Maximizing Exergy Efficiency in Multi-Carrier Energy Systems

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***Abstract*—In this paper a model for maximizing exergy efficiency in multi-carrier energy systems is introduced. Based on modeling concepts developed in the project "Vision of Future Energy Network", e.g. the Energy Hub concept, exergy is modeled in the context of energy systems that involve multiple energy carriers such as electricity, natural gas and heat. In the context of this integrated consideration of multiple energy carriers, the exergy approach allows to take into account the quality of different energy carriers. Hence this modeling approach provides the possibility to identify from a system perspective how the available energy content of different energy carriers can be exploited as efficiently as possible to satisfy a given demand for final energy carriers. In order to illustrate the proposed exergy analysis method, we compare it with a previously developed cost optimization and apply both methods to an example system consisting of an electricity and natural gas system interconnected by Energy Hubs.**

***Index Terms*—Exergy efficiency, multiple energy carriers, Energy Hub, cogeneration.**

I. Introduction

Making more efficient use of energy is considered one of the crucial levers to increase sustainability of energy systems. Efficiency measures can make important contributions to cope with the growing demand for energy on a worldwide scale and to mitigate climate change. Several studies have identified energy efficiency opportunities for a number of countries [1]. The energy strategy of ETH Zurich [2], which formulates a vision of a transformation path for the energy system in the 21st century, also emphasizes the importance of an efficient conversion of primary energy carriers to final energy carriers. These conversion processes involve a number of different energy carriers of diverse quality.

In order to characterize the quality of different forms of energy, exergy is a meaningful measure. Exergy is derived from the Second Law of Thermodynamics and describes the available useful energy. In contrast to the First Law of Thermodynamics stating the conservation of energy, exergy accounts for the irreversibility of conversion processes. Therefore, the exergy concept can be used to identify how the available energy content of different energy carriers can be exploited as efficiently as possible. Considering exergy is a powerful approach, which has already been introduced in legal frameworks and directives related to energy planning [3].

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The energy supply chain from production over transmission and distribution to final consumption generally involves several energy carriers. The project "Vision of Future Energy Networks" (VoFEN) at ETH Zurich aims at systematically analyzing multi-carrier energy systems in order to design optimal structures for future sustainable energy systems. Models for the representation of power flow, conversion and storage of multiple energy carriers have been developed within the project. Among them the Energy Hub model is the key concept [4].

Previous applications of exergy analysis have addressed the operation of chemical plants [5], sustainable building design [6], [7], the analysis and optimization of power as well as combined heat and power (CHP) generation [8], [9], and the balancing of electricity supply and demand [10]. Even industry sectors and whole countries have been analyzed using exergy as comparative measure [11], [12]. In this paper, the exergy approach is applied to multi-carrier energy systems. The exergy concept describing the quality of different energy carriers is integrated as additional criterion into the modeling tools of the VoFEN project which enable a systematic description of interactions between multiple energy carriers. The contribution of this paper is the formulation of a generic modeling framework that allows the analysis of exergy efficiency in multi-carrier energy systems from a systems perspective taking into account the physical properties of the energy networks.

The remainder of this paper is organized as follows: Section II describes how we model exergy efficiency in multi-carrier energy systems. In section III the optimization problem for maximizing exergy efficiency in multi-carrier systems is formulated. The proposed method is illustrated in section IV providing several application examples. Section V concludes the paper.

II. Modeling of Exergy Efficiency in Multi-Carrier Energy Systems

The following section describes the various "building blocks" constituting the overall framework for maximizing exergy efficiency in multi-carrier energy systems. Generally, our approach takes a multi-carrier optimal power flow as basis (in the following abbreviated MCOPF) as formulated in [13]. The MCOPF relies on so-called Energy Hubs. In that, we first outline the basic theory of Energy Hubs, followed by a short introduction to the notion of exergy. In a subsequent step we define exergy in the context of Energy Hubs.

