

# Single-machine infinite bus system

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The purpose of this report is to verify the model of synchronous machine in SIMULINK and MATLAB. The system of study is the one machine connected to infinite bus system through a transmission line having resistance  $r_e$  and inductance  $x_e$  shown in Figure 1.

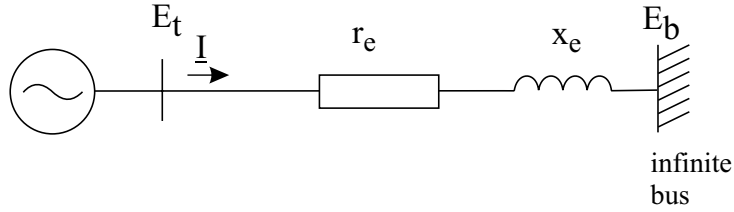


Figure 1: One machine to infinite bus system

The model of generator in SIMULINK is shown on Figure 2. The generator is modeled by transient model, according to the following equations. All system data can be found in Appendix.

**Stator winding equations:**

$$v_q = -r_s i_q - x_d' i_d + E_q' \quad (1)$$

$$v_d = -r_s i_d + x_q' i_q + E_d' \quad (2)$$

where

$r_s$  is the stator winding resistance

$x_d'$  is the  $d$ -axis transient resistance

$x_q'$  is the  $q$ -axis transient resistance

$E_q'$  is the  $q$ -axis transient voltage

$E_d'$  is the  $d$ -axis transient voltage

**Rotor winding equations:**

$$T_{do}' \frac{dE_q'}{dt} + E_q' = E_f - (x_d - x_d') i_d \quad (3)$$

$$T_{qo}' \frac{dE_d'}{dt} + E_d' = (x_q - x_q') i_q \quad (4)$$

where

$T_{do}'$  is the  $d$ -axis open circuit transient time constant

$T_{qo}'$  is the  $q$ -axis open circuit transient time constant  
 $E_f$  is the field voltage

**Torque equation:**

$$T_{el} = E_q' i_q + E_d' i_d + (x_q' - x_d') i_d i_q \quad (5)$$

**Rotor equation:**

$$2H \frac{d\omega}{dt} = T_{mech} - T_{el} - T_{damp} \quad (6)$$

$$T_{damp} = D\Delta\omega \quad (7)$$

where

$T_{mech}$  is the mechanical torque, which is constant in this model

$T_{el}$  is the electrical torque

$T_{damp}$  is the damping torque

$D$  is the damping coefficient.

There are two blocks in Figure 2, named "qde2qdr" and "qdr2qde". These blocks represent the transformation of the synchronously rotating reference input value to the reference frame rotating with the rotor, and vice versa. The transformation matrices are:

$$T = \begin{bmatrix} \cos \delta & -\sin \delta \\ \sin \delta & \cos \delta \end{bmatrix} \quad \text{and} \quad T^{-1} = \begin{bmatrix} \cos \delta & \sin \delta \\ -\sin \delta & \cos \delta \end{bmatrix} \quad (8)$$

where  $\delta$  is the rotor angle.

The investigation of the behavior of the generator is done in two ways. In the first case the inputs are the infinite bus voltage, transformed into rotating frame, the field voltage and the mechanical torque, Figure 3. The machine terminal and infinite bus voltages in terms of the  $d$  and  $q$  components are

$$\tilde{E}_t = v_d + jv_q \quad (9)$$

$$\tilde{E}_b = E_{bd} + jE_{bq} \quad (10)$$

Referring to Figure 1, the network constraint equation is

$$\tilde{E}_t = \tilde{E}_b + (r_e + jx_e) \quad (11)$$

$$(v_d + jv_q) = (E_{bd} + jE_{bq}) + (r_e + jx_e)(i_d + ji_q) \quad (12)$$

Resolving into  $d$  and  $q$  components gives

$$v_d = r_e i_d - x_e i_q + E_{bd} \quad (13)$$

$$v_d = r_e i_q + x_e i_d + E_{bq} \quad (14)$$

where the infinite bus voltage is transformed into rotating reference form by block  $qde2qdr$ .

