Comparative Assessment of Prediction Models in Voltage Control

A.G. Beccuti, T. Demiray, M. Zima, G. Andersson and M. Morari

***Abstract*—Emergency voltage control problems in electric power networks have stimulated the interest for the implementation of online optimal control techniques. In this sense there have been a number of publications in the past few years concerning the employment of predictive control schemes to counteract the possibility of voltage collapses, all of which inherently rely on a prediction model of the network to choose the control action to apply onto the system. The tradeoff between complexity and accuracy of the chosen model is therefore relevant in the derivation of the control scheme, as for such a system-critical real-time application it must yield reliable predicted values whilst avoiding an excessively onerous degree of complexity.**

I. Introduction

The last decade or so has witnessed a common trend in the restructuring and privatization of electrical power systems taking place at an international level, also entailing that electrical networks are nowadays placed under more physical strain to fully economically exploit the system infrastructure; at the same time, due to environmental reasons, it is usually difficult to expand the network by means of new transmission lines. All this accounts for the fact that nowadays electrical networks are placed under greater strain than before. Therefore the risk of malfunctioning and/or system collapse has increased, as also confirmed by the recent large scale blackouts throughout the world [1]. The present situation calls for appropriate countermeasures to be taken in the direction of superseding still commonly used, local control schemes (such as under-voltage load shedding) with schemes capable of capturing the complexity of the global system dynamics. This is in line with new research trends dating back to the last decade, on the basis of which it was foreseen that "the time taken by the... (voltage) instability to develop, while short for a human operator, would be ample for a modern computer executing efficient software to identify the problem, warn the operator and suggest or trigger corrective actions. Such protection, based upon on-line system analysis and adapting its decision to the disturbance of concern, deserves attention, although it is still beyond the state of the art" [2].

Accordingly, the recent past has seen an increasing interest in novel voltage control methods employing on-line measurement and computation procedures that collect available measured data, analyze the predicted evolution of the system based on an analytic model of the network itself and therefrom deduce an optimal control input, namely in *model predictive control* (MPC) schemes. For such schemes it is clear that the type of the employed control model is of the utmost importance: it must be sufficiently accurate to capture the behaviour of the network's composite and variegated nonlinearities but simple enough to remain a viable control tool for application in such a critical setting. A number of predictive control schemes based on different control models and objectives has been presented and proposed in the past few years including, for example, [3], [4], [5], [6], [7], [8], [9]. The proposed work draws on previous results obtained in the aforementioned publications and assesses the comparative performance of alternative control models in terms of both accuracy and complexity for a selected network benchmark.

This paper is organized as follows. Section II gives an overview of the considered network benchmark, while Section III presents the control problem. In Section IV, MPC and its application in the context of voltage control according to the different modelling techniques are briefly reviewed. The proposed scenario and simulation results are shown in Section V and the conclusions outlined in Section VI summarize the main results obtained.

II. Network System and Dynamics

The case study under consideration is based on a simplified model of the Nordel grid describing the interconnected transmission system of Sweden, Norway, Finland and eastern Denmark [6] and is shown in Fig.1; a complete description can be found at [10]. For the purpose of the proposed work first-order load dynamics have been added to loads at buses 7 through 11 [11]. Indeed, it is these load dynamics which are typically responsible for classical voltage decay phenomena. Following a disturbance, such as the outage of a component in the grid, the load dynamics act to restore the power demand, possibly beyond the capability limits of the damaged system. As the self-restoring dynamics of the load take place generators try to sustain voltages in their vicinity by means of their automatic voltage regulator (AVR) [2]. After the capability limits have been reached the AVR controller saturates and appropriate control actions must be taken if voltages have not yet been adequately stabilized. Local controllers are included for all generators, that is the turbine governors, the AVRs and the Power System Stabilizers (PSS) modulating the AVR reference signal in order to dampen frequency oscillations. The synchronous machines are represented by a fourth order transient model describing the mechanical states plus the field and damper winding dynamics [12]. The network model overall has over 800 differential and algebraic variables. The available controls are the reference voltages of the AVRs, which lie in the interval 0.90-1.05 p.u., and the selected shedding factors of the loads, varying between 0 (no load shedding at all) and 1 (the whole load is shed).