A. The Energy Hub Approach

In the industrialized part of the world, industrial, commercial, and residential consumers often require various forms of energy services provided by different infrastructures. Typically, coal, petroleum products, biomass, and grid-bound energy carriers such as electricity, natural gas, and district heating/cooling are used. In terms of the conversion and storage of the different energy carriers, various interactions may arise as for instance observed in co- or tri-generation facilities [14], [15]. Using e.g. CHPs or microturbines it is possible to produce electricity and heat out of natural or bio gas, biomass etc. Together with the deployment of distributed storage technologies such multi-carrier energy systems are characterized by complex dependencies. One concept to model and analyze these dependencies is the so-called Energy Hub approach, where "an Energy Hub generally represents an interface between energy producers, consumers, and the transportation infrastructure.[16]" From a system point of view, an Energy Hub provides the functions of in- and output, conversion and storage of multiple energy carriers. Fig. 1 displays a schematic picture of an exemplary Energy Hub.

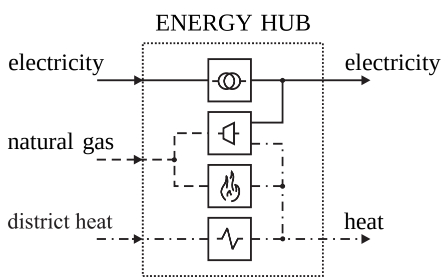


Fig. 1. Example of a hybrid Energy Hub that contains an electrical transformer, a gas turbine, a gas furnace, and a heat exchanger [17].

Formally, a hub can be described through a set of energy carriers $\alpha, \beta, \ldots \in \mathcal{E} = \{electricity,\ natural\ gas,\ heat,\ \ldots\}$, where each energy carrier may be a hub input and/or a hub output. The set of input powers (output powers) is defined by $P_\alpha, P_\beta, \ldots, P_\omega$ ($L_\alpha, L_\beta, \ldots, L_\omega$). As described in the above example, specific technologies inside the hub allow for the conversion of energy carriers. A simple conversion device with one input and one output can be described as follows:

$$L_\beta = c_{\alpha,\beta} P_\alpha \tag{1}$$

The device converts an energy carrier (α) into (β), where input and output powers are coupled through the coupling factor ($c_{\alpha,\beta}$) given by the converter's steady state energy efficiency. Accordingly, an Energy Hub can be modeled as a combination of different converters covering multiple in- and outputs. Equation (2) provides the mathematical description of the Energy Hub.

$$\underbrace{\begin{bmatrix} L_\alpha \\ L_\beta \\ \vdots \\ L_\omega \end{bmatrix}}_{\mathbf{L}} = \underbrace{\begin{bmatrix} c_{\alpha,\alpha} & c_{\beta,\alpha} & \cdots & c_{\omega,\alpha} \\ c_{\alpha,\beta} & c_{\beta,\beta} & \cdots & c_{\omega,\beta} \\ \vdots & \vdots & \ddots & \vdots \\ c_{\alpha,\omega} & c_{\beta,\omega} & \cdots & c_{\omega,\omega} \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} P_\alpha \\ P_\beta \\ \vdots \\ P_\omega \end{bmatrix}}_{\mathbf{P}} \tag{2}$$

The entries of the coupling matrix ($\mathbf{C}$) are the converter coupling factors. Each coupling factor relates one particular input to a certain output incorporating the specific conversion efficiencies. For a more detailed discussion on the definition, modeling and use of Energy Hubs the reader is referred to [13], [17].

In section III we will optimize an interconnected network of Energy Hubs, first focussing on cost-efficiency and afterwards maximizing the exergy efficiency of the systems. The notions of exergy and exergy efficiency are briefly sketched in the following subsection.

B. Definition of Exergy and Exergetic Efficiency

Typically, thermodynamic systems contain several different energy carriers, where a common basis for comparison is required. One comprehensive concept to develop such a comparison basis is exergy. Generally, energy (E) consists of exergy (B) and anergy (A) where exergy is the theoretically usable part of energy, thus quantifying the amount of work available from a thermodynamic system. On the contrary, anergy is energy which cannot be used. Equation (3) describes this basal relation.

