An Application of the Energy and Transmission Price Conjecture in an Oligopolistic Power Market

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***Abstract*— In network-constrained electricity market models of oligopolistic competition among power producers, strategically behaving generators are likely to manipulate prices in the energy and transmission market in order to gain higher profits. As for the energy market, several game theoretical approaches have been applied to represent strategic generators' behavior. Another approach is used in this paper which needs external sensitivity parameters to be estimated known as Conjectural Variations (CV). Additionally, the strategic generators' effect on transmission prices is taken into account by means of introducing Conjectured Transmission Price Response (CTPR) parameters. An estimation procedure of the CV- and CTPR- parameters will be within the scope of this paper. The results will show that a more accurate market outcome can be achieved when additionally accounting for strategic behavior in the transmission market. The overall market design forms an equilibrium model of bilateral trading with perfect arbitrage based on a point-to-point transmission pricing scheme and is formulated as a mixed linear complementarity (MLCP) problem.**

***Index Terms*— Conjectured transmission price response, transmission market, point-to-point transmission pricing, conjectural variations, mixed complementarity problem.**

I. Introduction

ONE of the goals with the introduction of liberalization in the electricity market sector is to enhance competition among power producers, which is supposed to result in reduced electricity prices. However, within an oligopolistic market structure strategically behaving companies are able to hold or further increase electricity prices above competitive levels by exerting their market power in one or more of the following submarkets: the market for electrical energy, the transmission market, the spinning reserve market, the market of financial transmission rights.

In the literature a wide range of oligopolistic price equilibrium models exists aiming at studying generator behavior in order to help analyze market designs and regulatory policies. Different game theoretical approaches are being used to provide strategic generators with the essential knowledge and information to affect market prices profitably. However, the implementation of strategic generator behavior comes along with an increased mathematical complexity which puts a constraint on those models' realization. After all, computational tractability is becoming increasingly important, especially when analyzing large-scale market models.

The assessment of strategic generator behavior cannot only be confined to the market of electrical energy. As can be seen throughout northwestern Europe electricity power markets often consist of several regional subnetworks which are connected by frequently congested cross-border transmission lines. Consequently, congestion decouples those markets, reduces overall competition and results in different electricity prices in the respective power markets. Therefore, depending in which price area their generating units are located, strategic generators are likely to pursue different strategies in the transmission market in an effort to either decrease or increase congestion prices.

Several authors overcome the difficulties of solving non-convex oligopolistic market models formulated as EPECs (Equilibrium Problem with Equilibrium Constraints) by introducing the Conjectural Variations (CV) approach for each firm acting strategically in the energy market. Regarding the transmission market Hobbs and co-authors [1] set up Conjectured Transmission Price Response (CTPR) functions which enable strategic generators to have an additional effect on transmission prices. In mathematical terms, both approaches define a first-order Taylor approximation whose coefficients need to be solved exogenously. While a lot of work has been done on estimating the CV-parameters [3] corresponding estimation procedures relating to the CTRP-parameter hardly exist in the literature.

This paper will show that both parameters can be determined by taking advantage of (available) historic market data. Based on the generators' KKT (Karush-Kuhn-Tucker) -conditions a set of linear equations is set up to determine the CV- and CTPR-parameters uniquely. Synthetic market will be generated in order to estimate both parameters and to test the application of this procedure within a transmission constrained market model environment. Furthermore, the advantage of additionally accounting for strategic generator behavior in the transmission market will be shown by comparing the results with classical game theoretical approaches such as Cournot and Bertrand. The results will show that including strategic generator behavior in the transmission market allows for a more realistic modelling of electricity markets.

The structure of the paper is as follows: Section II highlights the advantages of modelling strategic generator behavior by introducing the CV approach in the energy market and the CTRP function in the transmission market. The mathematical formulation of the market model is given in Section III followed by Section IV which presents the linear system of equations for the determination of the CV and CTRP parameters. Section V shows an application of the strategic generator model to the Northwestern European model used in [2]. The paper concludes with a summary and an outlook in Section VI.