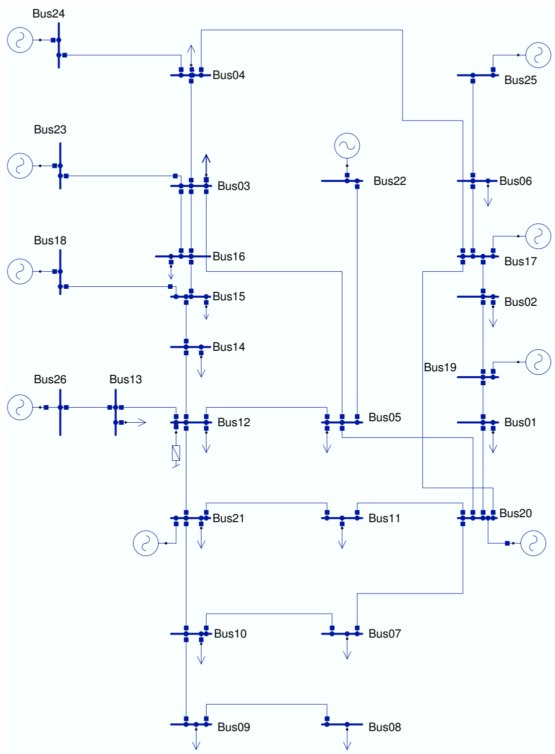


Fig. 1. Topology of the considered network

III. Control Problem

Following a disruptive event, such as an outage in the network, the primary aim of the emergency control scheme is to keep all voltages at values between 0.9 p.u and 1.15 p.u, that is sufficiently close to the nominal point of operation to ensure that the system is adequately far away from the point of collapse. The secondary aim is to minimize switching of the control inputs and to effectively restore the system to a steady state point of operation with constant input settings. Load shedding must be furthermore avoided unless required to fulfill the primary objective. Finally, it is also desirable to maintain the bus voltages relatively close to their nominal value 1 p.u..

IV. Control Scheme

A. Model Predictive Control

Model Predictive Control (MPC) has been traditionally and successfully employed in the process industry and has more recently been the object of study and investigation in power system control research [3], as it is a systematic control method allowing to define complex, multivariable systems subject to constraints by directly formulating an approximate *discrete-time control model* of the plant. The desired control aims are replicated in an appropriately selected cost function mirroring the order of importance of the imposed objectives. The control action is obtained by minimizing the cost function at each discrete time sampling instant over a finite horizon subject to the equations and constraints of the model, that is by solving the associated optimization problem. The first control move in the optimal sequence is then applied to the system and the procedure repeated at the successive sampling instant when the updated value of the system state is measured and taken as the new starting point for the optimal control problem. Further details about MPC can be found in [13].

B. Discrete Time Control Model

Due to the overall complexity and nonlinearity of the grid dynamics for a power network of realistic dimensions a common way of deriving a workable control model is online (i.e. at the specific sampling instant) linearization and time-discretisation of the system equations at the given operating point. The result is a model of the form

$$x(k+1) = A_k x(k) + B_k u(k) + f_k \tag{1a}$$

$$y(k) = C_k x(k) + D_k u(k) + e_k \tag{1b}$$

$$u(k) \in \mathcal{U} \tag{1c}$$

wherein $x(k)$ denotes the system states (describing the dynamics of the synchronous machines with associated controllers and of the loads), $y(k)$ the system ouputs (comprising among other values the bus voltage magnitudes), $\mathcal{U}$ the set of constraints on inputs $u(k)$ (AVR references and load shedding settings) and k the discrete time instant in the prediction horizon of the optimal control problem described in the following subsection; since the linearization is performed online matrices and vectors A_k, B_k, f_k, C_k, D_k and e_k are in general time-varying, as indicated by their subscript.