$$E = B + A \tag{3}$$

A well-known example of anergy is the (unused) waste heat of combustion processes. It has to be noted that exergy is always related to an exergy reference environment which is normally the standard atmosphere. Thus, exergy is the available work from a thermodynamic system which is brought to equilibrium with its surrounding exergy reference environment. Consequently, exergetic efficiency denotes the portion of the available work which is actually used. According to [18] for a single device or a system, the exergetic efficiency ψ is defined by the exergy flowing into the system ($\dot{B}_{used}$) and the exergy flow being actually used ($\dot{B}_{in}$):

$$\psi = \frac{\dot{B}_{used}}{\dot{B}_{in}} \tag{4}$$

Alternatively, according to [19] the exergetic efficiency of a device is calculated using first- and second-law thermal efficiencies $\eta_{th,1}$ and $\eta_{th,2}$ and the mechanical efficiency η_m

$$\psi = \eta_{th,1} \cdot \eta_{th,2} + \eta_m \tag{5}$$

where $\eta_{th,2}$ depends on the temperature at which heat is transferred from one reservoir to another, and is also known as the Carnot efficiency:[1]

$$\eta_{th,2} = \eta_{Carnot} = \frac{T - T_{ref}}{T} = 1 - \frac{T_{ref}}{T} \tag{6}$$

Example: A CHP device converts incoming natural gas power of 70 kW to 24.5 kW electric and 33.6 kW thermal power. The latter is used for room heating. In this case, the mechanical (electric) efficiency of the device is:

$$\eta_m = \frac{24.5 \text{ kW}}{70 \text{ kW}} = 0.35 \tag{7}$$

[1] see, e.g., [18], p.340.

Conversion to heat is performed with a first-law thermal efficiency of:

$$\eta_{th,1} = \frac{33.6\text{ kW}}{70\text{ kW}} = 0.48 \tag{8}$$

Assuming that the heat leaving the device with a temperature of $T = 353$ K is transferred to a room, the second-law efficiency depends on the temperature of the exergy reference environment. Let the environment be a cold winter atmosphere with $T_{ref} = 258$ K. Then, the efficiency relative to the cold winter environment is:[2]

$$\eta_{th,2} = 1 - \frac{258\text{ K}}{353\text{ K}} = 0.2691 \tag{9}$$

Applying (5), the exergetic efficiency amounts to

$$\psi = \eta_{th,1} \cdot \eta_{th,2} + \eta_m \tag{10}$$
$$= 0.48 \cdot 0.2691 + 0.35 = 0.4791 \tag{11}$$

The simple example above showed how to calculate the exergy efficiency of a CHP device. In the next subsection we will adopt the framework for determining the exergy flows related to Energy Hubs.

C. Exergy in the Context of Multi-Carrier Energy Systems

In order to use (4) to calculate the exergetic efficiency, it is necessary to know the exergy flowing into a system as well as the used exergy. To further detail this concept, we use an exemplary Energy Hub as depicted in Fig. 2.

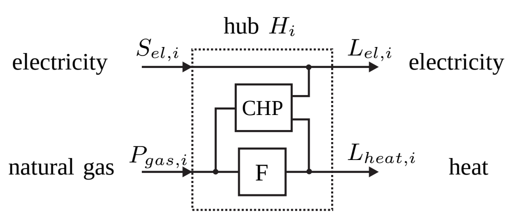


Fig. 2. Energy hub comprising a combined heat and power (CHP) device and a furnace. $S_{el,i}$ denotes the electricity input (apparent power), $P_{gas,i}$ the gas input. $L_{el,i}$ refers to the electric load, $L_{heat,i}$ to the heat load. [17]

Although in the later OPF formulation the apparent power $S_{el,i}$ is considered (see (12)), we neglect the reactive power ($jQ_{el,i}$) for exergy efficiency calculations. The exergy concept takes into account irreversibilities of conversion processes, where reactive power can be seen as being reversibly fluctuating in the system. We furthermore assume lossless transmission inside the hub. For short distances, e.g. in a house or in a building complex, this simplification appears reasonable.