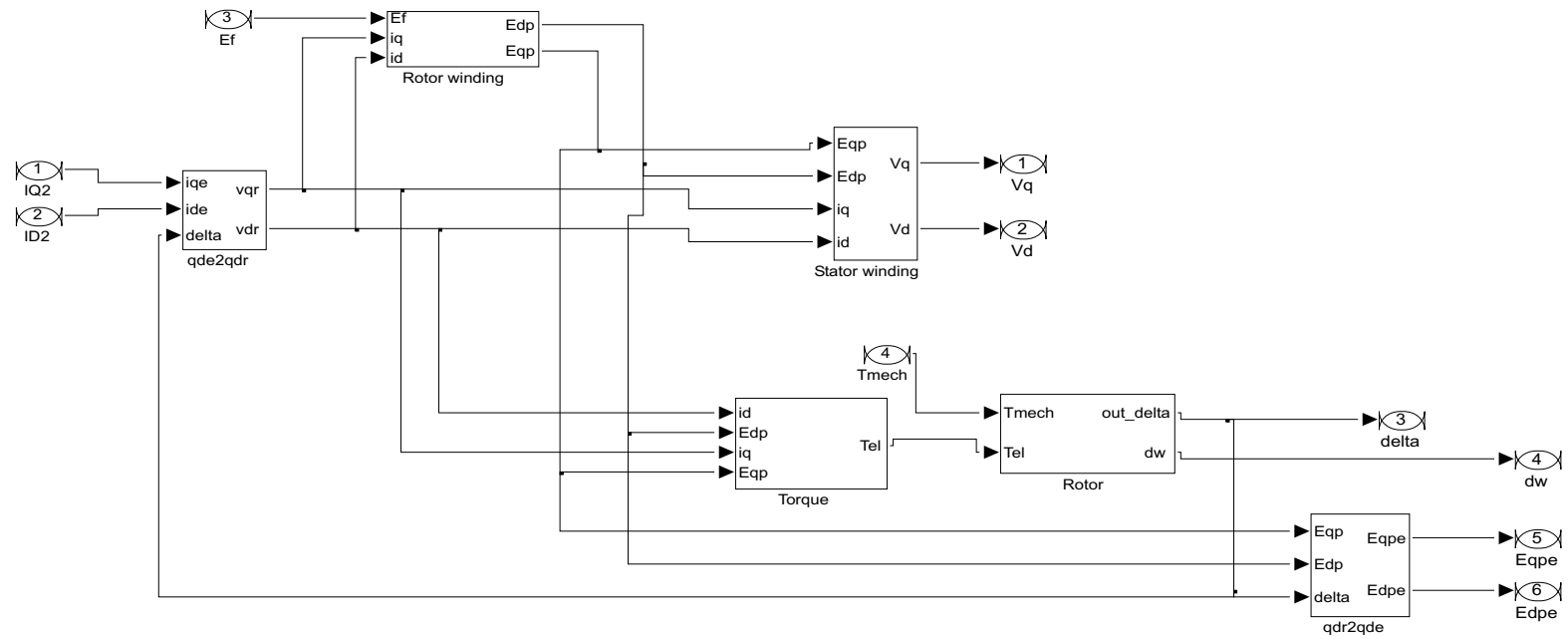


Figure 2: Generator model in SIMULINK

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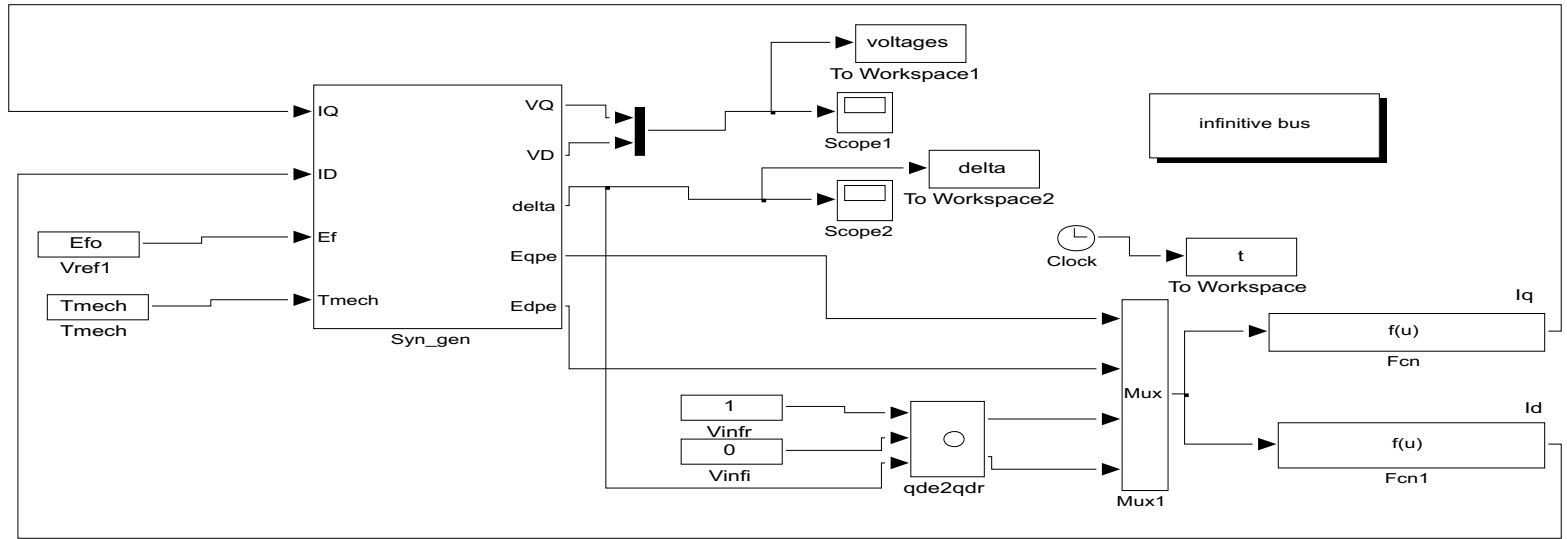