The formulation of an adequate discrete time control model is of paramount importance in the development of a consistent and effective optimal control scheme. It must be sufficiently accurate to capture the system dynamics but should be simple enough to remain viable for online derivation in such a safety-critical application. In view of these requirements a set of alternative control models of varying complexity/accuracy will be constructed and employed in the proposed simulations in order to assess their comparative performance, based on the following approaches: i) trajectory sensitivities ii) zero-order-hold (ZOH) iii) trapezoidal or backward Euler and iv) forward Euler. The latter three methods are standard integration techniques [14] employed for obtaining discrete time representations of differential-algebraic systems. The first approach is based on the work presented in [15] and employed in [16], [4], [6] in the context of both power systems analysis and controller synthesis. Trajectory sensitivities are time varying sensitivities derived along a predicted nominal trajectory of the system, computed by considering the full continuous time differential dynamics. They describe possible changes of conditions from which the nominal trajectory is predicted, as would be the case for example for a modification in the control inputs. By exploiting perturbation analysis theory and specific properties of the system's Jacobian matrix trajectory sensitivities afford an accurate reproduction of the nonlinear system behavior using a considerably reduced computational burden with respect to the full non-linear integration of the system trajectories; however, they are still more onerous to compute if compared with any of the other foregoing approaches ii)-iv).

Additionally, approaches ii)-iv) have also been applied to a simplified representation of the network equations; specifically in the context of a wide variety of voltage stability related issues and as mentioned in Section II, the primary driving force behind voltage decays is the behaviour of the load dynamics. These typically act on a slower time scale than the dynamics of the synchronous machines with associated local controllers, so that the equations describing the latter may be assumed to be at steady state and only the former retained in a reduced order differential-algebraic description of the system wherefrom the control model is derived. Correspondingly, the thus obtained approximation will be referred to as the *reduced* control model, whereas the *full* control model shall indicate that the complete system equations have been employed in the linearization.

C. Optimal Control Problem Formulation

The implementation of the MPC scheme entails the formulation of a cost function

$$J = \sum_{\ell=0}^{N-1} S(k+\ell|k) \tag{2}$$

penalizing the *predicted* evolution of the stage cost $S(k+\ell|k)$ from the discrete time sampling instant k over the finite horizon N by employing the quadratic norm. For the control problem at hand the stage cost is chosen to be the expression

$$\begin{aligned} S(k) = s(k)^T Q_s s(k) + u(k)^T Q_u u(k) + \\ \Delta u(k)^T Q_{\Delta u} \Delta u(k) + v_{err}(k)^T Q_v v_{err}(k) \end{aligned} \tag{3}$$

and thus comprises four terms respectively penalizing the slack variables $s(k)$ (which measure how far below or above each of the bus voltages is beyond the prescribed bounds), $\Delta u(k) = u(k) - u(k-1)$ (i.e. the variation of the control inputs between successive time instants), $u(k)$ (the control inputs themselves, in this case actually only the load shedding employed) and $v_{err}(k)$ (the distance of each bus voltage to the nominal per unit value); the (positive) penalty matrices Q_s, $Q_{\Delta u}$, Q_u and Q_v mirror the different priority levels of the control objectives.

The control law at time-instant k is then obtained by building online a model of the system of the form (1) and minimizing the objective function (2) over the sequence of control inputs $U = [u(k), \ldots, u(k+N-1)]^T$ subject to the derived model equations and constraints.

Notice that the sampling interval need not necessarily be equal to the prediction step employed in the horizon, that is the successive sampling instant and thus the successive optimal control problem instance do not have to temporally coincide with the first predicted step. With a slight abuse of notation this is not explicitly featured in the above formulation, the substance of which however remains unvaried: a set of optimal inputs is computed at a given sampling instant over the chosen horizon and the *first few control moves up to the following sampling instant* are then applied to the physical system.