II. Strategic Generator Behavior

Building up reliable models of liberalized electricity markets in an effort to achieve reasonably market outcomes such as electricity and transmission prices, demand and generation levels remains a challenging task. Several influencing factors such as the choice of the market design, the congestion management scheme, the generating companies' bidding behavior and the network representation may have a major impact on the resulting market outcomes and consequently need to be modelled adequately. However, when accounting for the interrelation between all those factors, the mathematical complexity of the electricity market model increases significantly and a unique solution cannot be guaranteed anymore.
One of the mathematically most demanding impact factors on market prices concerns the modelling of strategic generator behavior. In an oligopolistic market structure large generating companies can abuse their position in the market to affect electricity and transmission prices profitably by pursuing different generation and sales strategies than in a perfectly competitive manner. As far as the energy market is concerned strategically behaving companies have an incentive to restrict their generation output in order to raise the offer price on marginal units. Furthermore, they can influence the transmission price by taking advantage of their location with respect to the congested transmission line and redistribute their sales [4]. In both cases those generators exploit their price making ability to influence the market equilibrium point. In general, it is said of those generators to posses a potential of market power which they can exert through strategic bidding as a result of their market dominance and transmission constraints in the network [5].
In the following the strategic behavior model used in this paper will be presented. As to the energy market the widely known CV approach is being used and will therefore be described in a short way. The focus will be to introduce the benefits of the CTPR approach which enables to model several degrees of competition in the transmission market. In mathematical terms both approaches are first-order Taylor approximations and need to require the determination of an exogenous parameter.

A. The concept of Conjectural Variations (CV)

The CV method considers the reaction of competitive firms when a strategic firm is deciding on its optimal sales strategy. It is a more sophisticated modelling approach of strategic behavior than the widely used Cournot competition because it takes into account how competitors change their sales as a result of a change in the firm's sales output.
Mathematically, the CV equilibrium can be stated as

$$s_{-f,i} = s^*_{-f,i} + CV_{f,i} \cdot (s_{f,i} - s^*_{f,i}), \tag{1}$$

where $s_{f,i}$ denotes firm f's sales and $s_{-f,i}$ all competing firms' total sales at node i, which is supposed to be a load center ($i \in L(i)$). All variables with an asterisk ($*$) indicate market equilibrium values of that variables.
Modelling each load as a linear decreasing demand function, nodal price π_i and firm f's sold energy $s_{f,i}$ are interrelated by

$$\pi_i(s_{f,i}) = \alpha_i + \beta_i \cdot (s_{f,i} + s_{-f,i} + a_i), \tag{2}$$

where α_i (> 0) and β_i (< 0) denote intercept and slope of the demand function and a_i the sold energy by arbitrager at node i [1]. Inserting (1) in (2) one gets $\forall i \in L(i)$

$$\pi_i(s_{f,i}) = \alpha_i + \beta_i \cdot \big(s^*_{-f,i} - CV_{f,i} \cdot s^*_{f,i} + s_{f,i} \cdot (1 + CV_{f,i}) + a_i \big). \tag{3}$$

Different levels of competition can then be achieved by assigning values to the CV parameter ranging from -1 ("perfect competition") to 0 ("Cournot competition") [3].
Difficulties arise in the empirical determination of the CV parameter. Some development on this area has been done by Garcia-Alcade et al. [3].

B. The Conjectured Transmission Price Response (CTPR)

A real advantage of the CTPR approach is that it overcomes the difficulties of solving generator models formulated as a Mathematical Program with Equilibrium Constraints (MPEC) which then lead to a market model known as an Equilibrium Problem with Equilibrium Constraints (EPEC). Furthermore, one can easily expand the application to other submarkets such as the capacity market [11], emission allowances [12] without significantly complicating the resulting model while retaining good convexity properties. However, one of the main drawbacks is that it requires to specify the CTPR's coefficient, which remains a challenging task for future research.
Similar to the Conjectured Supply Function approach [10] a CTPR function can be introduced in order to capture the generators ability to influence the transmission prices. In contrast, models in which a generator correctly anticipates the influence of its generation decisions on the costs of transmission require the first-order conditions for the TSO (Karush-Kuhn-Tucker conditions) to be imbedded in the generators' constraint set [7]. In this way, strategic generators foresee the impact of their actions on the transmission price. However, the profit-maximization problem for each generator becomes an MPEC, which is an optimization problem with a non-convex feasible region, and for that reason such a model is generally difficult to compute for large systems.
One way to avoid the related difficulties is to use a Bertrand or a Cournot assumption on transmission prices, which simplifies the market model considerably. In the Bertrand model strategic generators anticipate that prices of transmission services or locational price differences cannot be affected by their actions whereby a Cournot-like behavior towards the ISO's actions makes the strategic generators believe that ISO's injections/withdrawels into/from a bus are constant [8]. In [9] a hybrid Bertrand-Cournot model with respect to transmission decisions by the ISO is presented, in which, depending on the congestion pattern within individual subnetworks, the choice for one approach is more defensible than for the other.
An alternative model to ease the problem is to include smooth functions for modelling the manipulation of the transmission