Figure 3: Generator model in SIMULINK connected to the infinite bus

Using stator Equations (1) and (2) to eliminate  $e_d$  and  $e_q$  in Equations (12) and (13) yields the following expressions for  $i_d$  and  $i_q$  in terms of the state variables  $E_d'$ ,  $E_q'$ ,  $\delta$  and infinite bus voltage:

$$i_d = \frac{(E_d' - E_{bd})(r_s + r_e) + (x_e + x_q')E_q'}{(x_e + x_q)(x_e + x_d') + (r_e + r_s)^2} \quad (15)$$

$$i_q = \frac{(E_q' - E_{bq})(r_s + r_e) - (x_e + x_d')(E_d' - E_{bd})}{(x_e + x_q)(x_e + x_d') + (r_e + r_s)^2} \quad (16)$$

The stator voltage equations (1) and (2), without the external RL line parameters, are used to compute the terminal voltage of the generator within the block "stator winding".

The block "infinite bus", from Figure 3 performs the steady state values of machine variables. The input equations for steady state values are:

$$I_t = \text{conj} \frac{S_b}{E_b} \quad (17)$$

$$E_q = E_b + [(r_s + r_e) + j(x_q + x_e)]I_t \quad (18)$$

$$\text{delt}_0 = \text{angle}(E_q) \quad (19)$$

$$E_{q0} = \text{abs}(E_q) \quad (20)$$

$$I = I_t(\cos(\delta_0) + j\sin(\delta_0)) \quad (21)$$

$$I_{qo} = \text{real}(I) \quad (22)$$

$$I_{do} = \text{imag}(I) \quad (23)$$

$$E_{f0} = E_{q0} + (x_d - x_q)I_{d0} \quad (24)$$

$$V_t = V_i + I_t(r_e + jx_e) \quad (25)$$

$$S_{t0} = V_t \text{conj}(I) \quad (26)$$

$$T_{mech} = \text{real}(S_{t0}) \quad (27)$$

where

$E_b$  - infinite bus voltage phasor

$S_b$  - complex power delivered to infinite bus

$I_t$  - phasor current of generator

$E_t$  - terminal voltage phasor

$E_q$  - Voltage behind q-axis reactance

$I$  - generator current in its rotor reference frame

$\delta_0$  - internal rotor angle

$E_{f0}$  - referent input voltage presenting the field voltage

$V_t$  - terminal voltage of generator

Figures 4 and 5 present the results of SIMULINK simulations of the generator voltage and rotor angle, based on above mentioned method.

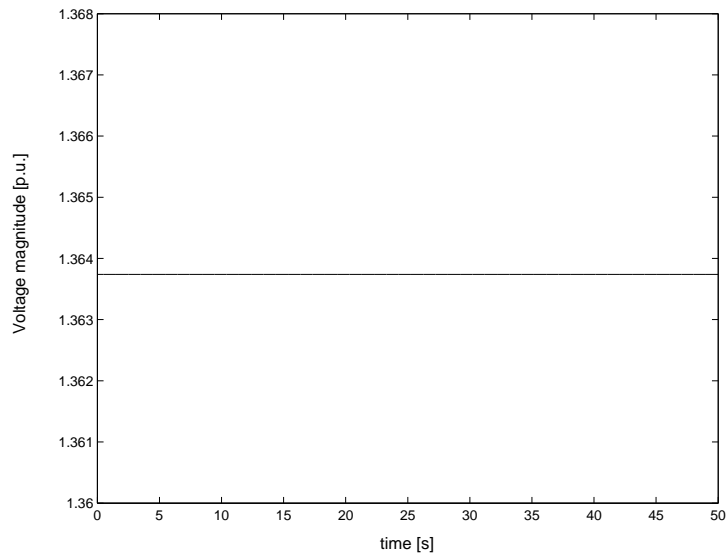


Figure 4: Generator terminal voltage

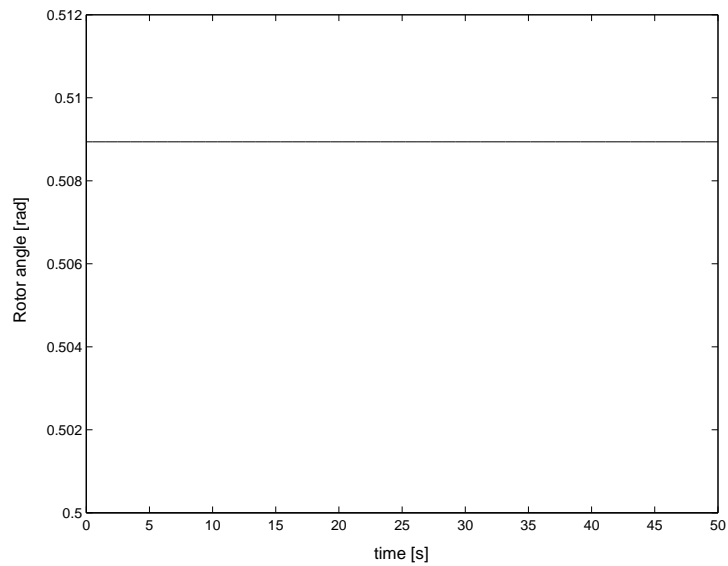


Figure 5: Rotor angle with  $\delta_0$  initial conditions

In the Figure 6 generator model with the dynamic equations interfaced to the algebraic equations of the static network is presented. This model is obtained by equalized stator voltage equations, Eq.1 and Eq.2, and network constraint equation, Eq. 11, with transient saliency ignored ( $x_d' = x_q'$ ). The resulting equation is:

$$(E_d' + jE_q') - (E_{bd} + jE_{bq}) = [(r_e + jx_e) + (r_s + jx_d)](i_d + ji_q) \quad (28)$$