V. Simulation Results

As a sample case study the double outage at $t = 2$ and $t = 12$ seconds of the lines connecting nodes 11-21 and 10-21 is considered, thereby creating a radial connection from nodes 20 to 8. Fig. 2 depicts the subsequent evolution of the network voltages if no control action is taken. As can be seen following the disturbance fast oscillations initially arise in the network voltages due to the generators' dynamics, but are ultimately dampened out by the local controllers; subsequently the loads' slower self-restoring dynamics come into play and progressively evolve causing a gradual decay in the voltages of buses 7 through 11, until after about a minute the configuration of the grid is physically unsustainable, ultimately leading to collapse. Adequate control action must be therefore be taken to countervail the possibility of system blackout and restore the grid to an acceptable operating state.

For the MPC scheme a sampling time of 4 seconds has been chosen, that is the optimal control problem is formulated and solved with this periodicity. The prediction step length of the horizon is set to 1 second and the length L of the horizon is chosen to be 12, so that the future 12 seconds of the system behaviour are predicted and the first 4 optimal control moves are applied to the grid; for consistency these are constrained to be equal in the prediction, as this reflects the effective control setup. Concerning the weights to be used in the cost function if not otherwise indicated the entries in the diagonal penalty matrices Q_s, $Q_{\Delta u}$, Q_u (used to actually penalize only the level of load shedding employed) and Q_v are respectively set to be 10^8, 1000, 5000 and 1.

A set of MPC controllers was derived based on the considerations made in Section IV starting from different possible control models, and their performance is illustrated in the following. In the presented case studies control models based on ZOH/forward Euler/backward Euler approaches yielded identical or near-identical results, so that only the results corresponding to the ZOH approach are shown in the plots.

Case 1: Simulation results stemming from the first controller to be considered, based on a model derived from trajectory sensitivity analysis, are depicted in Fig.3. As can be seen following the outage control action is taken on the load shedding, specifically at buses 8 and 9 (i.e. at the extremity of the radial connection), in order to alleviate the strain under which the network would otherwise be placed and restore system voltages to acceptable levels. Subsequently the AVR settings of the generators are adjusted to try to maintain voltages just on the border of the acceptable region of operation.

Case 2: Employing a control model constructed through ZOH time-discretisation of the full system equations yields the results shown in Fig.4. Again, immediate action is taken to increase voltage levels; notice that in this case load is shed at practically all buses of the radial connection, in contrast to Case 1. Furthermore the load shedding employed is adjusted in order to try to reduce the overall amount of load disconnected whilst keeping voltages at their prescribed limits, but because of the reduced accuracy of the model this induces extensive

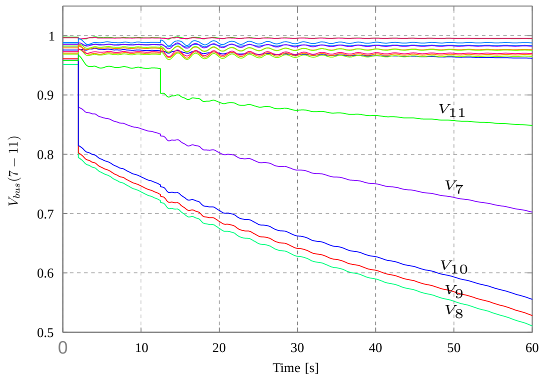
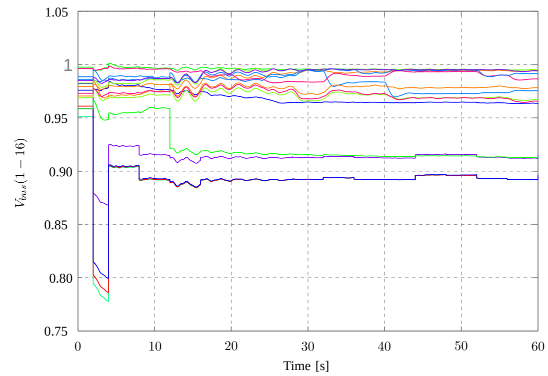
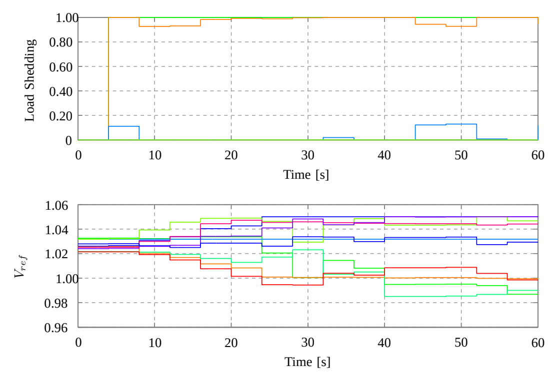