prices. This has been done by introducing the CTPR function which makes the problem to be treated as being convex and modelled as a Mixed Linear Complementarity Problem (MLCP) [8]. It allows strategic generators to behave in a more sophisticated manner, such that they anticipate how the prices of point-to-point transmission or the prices resulting from auctions of transmission interfaces will change as a result of their demand for transmission services. According to a point-to-point transmission scheme the CTPR function can be defined as

$$\begin{aligned}\omega_i(s_{f,i}, g_{f,h,i}) = \omega_i^* + \\ CTPR_{f,i} \cdot [\ (s_{f,i} - \textstyle\sum_h g_{f,h,i}) - \\ (s_{f,i}^* - \textstyle\sum_h g_{f,h,i}^*)\ ],\end{aligned} \quad (4)$$

$\forall i \in L(i) \cup P(i)$ (demand and/or generation nodes). Suppose a firm f believes that a change of its sales $s_{f,i}$ or generation output $\sum_h g_{f,h,i}$ of all owned units h located at node i induces a change of the transmission price ω_i. If that change is given through a relation as described in (4), then the $CTPR_{f,i}$ value represents the modeler's judgment about how each firm might anticipate that the transmission price will change.

Unlike the MPEC-based formulation of each strategic generator's profit maximizing problem, which results in an endogenous and correct determination of the transmission price, the CTPR parameter is an exogenous assumption. Furthermore, it allows to the modelling of large generators reasonably insofar as they are assigned a large response, while small ones might assume no response [1].

Since the correct determination of the CTPR parameter held by strategic generators is a challenging task which has not been addressed in an analytical way so far [1], [11], most authors treated this parameter to be either zero (”naive assumption on transmission prices”) or at random to permit a certain degree of influence on the transmission prices.

III. Model formulation

The formulation of the network-constrained market equilibrium model as a MLCP in this paper is quite similar to several ones done in previous works. It simulates a bilateral market with perfect arbitrage based on a point-to-point transmission service scheme, when strategic generators assume that their demand on transmission capacity will change transmission prices. In the following a general model formulation for each of the market participants will be given without going too much into the interpretation of the market actors' models. For more information the authors would like to refer to [1], [8], [9], [10].

A. The generator model

A strategic generator has to decide on its generation and sales level in order to maximize its profit. Within the following model environment generating companies have access to several different nodes (markets) in the network. Thus, their sales have to be evaluated in an optimal way for each of the markets they participate in. If the area of production differs from the area of sales, the respective generator needs to be charged for any point-to-point (market-to-market) transmission service that would change the amount of flow on any fully used transmission line (flowgate). Hence, the generating firm's profit function is formulated as follows:

$$\begin{aligned}\max \quad & \sum_{i \in L(i)} (\pi_i - \omega_i) \cdot s_{f,i} - \\ & \sum_{\substack{i \in P(i), \\ h \in H(f,i)}} (C_{f,h,i}(g_{f,h,i}) - \omega_i \cdot g_{f,h,i}),\end{aligned} \quad (5)$$

where π_i is the nodal price at node i, ω_i is the conjectured price of transmission service from the hub to node i by firm f and $C_{f,h,i}(g_{f,h,i})$ denotes the total generation cost by unit h located at node i.

The structure of the generator model is completed by adding constraints on generation capacity and energy balances as follows

$$g_{f,h,i} \le G_{f,h,i}, \quad (\mu_{f,h,i}), \quad \forall\ i, h \in P(i), H(f,i) \quad (6)$$

$$\sum_{i \in L(i)} s_{f,i} = \sum_{\substack{i \in P(i), \\ h \in H(f,i)}} g_{f,h,i}, \qquad (\theta_f) \quad (7)$$

$$\forall\ s_{f,i}, g_{f,h,i} \ge 0. \quad (8)$$

B. The TSO model

The TSO's objective is the efficient allocation of scarce transmission capacity to the most highly valued transmission services [1] subject to the DC load flow constraints

$$\max \quad \textstyle\sum_i \omega_i^* \cdot y_i \quad (9)$$

$$\text{s.t.:} \quad \textstyle\sum_i PTDF_{i,k} \cdot y_i \le T_{k+}, \quad (\lambda_{k+}) \quad \forall k \quad (10)$$

$$-\textstyle\sum_i PTDF_{i,k} \cdot y_i \le T_{k-}, \quad (\lambda_{k-}) \quad \forall k \quad (11)$$

The TSO behaves like a price taker with respect to transmission prices ω_i^* while deciding on the most valuable set of transmission services y_i to provide in order to maximize its profit. As done in several previous works a PTDF-based (Power Transfer Distribution Factor) representation of the physical network has been applied. For more details please refer to [1], [10].