According to this equation, Eq. 28, the impedance of the generator is easily added to network impedance. In the Figure 7 the network block is presented, where  $RZ$  and  $IZ$  are the real part and imaginary part respectively of the common admittance. Figures 8 and 9 present the results of SIMULINK simulations of the generator voltage and rotor angle, based on this method which is basically the same as the first method, but if there exist several generators in the network, the first method is useless.

As can be noticed, the rotor angles for both case are the same, but not the voltages. The reason is neglecting of saliency in the second case so the  $i_d$  and  $i_q$  currents have different values.

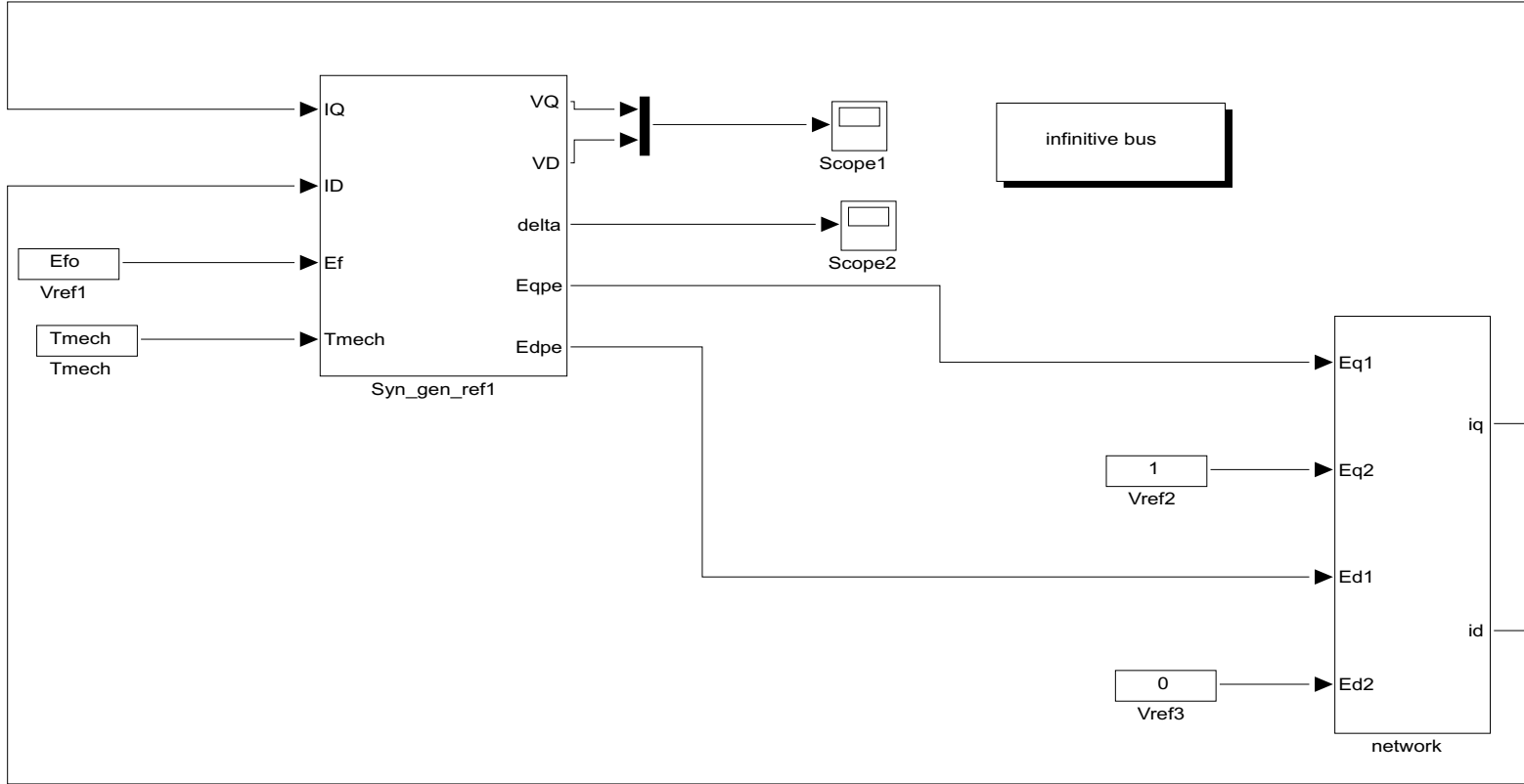


Figure 6: Generator model in SIMULINK connected to the infinite bus

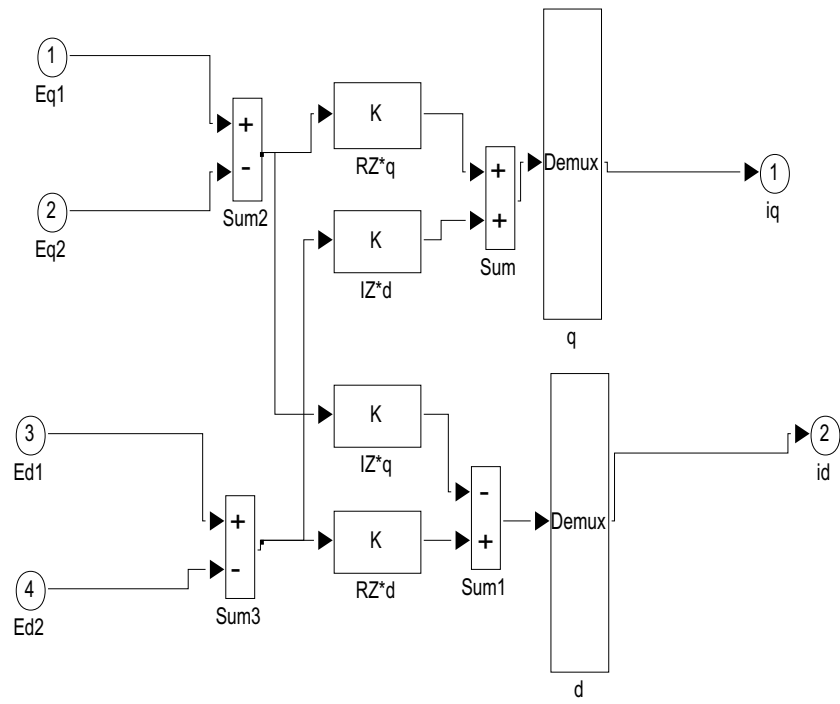


Figure 7: Inside the network block

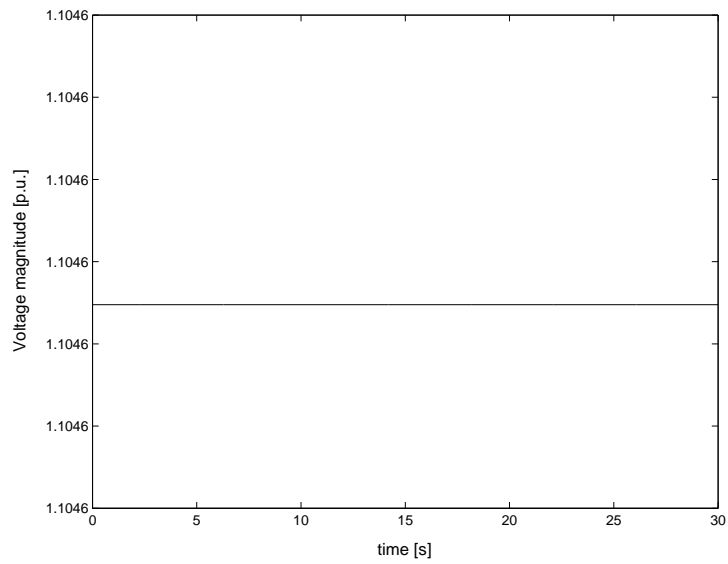


Figure 8: Generator terminal voltage

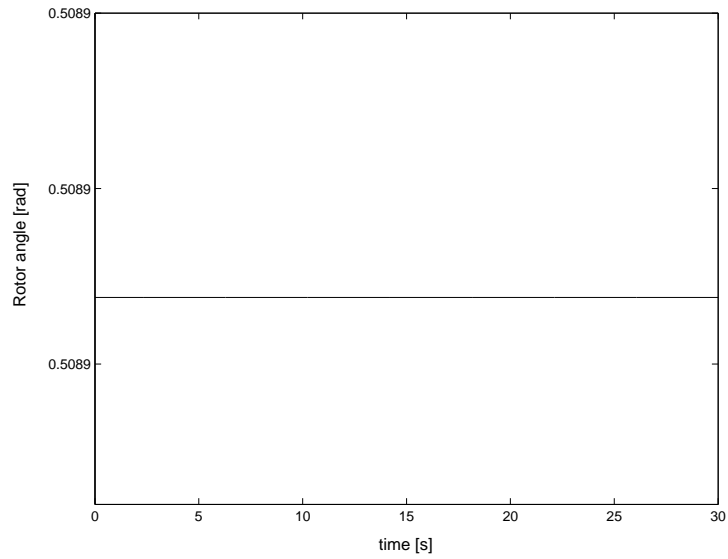


Figure 9: Rotor angle with  $\delta_0$  initial conditions

In Figure 10, one machine connected to the infinite bus system is shown, on the way how is presented in the previous case, shown in Figure 3, but with exciter and PSS included. Figures 11 and 12 represent the blocks of the exciter and PSS.

