are the vectors of upper and lower limits of the deadbands, respectively, and } w_1, w_2 \text{ are weighing factors.}
\end{align*}$$

where $P_{j}^d$ is the power rating of EWH $j$. Constraint (27) ensures that the LFC reserves are successfully provided. The outcome of the single-period AC-OPF is the set $\{P_{\text{CL},t}^*, P_{CL,i}, V^*, \delta_t^*\}$, where $P_{CL,i}$ represents the optimal allocation of the set-point among the MV/LV substations at the current time step.

2) Independent MILPs: In the second step, the algorithm further allocates $P_{\text{CL},t}^*$ to individual EWHs by solving the following problem separately for each MV/LV substation $i$:

$$\begin{align*}
\forall i \min_{u_t, s^-t, s^+t} & \quad w_1 |d^T u_t - P_{\text{CL},t}(i)| + w_2(|s^-| + |s^+|) \\
\text{s.t.} & \quad T_{t+1} = A_t T_t + B_t u_t + E_t \\
& \quad T_{\text{min}} - s^- \leq T_{t+1} \leq T_{\text{max}} + s^+ \\
& \quad s^- \geq 0, \quad s^+ \geq 0,
\end{align*}$$

where $u_t \in \{0, 1\}$ is the vector of ON/OFF actions, $d$ is the vector of the power ratings of the EWHs that belong to $S_{CL,i}$, $A_t$, $B_t$ and $E_t$ are taken from (5), $T_{\text{min}}$ and $T_{\text{max}}$ are the vectors of upper and lower limits of the deadbands, respectively, and $w_1, w_2$ are weighing factors.

Note that the slack variables $s^-$ and $s^+$ are introduced to obtain soft constraints on temperature, which guarantees feasibility of the problem under all circumstances. Since the EWH states at the end of each time step are predicted internally in the optimization$^4$, solutions that would result in significant temperature constraint violations or undesired ON/OFF cycling can be excluded.

IV. CASE STUDY

The effectiveness of the proposed algorithm is shown considering the benchmark MV DN from [12]. The network has a nominal voltage of 20 kV and it is comprised of 11 buses and 10 lines. Its topology and parameters can be found in [12], but are omitted here due to space limitations. The network supplies a rural area with both residential and industrial customers and a peak demand of 3.55 MW. The load is allocated to the buses similarly to [12], while typical residential and industrial load profiles are adopted from [13].

A large PV penetration is assumed in the network. According to [14], in rural areas the average PV potential is 13.7 kWp for residential buildings and 53.9 kWp for agricultural buildings. Based on this information, an optimistic scenario of 1.5 MWp installed PV power at MV/LV substations 3, 4, 5, 8, 9 and 11, adding up to a total of 9 MWp, is used in our simulations. The standard deviation of PV day-ahead prediction errors is assumed to be 12%.

A population of 500 EWHs, which is uniformly distributed among the MV/LV substations, is considered in this paper. The statistical method proposed in [4] is used to generate the population parameters and the water draw time series. Finally, we assume $\alpha = 40\%$ and $\beta = 2\%$.

V. SIMULATION RESULTS

A. Day-ahead Scheduling

In this section, we present results from the day-ahead scheduling phase of the algorithm considering the three cases of Table I. Case A is the uncontrolled case where only constraints (11)-(17) are considered in the day-ahead AC-OPF and there is no real-time control. Cases B and C are considered to illustrate the effect of different levels of robustness on algorithm performance.

Figure 1 shows the optimal EWH dispatch, Figure 2 the resulting SOC profile, and Figure 3 shows the total PV power that can be injected in the grid without violating network

$^4$The problem is basically a Model Predictive Control (MPC) problem with a prediction horizon of 2 steps.
constraints. To maximize PV integration a major part of the EWH consumption is shifted to hours 13.00-15.00, when the available PV energy is at maximum. Note that at hour 13.00 the scheduled EWH power in cases B and C is more than 2 times higher than in case A. The significant energy consumption during hours 20.00-24.00 is due to the large amount of water draws at this time but also due to constraint (20). Note that the scheduled SOC keeps far from $SOC_{\text{min}}$ and $SOC_{\text{max}}$ to satisfy constraints (22) and (23). In case C the SOC keeps closer to 50%, which further robustifies the algorithm against aggregated EWH modeling errors.

A comparison of the absorbed PV energy among the three cases in presented in Table II. The day-ahead scheduling algorithm increases the PV energy yield by 3.80% and 3.04% in cases B and C, respectively, compared to case A. This is achieved with an increase of only 0.34% and 1.53% in EWH energy consumption for cases B and C, respectively.