C. The arbitrager model

The model in this paper assumes perfect arbitrager to have access to all the markets. They can buy and sell power elsewhere in order to eliminate any price difference among the nodes (markets) that are beyond transmission cost. Finally, transmission prices will always be equal to nodal price differences. Thus, the profit function for arbitrager yields

$$\max \quad \sum_{i \in L(i)} (\pi_i^* - \omega_i^*) \cdot a_i \quad (12)$$

$$\text{s.t.:} \quad \sum_{i \in L(i)} a_i = 0. \qquad (\pi_{hub}) \quad (13)$$

Constraint (13) ensures that arbitrager are neither net producers nor net consumers.

D. The market clearing conditions

Following market clearing conditions are needed in order to couple the optimization problems of all market participants and to enforce a market equilibrium:

$$y_i = \sum_f s_{f,i} - \sum_{f,h} g_{f,h,i} + a_i^*, \quad \forall\, i \tag{14}$$

$$\pi_i^* = \pi_{f,i}(\sum_f s_{f,i} + a_i^*), \quad \forall\, i \in L(i) \tag{15}$$

$$a_i^* = a_i, \quad \forall\, i \in L(i) \tag{16}$$

$$\omega_i^* = \omega_i. \quad \forall\, i \in L(i) \cup P(i) \tag{17}$$

E. Solution approach

The equilibrium model of the power market is formulated mathematically as a MLCP, which is composed of all market players' KKT conditions linked with the market clearing equations as described in detail in [1]. For this resulting set of equations to be solved, one has to ensure the number of equations to be equal the number of variables. Finally, the mathematical problem is solved with the PATH-solver, which is available for GAMS [13].

IV. Determination of CV and CTPR parameters

In the following a system of linear equations will be derived in an effort to determine empirically the CV and CTPR parameters in a transmission constrained network. It is important to point out that those equations strictly hold for the bilateral trading model based on a point-to-point transmission pricing system.

A. Methodology

The lagrangian function of a firm's profit maximization problem as introduced in Section III-A is as follows:

$$\begin{aligned} &\ell(s_{f,i}, g_{f,h,i}, \mu_{f,h,i}, \theta_f) = \\ &\sum_{i \in L(i)} (\pi_i - \omega_i) \cdot s_{f,i} - \\ &\sum_{\substack{i \in P(i),\\ h \in H(f,i)}} (C_{f,h,i}(g_{f,h,i}) - \omega_i \cdot g_{f,h,i}) + \\ &\sum_{\substack{i \in P(i),\\ h \in H(f,i)}} \mu_{f,h,i} \cdot (G_{f,h,i} - g_{f,h,i}) - \\ &\theta_f \cdot (\sum_{i \in L(i)} s_{f,i} - \sum_{\substack{i \in P(i),\\ h \in H(f,i)}} g_{f,h,i}). \end{aligned} \tag{18}$$

Then, the KKT condition with respect to the variable $s_{f,i}$ is defined as

$$\begin{aligned} 0 \le s_{f,i} \perp \; &(\beta_i \cdot (1 + CV_{f,i}) \cdot s_{f,i} + \pi_i) - \\ &[\omega_i^* + CTPR_{f,i} \cdot (s_{f,i} - \textstyle\sum_{h \in H(f,i)} g_{f,h,i})] - \\ &\theta_f \le 0. \end{aligned}$$

If one notes that the above KKT condition equals to zero for $s_{f,i} \neq 0, \forall i \in L(i)$ and that θ_f describes each firm's marginal costs at the hub node MC_f^{hub}, then that condition can be split up in the following two equations:

$$\textbf{I}. \quad \pi_i = MC_{f,i}^{eff} - \beta_i \cdot (1 + CV_{f,i}) \cdot s_{f,i} \tag{19}$$

$$\begin{aligned} \textbf{II}. \quad &MC_{f,i}^{eff} = \omega_i^* + MC_f^{hub} + \\ &CTPR_{f,i} \cdot (s_{f,i} - \sum_{h \in H(f,i)} g_{f,h,i}). \end{aligned} \tag{20}$$