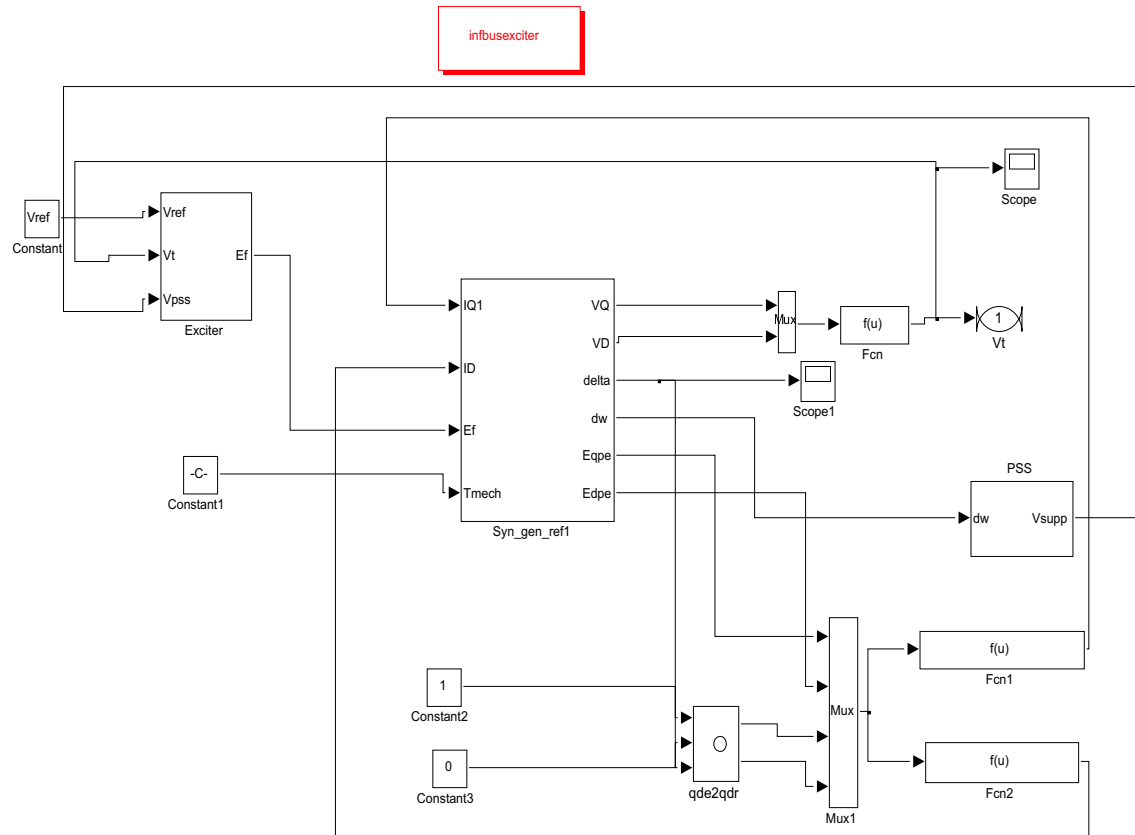


Figure 10: SIMULINK model of generator with the exciter and PSS

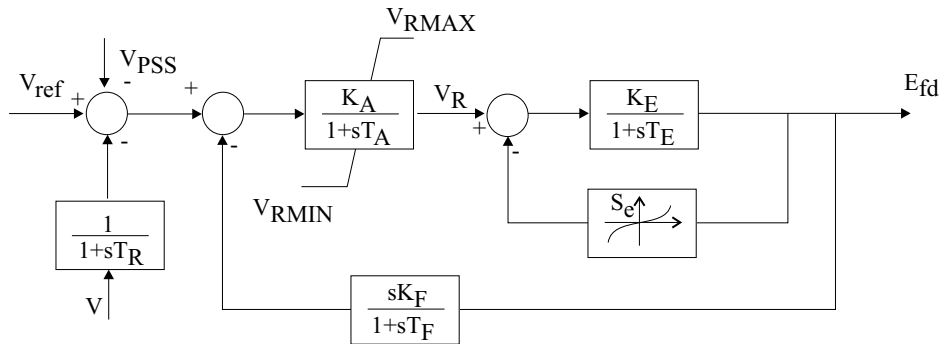


Figure 11: SIMULINK model of AVR and exciter

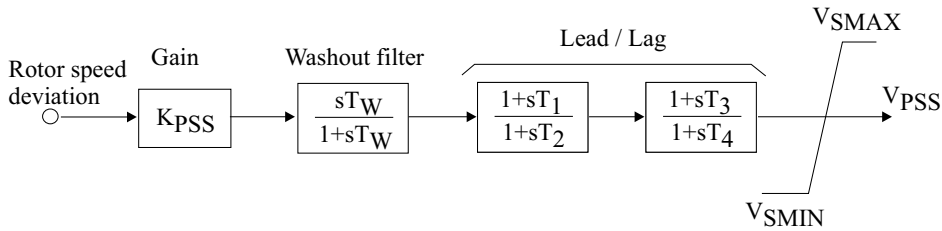


Figure 12: SIMULINK model of PSS

The absolute value of the terminal voltage and the response of the rotor angle for the system with the exciter and PSS are shown in Figure 13.