Fig. 2. Proposed simulation scenario



(a) Bus voltage magnitudes



(b) Inputs

Fig. 3. Simulation results for Case 1

Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
2.003	2.067	1.999	1.961	1.945	2.119

TABLE I
AVERAGE AMOUNT OF OVERALL LOAD SHED

chattering in the control inputs, which is undesirable especially in the case of load shedding.

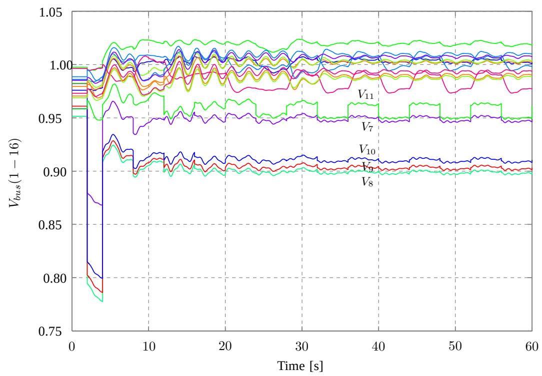
Case 3: Using a control model deduced through ZOH approximation of the *reduced* system equations yields a similar outcome to that of Case 2, as portrayed in Fig.5, illustrating that at least for the specific control problem neglecting the faster dynamics does not introduce any significant additional errors.

Case 4: By slightly modifying the cost function the performance of the controller derived in Case 2 can be improved, specifically by increasing the penalization for the deviation of the load shedding settings by a factor of 20. A controller based on the same model utilized therein then yields the behaviour illustrated in Fig.6. Load shedding is now only slightly adjusted in order to reduce the amount of load disconnected whilst maintaining voltages at their lower bounds.

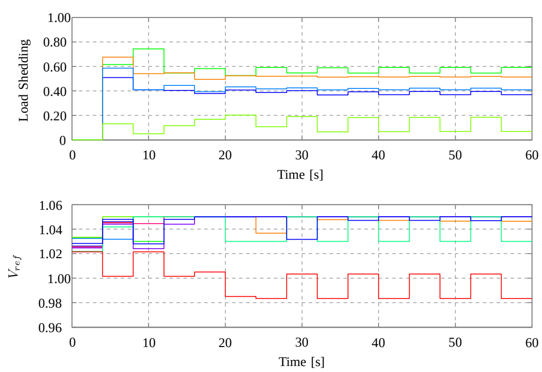
Case 5: The same modification in the cost function has been used for the model considered in Case 3. Similar results are obtained as in Case 4, see Fig.7, showing that with the appropriate adjustments similar performance can be obtained with simpler dynamical models.

Case 6: Lastly, simulation results achieved with a further slightly modified cost function based on a ZOH approximation of the full system equations are presented in Fig. 8. For this case the cost function has been changed so as not to include the weighting term penalizing the absolute value of load shedding employed, yielding a controller which simply keeps voltages within the imposed bounds and does not try to reduce the amount of load shed; this would implicitly push some bus voltages just at the lower admissible limit as illustrated in the foregoing case studies. Rather, a slightly higher amount of load is disconnected than before and all the AVR references are kept to their upper limits so that voltages are restored to a safe range of values; subsequently no further action is taken since maintaining the given input profile constant constitutes the cheapest control strategy according to the modified cost function. The same results are obtained also with a reduced control model.