The new variable $MC_{f,i}^{eff}$ has been introduced, which represents firm f's effective marginal costs at node $i \in L(i)$. Eq. (19) resembles to what has been already found out in [3] for electricity market models disregarding the transmission network. There, each firm's marginal costs are simply derived from the marginal generating unit of its whole power plant fleet. Furthermore, eq. (19) can also be interpreted as firm f's bidding function based on a specific level of competition depending on the value of the CV parameter. It then says that for $-1 < CV_f \le 0$ firm f is willing to sell a specific amount of energy s_f for a price π higher than its marginal generation costs MC_f.
However, when considering the transmission constrained power network different price levels arise at nodes $i \in L(i)$ when congestion occurs. Hence, from the strategic firm's perspective its optimal sales output at a demand node i depends considerably on the marginal cost occurring for that firm related to that node. Those costs arise even if a firm does not own any generation units located at the same node it sells energy to. However, marginal costs can be assigned to each of the demand nodes by means of the marginal costs at the hub MC_f^{hub} and the nodal price for transmission capacity ω_i^* (eq. (20)). It then represents firm f's total marginal costs assigned to that node arising from generation and transmission costs of the specific amount of energy to that location ($\omega_i^* + MC_f^{hub}$). When additionally accounting for each strategic firm's impact on the transmission price a second term is being added to the firm's marginal cost according to eq. (20), which includes the CTPR parameter ranging from Bertrand- ($CTPR_{f,i} = 0$) to Cournot-like ($CTPR_{f,i} = \infty$) behavior in the transmission market. It accounts for the impact of the weighted net energy injection/withdrawal caused by a firm f at node $i \in P(i) \cup L(i)$ on its marginal costs.
An empirical determination of the CV and CTPR parameters by means of historical market data seems to be possible as a result of the linear characteristic of eqs. (19) and (20). In case all parameters ($\pi, \beta_i, \omega_i, s_{f,i}, g_{f,h,i}, MC_f^{hub}, MC_{f,i}^{eff}$) are known this system of equations can be solved uniquely for $CV_{f,i}$ and $CTPR_{f,i}$. However, the requirement of some sensitive data like $MC_{f,i}^{eff}$ and $s_{f,i}$ makes this approach quite difficult to be followed. As a result of that some of the needed data will here be acquired by simulation.

B. Synthetic Market Data (SMD)

The determination of the CV and CTPR parameters based on historical market data analysis allows one to identify each strategic firm's impact on energy and transmission prices. It then enables to model electricity markets by producing more realistic market outcomes than standard game-theoretical

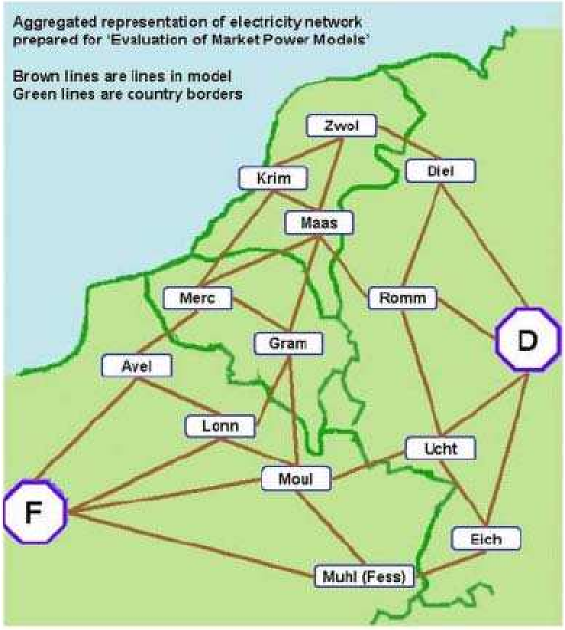

Fig. 1. Linearized DC network of Northwestern Europe

approaches like Bertrand and Cournot can do. In an effort to test this ability and because of the difficulty in acquiring the needed market data for the determination of the CV and CTPR parameters, a set of *Synthetic Market Data (SMD)* will be generated as a result of several simulation runs of the test system. These simulation runs include Bertrand- and Cournot-like behavior as well as estimated values of the CV and CTPR parameters for each strategic firm[1]. Then, the simulation results of all parameters are averaged and defined as SMD, which provides the set of data upon which CV and CTPR parameters are to be determined.

V. Case Study

The underlying electricity market model used to test the combined application of the CV and CTPR approach shown in Fig.1 is known as the COMPETES market model [14].