To check is this model correct or not, input referent voltage for the exciter is set on 1[pu]. The response of the system is in Figure 14.

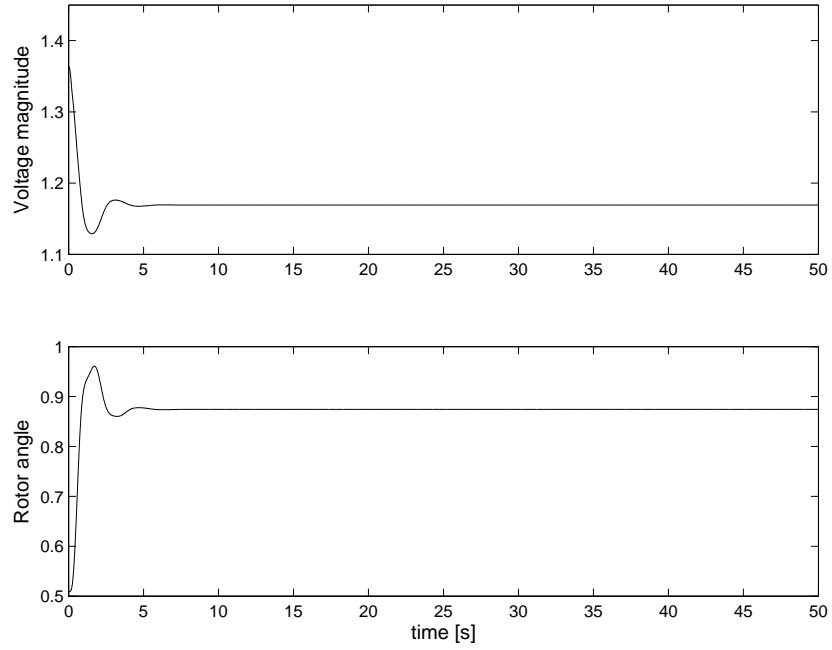


Figure 13: Response of generator equipped with the exciter and PSS with initial condition

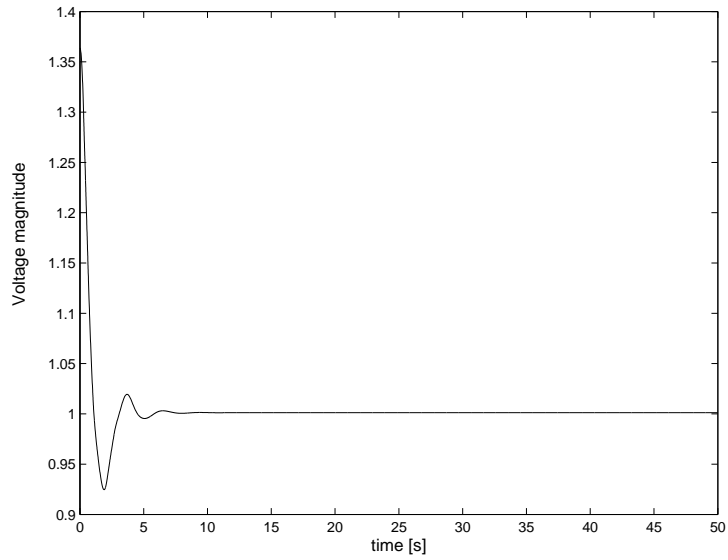


Figure 14: Terminal generator voltage with  $V_{ref}$  of 1 [pu]

## Terminal faults on a synchronous machine

The purpose of this part is to examine the response of a synchronous machine to electrical faults applied at its stator terminals. For the synchronous machine connected to the infinite bus according to Figure 3, the response of the voltage and angle are shown in Figure 15 and in Figure 16.