Assessment of Results: The different control models yield varying degrees of accuracy in the prediction which are in turn mirrored in the type of controller response achieved. The controller based on the trajectory sensitivities approach (Case 1) captures the multi-objective nature of the problem and possesses the precision required to tune the AVR settings in order to keep the voltage values at the limits; controllers based on models of reduced accuracy (Cases 2 and 3) seem to push the references to their upper bound and subsequently can be prone to generating some undesirable chattering in the load shedding, but by adjusting the cost function coefficients (Case 4 and 5) this phenomenon can be considerably attenuated. Finally, by eliminating entirely one of the objectives, the simplified control problem can be solved by just restoring the system to a safe region of operation. Concerning the complexity of the aforementioned methods, an elementary analysis of the computational burden related to each has been performed by simply recording the average amount of time needed to build the control model online on a standard Pentium IV 3GHz machine running Matlab 7.0: the model based on trajectory sensitivities requires about 4.9 seconds, the ZOH integration takes about 0.47/0.36 seconds and the forward/backward Euler



(a) Bus voltage magnitudes



(b) Inputs

Fig. 4. Simulation results for Case 2

methods take 0.27/0.16 seconds (full/reduced system equations).

It should be underlined that the above considerations specifically exploit the fact that classic voltage decay behaviour typically manifests itself through a relatively slow, monotonically decreasing evolution of system variables; the presented approach would presumably not be applicable for example to power system oscillations or related phenomena.

VI. Conclusions

A comparative assessment of prediction models in MPC control approaches for dealing with voltage stability issues has been presented and discussed. Depending on the control setup and required objectives, also simplified models based on standard integration procedures and neglecting faster dynamics can still yield satisfactory performance.

Acknowledgments

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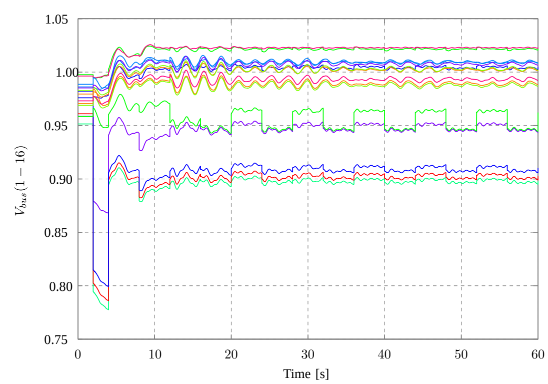
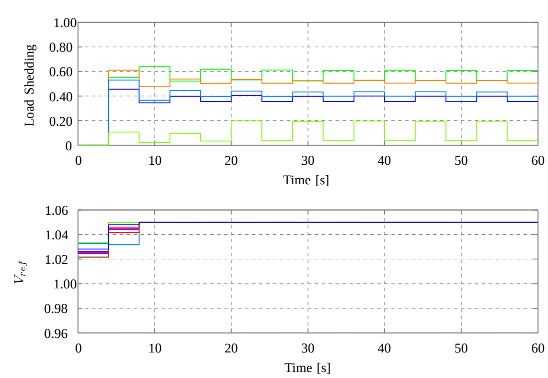
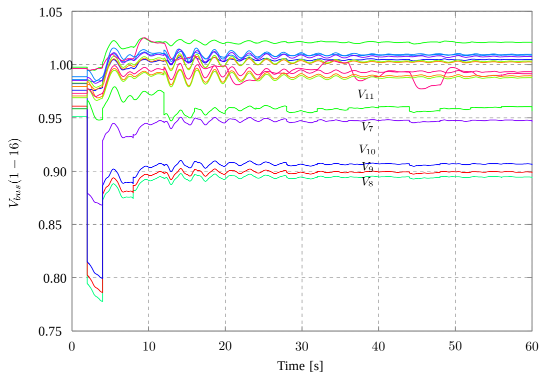