It covers the northwestern European electricity markets of the Netherlands, Belgium, Germany and France and is applied in several case studies [1], [2] to simulate strategic generator behavior in a transmission constrained network.
Several assumptions made to the application in this paper will be summarized in the following. In terms of input data regarding maximum flowgate capacities and reactances, generating unit characteristics and load data please refer to [14].
The electricity market consists of seven load and generation centers located at the nodes D, Zwol, Krim, Maas, Merc, Gram and F. Each generating company in the four countries owns several production units and can either be modelled as a strategic firm or as a competitive fringe firm. As in [2] the largest generating companies in the four countries are modelled as strategic players since they are likely to affect the energy and transmission prices by their actions.
The model considers demand loads for five time periods, denoted as $Sz_1 - Sz_5$, for each of the four countries. For each of the load centers demand is modelled as a linear decreasing curve based on a demand elasticity of -0.2 at the price quantity pair for the competitive solution.

[1]CV values are determined approximately as a function each firm's market share while CTPR values are estimated as a function of each firm's nodal injection based on the solution of a perfectly competitive market.

A. The reference case

Each strategic player's level of competition is described by CV values in the energy and CTPR values in the transmission markets. Their determination results from the application of the linear eqs. (19) and (20) for which available market data is needed. As some data is very sensitive to acquire a set of SMD will be generated for each of the five load scenarios according to the procedure explained in Section IV-B.
Tab. I and II show the resulting energy and transmission prices while Fig. 2 depicts the course of load. Since Germany (D) is treated as the hub-node the transmission price ω_D is equal to zero for all scenarios. Furthermore, since the model used in this paper assumes perfect arbitragers to have access to all load centers (markets), energy price differences will always be reflected by the difference of transmission prices of the corresponding nodes. Thus, it holds $\forall i, j \in L(i), L(j) \wedge i \neq j$:

$$\pi_i - \pi_j = \omega_i - \omega_j. \tag{21}$$

In addition to that Tab. III displays all congested transmission lines for each scenario. This is an important issue since the market outcomes of scenarios Sz_1 to Sz_4 provide the basis for the calculation of the conjectured parameters. As a result of that it will be interesting to analyze the accuracy of the combined CV and CTPR approach related to electricity price modelling when being determined based on different congestion scenarios than the ones to be applied to.

TABLE I
Energy prices in €$/MWh$

	D	Zwol	Krim	Maas	Merc	Gram	F
Sz_1	21,91	33,94	40,37	39,94	47,52	34,17	12,16
Sz_2	15,77	27,42	32,85	33,72	37,44	21,26	4,23
Sz_3	17,57	31,25	38,66	37,35	47,85	31,01	4,23
Sz_4	16,03	27,84	32,45	35,37	33,80	15,59	4,23
Sz_5	16,28	28,16	33,15	35,24	35,87	18,08	4,23

TABLE II
Transmission prices in €$/MWh$

	D	Zwol	Krim	Maas	Merc	Gram	F
Sz_1	0,00	12,03	18,46	18,03	25,61	12,25	-9,75
Sz_2	0,00	11,65	17,08	17,95	21,67	5,49	-11,54
Sz_3	0,00	13,68	21,09	19,78	30,28	13,45	-13,34
Sz_4	0,00	11,81	16,42	19,34	17,77	-0,44	-11,80
Sz_5	0,00	11,88	16,87	18,96	19,59	1,80	-12,05

B. Modus operandi

The market outcomes (Tab. I and II) of all five load scenarios will serve as reference values and compared to

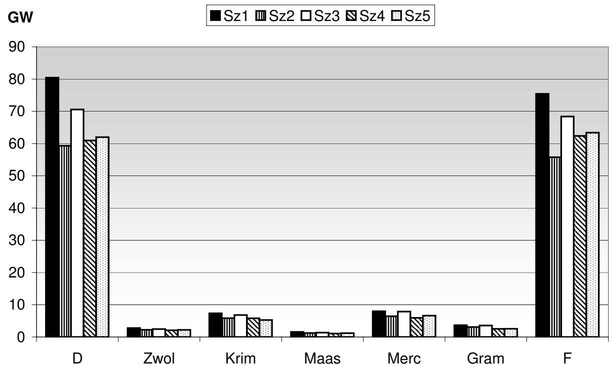

Fig. 2. Load levels for all scenarios

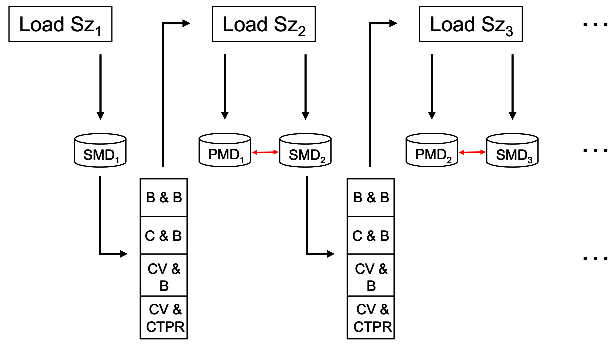

Fig. 3. Procedure to test different strategic modes

TABLE III

CONGESTED TRANSMISSION LINES

Sz_1	Romm→Maas	-	Avel→Merc
Sz_2	Romm→Maas	Gram→Maas	Avel→Merc
Sz_3	Romm→Maas	-	Avel→Merc
Sz_4	Romm→Maas	Gram→Maas	Avel→Merc
Sz_5	Romm→Maas	Gram→Maas	Avel→Merc