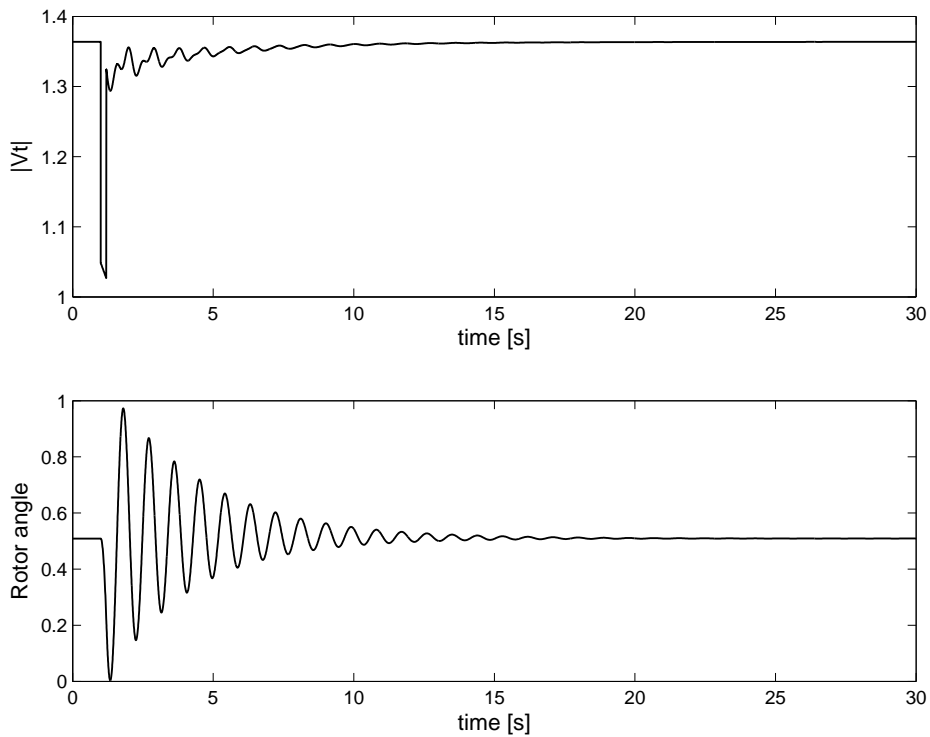


Figure 15: Response of generator to 0.2-sec duration fault at its terminal

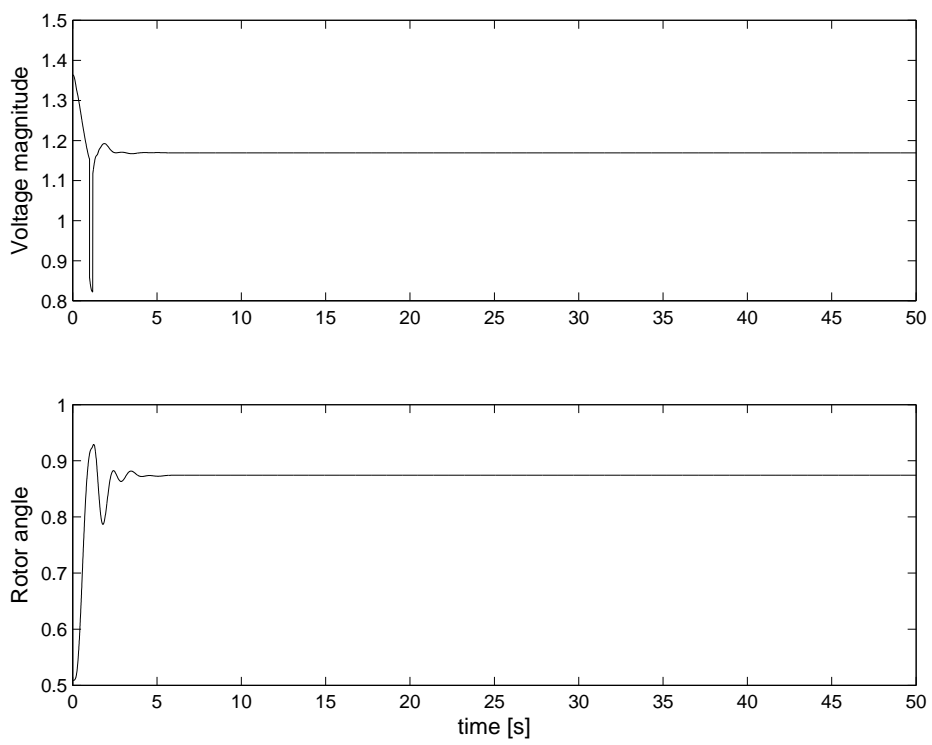


Figure 16: Response of generator equipped with exciter and PSS to 0.2-sec duration fault on its stator terminal

## References

- [1] Chee-Mun Ong, Dynamic Simulation of Electric Machinery, Prentice Hall, 1998
- [2] P. Kundur, Power System Stability and Control, McGraw-Hill, Inc., 1993

## Appendix

The generator parameters in per unit are as follows:

$$\begin{array}{llllll} X_d = 1.79 & X_q = 1.66 & X'_d = 0.355 & X'_q = 0.57 & R_s = 0.0048 & \\ T'_{d0} = 7.9s & T'_{q0} = 0.41s & H = 3.77 & D_w = 2 & T_{mech} = 0.8s & \end{array}$$

The exciter parameters in per unit are as follows:

$$\begin{array}{llllll} K_A = 50 & T_A = 0.06s & T_E = 0.052s & K_E = -0.0465 & & \\ T_F = 1.0s & K_F = 0.0832 & A_E = 0.0012 & B_E = 1.264 & & \\ V_R^{max} = 1 & V_R^{min} = -1 & & & & \end{array}$$

The PSS parameters are:

$$\begin{array}{llll} \text{wash-out network:} & K_s = 120 & T_w = 1 & \\ \text{lead-lag network:} & T_1 = 0.024 & T_2 = 0.002 & \\ \text{lag-lead network:} & T_3 = 0.024 & T_4 = 0.24 & \end{array}$$

The external line parameters are:

$$r_e = 0.2 \quad x_e = 0$$