(a) Bus voltage magnitudes

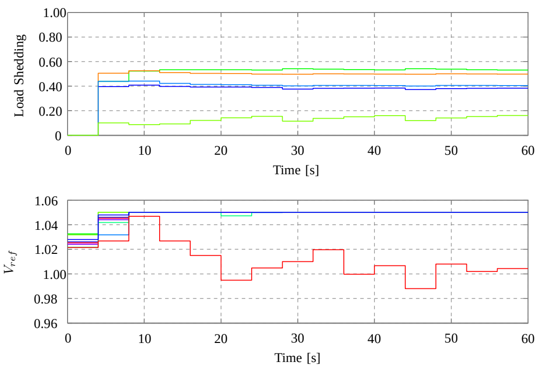


(b) Inputs

Fig. 5. Simulation results for Case 3

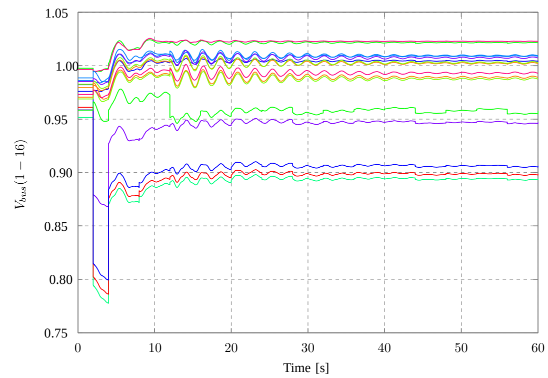


(a) Bus voltage magnitudes

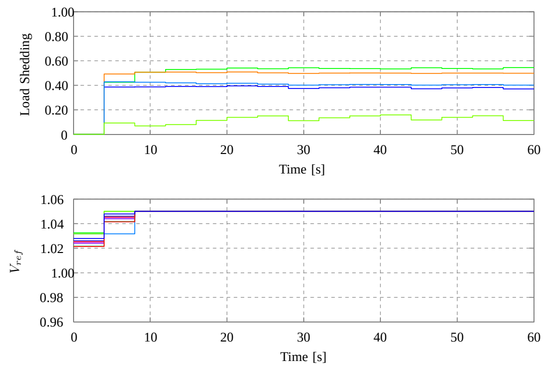


(b) Inputs

Fig. 6. Simulation results for Case 4



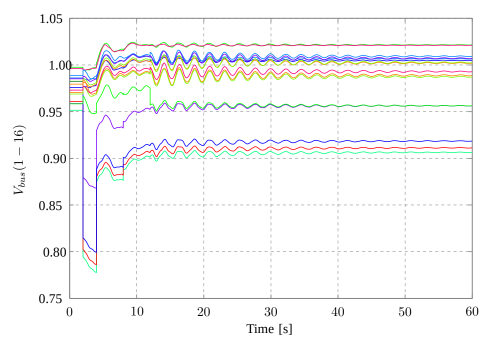
(a) Bus voltage magnitudes



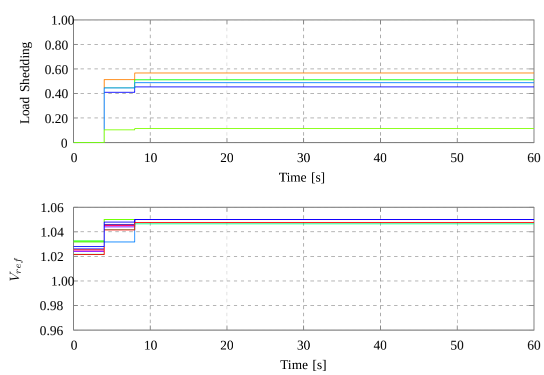
(b) Inputs

Fig. 7. Simulation results for Case 5

of january 1st, 1997 using trajectory sensitivities," *IEEE Transactions on Power Systems*, vol. 14, no. 3, August 1999.



(a) Bus voltage magnitudes



(b) Inputs

Fig. 8. Simulation results for Case 6