TABLE IV

DIFFERENT STRATEGIC MODES

strategic mode	energy market	transmission market
I	Bertrand $CV = -1$	Bertrand $CTPR = 0$
II	Cournot $CV = 0$	Bertrand $CTPR = 0$
III	Conjectural Variation $CV = ?$	Bertrand $CTPR = 0$
IV	Conjectural Variation $CV = ?$	Conject. Transm. Price Resp. $CTPR = ?$

those resulting from the application of the game-theoretical approaches listed in Tab. IV.

In addition to the often used standard game-theoretical models of strategic generator behavior like the popular Bertrand and Cournot approaches in energy markets the more sophisticated CV approach will also be evaluated [3]. However, all three approaches lack in terms of accounting for the strategic firm's impact on transmission prices and are likely to fail at modelling electricity prices appropriately.
Fig. 3 explains the procedural method. Starting from Sz_1 each scenario's market outcome is available to be used by all firms in order to set up their strategies in the energy and transmission market to be applied to the subsequent scenario. Thus, four different cases, namely $Sz_{\overrightarrow{12}}$, $Sz_{\overrightarrow{23}}$, $Sz_{\overrightarrow{34}}$ and $Sz_{\overrightarrow{45}}$, will be under consideration. Since load and demand elasticity for each of the following scenarios is not known to the firms in advance the application of their strategies leads to market outcomes, designated as *Predicted Market Data (PMD)*, which is then compared to SMD. Here, the practicability of each strategic mode will be evaluated by means of the deviation of PMD from SMD in terms of electricity and transmission prices.

As far as strategies I and II are concerned (Tab. IV) an analysis of a previous scenario is not needed since they are modelled by assigning constant CV and CTPR values to all firms. Focussing on strategies III and IV SMD from the previous scenario is used to refine the firms' strategies in the energy and transmission market by determining their CV and CTPR values by means of eq. (19) and (20)[2].

C. Results

Tables V and VI present the relative errors by means of the electricity and transmission prices resulting from each applied strategic mode according to the procedure shown in Fig. 3.

TABLE V

RELATIVE ERRORS OF THE ELECTRICITY PRICES IN %

Sz	mode	D	Zwol	Krim	Maas	Merc	Gram	F
$\overrightarrow{12}$	I	7,9	12,5	14,7	12,9	18,2	24,8	0,1
	II	2,2	8,3	12,2	11,4	16,0	26,8	0,1
	III	5,9	11,4	13,9	12,1	17,6	25,3	0,0
	IV	5,6	4,3	3,9	3,6	3,8	2,3	0,0
$\overrightarrow{23}$	I	15,9	16,5	16,6	16,1	17,2	16,1	0,1
	II	0,4	9,5	11,5	13,8	10,6	11,4	0,1
	III	9,8	14,0	14,9	15,6	14,9	14,5	0,0
	IV	0,4	1,8	2,0	2,4	1,9	2,0	0,0
$\overrightarrow{34}$	I	9,4	12,0	13,7	11,4	17,8	27,2	0,1
	II	0,8	8,4	12,0	10,9	16,3	31,5	0,1
	III	7,4	11,0	13,1	11,0	17,5	28,4	0,0
	IV	1,5	1,0	1,6	0,0	4,0	9,7	0,0
$\overrightarrow{45}$	I	10,8	13,6	15,5	13,0	19,8	28,3	0,1
	II	0,1	10,9	13,6	15,8	13,4	16,7	0,1
	III	10,6	13,3	14,8	13,1	18,1	24,2	0,0
	IV	0,2	0,1	0,0	0,1	0,4	1,2	0,0

Focussing on the case $Sz_{\overrightarrow{45}}$ the results for mode IV show a relative error in energy prices always less than 2%. Here, the combined application of the CV and CTPR approach seems to yield electricity prices almost identical to those of load scenario Sz_5. An analysis of the underlying energy and transmission prices, the load levels and the congested transmission lines of Sz_4 and Sz_5 shows no significant difference between both scenarios. Furthermore, the determination of both the conjectured parameters based on the market data analysis of Sz_4 and its application to the subsequent load scenario leads to market results similar to those of Sz_5. However, as can

[2]All non-competitive firms are assigned CV values of -1 and CTPR values of zero. The strategic firms' values of both conjectured parameters range from -1 to 0 (CV) and from 0 to 0.8 (CTPR).