B. Real-time Operation

Results from the real-time operation algorithm during a typical day are presented in this section. Figure 4 shows the LFC signal requested from the EWH aggregation as a percentage of its control band. Figure 5 shows the corresponding target power trajectory and the aggregated EWH power for cases B and C. The MAPE between the target trajectory and the aggregated EWH power can be found in Table II and the tracking accuracy is very high in both cases. Case C results in a lower MAPE due to the increased robustness against aggregated EWH modeling and water draw prediction errors. Note that in case B the tracking performance deteriorates whenever the aggregated SOC falls below 30% and the target power is much lower than the EWH aggregated power in the uncontrolled case A.

Although the LFC signal is available on 10-second time steps, a resolution of 1 minute is used in the simulations to decrease computation time.

Figure 6 shows the evolution of the aggregated EWH SOC per network substation in case C. The solid curves correspond to buses with installed PV power, the dashed curves to buses with load only, whereas the black curve indicates the SOC of the whole aggregation. Although the black curve generally follows the day-ahead optimal SOC curve from Figure 2, the two curves differ for two reasons: (a) LFC reserves are provided, and (b) the aggregated EWH model is not error-free. The EWHs situated at remote buses with installed PV power (e.g., buses 9 and 11) are around 100% SOC during most of the daytime to avoid overvoltages.

To assess the impacts of the proposed algorithm on user comfort and device operation we show the temperature deviations from the deadband and the average number of switching actions in Figure 7 and Table II. In cases B and C the worst case deviations from the lower and upper deadband limits are $-0.006^\circ\text{C}$ and $+0.027^\circ\text{C}$, respectively. The deviations are kept negligible by considering only the EWHs within their deadbands in problem (26)-(28) and by formulating the predictive optimization problem (29)-(32). Although the user comfort is nearly always respected, many EWHs have to operate at their upper deadband limit for several hours per day to maximize the PV energy integration. This, in combination to the LFC provision, drastically increases the average number of switching actions per EWH, which might increase the wear of the devices and reduce their lifetime.

VI. DISCUSSION

Based on our results, LFC and active DN management for PV integration can be simultaneously provided by TCL aggregations using the proposed algorithm. The more robust the algorithm, the higher the LFC tracking accuracy, but the less the PV energy yield and the higher the EWH consumption and the number of switching actions.

The computation time of the algorithm is reasonable and ensures implementability of the approach. Problems (10)-(24) and (26)-(28) were solved using the solver IPOPT, whereas
problems (29)-(32) were solved using the solver GLPK. Both solvers were called through a MATLAB-YALMIP interface [15] using a 4 core machine (2.83 GHz) with 8 GB RAM. Problem (10)-(24) takes 20 minutes on average, problem (26)-(28) takes around 2 seconds, whereas each of the problems (29)-(32) is solved in approximately 0.1 seconds. Note that single-period AC-OPF problems, like (26)-(28), can be solved using specialized packages, e.g. MATPOWER, in less than a second even for networks with hundreds of buses [16]. Also, problems (29)-(32) are independent and parallelizable, which indicates the scalability of the approach for larger networks.

In our simulations, the residual $|d^T u_r - P^*_CL(i)|$ in (29) is 0.025 kW on average with its maximum value being 1.256 kW. Therefore, problems (26)-(28) and (29)-(32) are independent and parallelizable, which indicates the scalability of the approach for larger networks.

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In this paper, we presented an hierarchical control algorithm to enable multi-tasking with TCLs, namely frequency and voltage regulation, assuming a two-way communication infrastructure between an aggregator and individual loads. The scheduling phase of the algorithm determines the optimal dispatch of the loads and is robust against uncertainties related to prediction errors and model mismatches. The real-time operation phase allocates control actions to loads to securely provide frequency reserves while respecting DN and user comfort constraints. Moreover, the optimization problems are tractable and therefore the proposed algorithm is implementable for realistic network sizes. Future work will focus on stochastic OPF formulations and benchmarking of the proposed method against simpler heuristic rules.

VII. CONCLUSION

In this paper, we presented an hierarchical control algorithm to enable multi-tasking with TCLs, namely frequency and voltage regulation, assuming a two-way communication infrastructure between an aggregator and individual loads. The scheduling phase of the algorithm determines the optimal dispatch of the loads and is robust against uncertainties related to prediction errors and model mismatches. The real-time operation phase allocates control actions to loads to securely provide frequency reserves while respecting DN and user comfort constraints. Moreover, the optimization problems are tractable and therefore the proposed algorithm is implementable for realistic network sizes. Future work will focus on stochastic OPF formulations and benchmarking of the proposed method against simpler heuristic rules.

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