TABLE VI
RELATIVE ERRORS OF THE TRANSMISSION PRICES IN %

Sz	mode	D	Zwol	Krim	Maas	Merc	Gram	F
$\overrightarrow{12}$	I	-	18,7	21,0	17,3	25,7	73,2	10,8
	II	-	22,6	25,5	23,3	29,3	110,3	3,0
	III	-	18,8	21,2	17,5	26,2	81,0	8,1
	IV	-	2,6	2,3	2,0	2,6	7,1	7,6
$\overrightarrow{23}$	I	-	17,2	17,3	16,3	18,0	16,3	20,9
	II	-	21,3	20,7	25,8	16,6	25,8	0,5
	III	-	19,3	19,1	20,7	17,9	20,6	13,0
	IV	-	3,5	3,4	4,1	2,8	4,1	0,5
$\overrightarrow{34}$	I	-	15,4	17,8	13,1	25,4	624,0	12,8
	II	-	20,9	24,4	20,6	31,7	1154,5	1,1
	III	-	16,0	18,7	13,9	26,6	743,6	10,0
	IV	-	0,4	1,7	1,2	6,2	291,3	2,0
$\overrightarrow{45}$	I	-	17,5	20,0	15,0	27,3	186,0	14,6
	II	-	26,0	26,9	29,4	24,6	168,9	0,1
	III	-	17,1	19,0	15,3	24,5	147,9	14,3
	IV	-	0,1	0,3	0,0	0,8	14,0	0,3

be seen by means of the relative error of the transmission price at the location Gram (14.0%) the accuracy of strategic mode IV to reproduce transmission prices correctly is not always guaranteed. Here, an increase of the energy price from scenario Sz_4 to Sz_5 might cause the inaccuracy in the relevant transmission price. However, the main reason for that is due to the small absolute value of the transmission price at the location Gram.

As far as strategic modes I to III are concerned none of them seems to be an adequate approach to model the firms' strategies in an effort to yield correct market prices. The results show that these approaches are far from reproducing market outcomes for the subsequent load scenario closely enough even though the market outcomes are similar to the previous scenario.

Let the focus be now on the cases $Sz_{\overrightarrow{12}}$, $Sz_{\overrightarrow{23}}$ and $Sz_{\overrightarrow{34}}$. Applying strategic mode IV the relative error of the electricity prices is higher on average than in the previously discussed case. Especially when considering $Sz_{\overrightarrow{12}}$ the higher relative error can be attributed to several reasons. Firstly, energy prices are much higher in Sz_1 than in Sz_2 as seen in Tab. I. Furthermore, as a result of the considerable load decrease (Fig. 2) the load flow situation in the network changes and, hence, the transmission congestion pattern. Tab. III shows the transmission line from Gram to Maas to be congested whose impact on transmission and electricity prices are considerable (Tab. I and II). Moreover, when considering all scenarios there is a correlation between energy and transmission price at the location Gram and whether transmission line Gram to Maas is congested or not. This contributes to a higher relative error in both the energy and transmission prices because strategic generators are not able foresee the transmission congestion pattern for the following scenario.

Generally, the results show that considering strategic generator behavior in both the energy and transmission market (strategic mode IV) leads to lower relative errors of the electricity and transmission prices compared to the modes I-III in each of the four studied cases. It further confirms the assumption that each strategic generator's impact on transmission prices has to be taken into account when modelling electricity markets in transmission constrained networks.

VI. CONCLUSION

When modelling liberalized electricity markets the physical transmission network has to be taken into account in order to achieve more accurate electricity prices. As a result of transmission congestion different levels of electricity prices arise throughout the network which can further be influenced by large market players' strategic behavior in the energy and transmission market.

The combined CV and CTPR approach accounts for the modelling of strategic generator behavior while retaining the mathematical feasibility. Its advantage over the widely used Bertrand and Cournot approach is in the modelling of each strategic firm's impact on transmission prices which leads to more accurate electricity prices as shown in the case study.

As to the determination of both the conjectured impact factors a system of linear equations has been derived, with a set of market parameters needed to be known in advance. While some of that market data is available in most of the present electricity markets, details about marginal generating costs and all firms' sales are very sensitive. Future work has to deal with the estimation of unavailable market data in order to test the validity of the combined CV and CTPR approach in realistic case studies.

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