$$S_{el,i} = P_{el,i} + jQ_{el,i} \tag{12}$$

With the hub displayed in Fig. 2 and the assumption on reactive power and hub-internal lossless transmission, the incoming as well as the used exergies for electricity, natural gas and heat have to be defined. First, electricity ($P_{el,i}$) is pure exergy ($\dot{B}^{in}_{el,i}$). The same applies for the electricity load at the output ($L_{el,i}$ and $\dot{B}^{out}_{el,i}$). For chemical energy carriers, such as natural gas, the following approximation is common: Their exergy is described by the lower heating value (LHV_{chem}) and the mass flow ($\dot{m}$). The product of the latter two equals the chemical power flow:

$$\dot{B}^{in}_{chem} \approx P_{chem} = \dot{m} \cdot LHV_{chem} \tag{13}$$

This approximation is valid as long as the combustion products enter an environment where the pressure does not show extreme deviations from the standard pressure. This is the case for most combustion processes being the reason why this approximation is commonly used. Finally, the exergy of heat depends on the temperature of the transferred heat (T) and on the temperature of the reference environment (T_{ref}). Table 1 gives an overview of the corresponding exergies:

	Incoming exergy	**Used exergy**
Electricity	$\dot{B}^{in}_{el} = P_{el}$	$\dot{B}^{out}_{el} = L_{el}$
Natural gas	$\dot{B}^{in}_{chem} \approx P_{chem}$ $= \dot{m} \cdot LHV_{chem}$	no gas output
Heat	no heat input	$\dot{B}^{out}_{heat} = L_{heat} \cdot \left(1 - \frac{T_{ref}}{T}\right)$

TABLE I
OVERVIEW OF INCOMING AND USED EXERGIES FOR THE ENERGY HUB IN FIG. 2. FOR THE ABBREVIATIONS, PLEASE SEE THE TEXT ABOVE.

The above section described the Energy Hub as basal modeling principle, followed by an introduction to the notions of exergy and exergetic efficiency, where in a last step these concepts have been adopted for the use in conjunction with Energy Hubs. Subsequently, section III presents the cost-based multi-carrier optimal power flow problem (MCOPF) as well as the model for the maximization of the exergy efficiency in multi-carrier energy systems. It has to be noted that the following MCOPF formulation is based on the research already presented, e.g. in [13], [17]. For the sake of readability and to provide a better understanding of the modified modeling framework in terms of exergy analysis, we include a synopsis of the previous work also in this paper.

III. Maximizing Exergy Efficiency in Multi-Carrier Energy Systems

A. Multi-Energy Carrier Optimal Power Flow

In order to formulate the multi-energy carrier optimal power flow (MCOPF) a set of hub numbers ($\mathcal{H}$) has to be defined, with $i, j, \ldots \in \mathcal{H} = \{1, 2, \ldots, N_H\}$. Each hub ($i$) is characterized by a coupling matrix (C_i), by the power inputs (P_i) and the hub loads (L_i). The network power flows are denoted by $\mathbf{F}_\alpha$. The term $\mathbf{G}_\alpha(\mathbf{P}_i)$ refers to the set of power flow equations of the hubs and the different networks (e.g. electricity and gas).[3] The hub input power might be constrained with lower limits $\underline{\mathbf{P}}_i$ or upper limits $\bar{\mathbf{P}}_i$. Networks flows are also likely to be constrained by lower flow limits $\underline{\mathbf{F}}_\alpha$ and upper flow limits $\bar{\mathbf{F}}_\alpha$. With the above nomenclature it is possible to formulate

[2] In [20] this value is recommended for exergy-based evaluations of heating systems in Germany. The reason is that heating systems have to be designed for this temperature.

[3] We refrain from presenting the flow equations as this exceeds the scope of this paper. In [17] the flow modeling of electricity and gas networks is described comprehensively.

the multiple-energy carrier optimal power flow as follows:
Minimize

$$f(\mathbf{P}_i, \mathbf{F}_\alpha) \tag{14}$$

subject to

$$\mathbf{L}_i - \mathbf{C}_i\mathbf{P}_i = 0, \ \ \forall i \in \mathcal{H} \tag{15a}$$

$$\mathbf{G}_\alpha(\mathbf{P}_i) = 0, \ \ \forall \alpha \in \mathcal{E} \tag{15b}$$

and

$$\underline{\mathbf{P}}_i \leq \mathbf{P}_i \leq \bar{\mathbf{P}}_i, \ \ \forall i \in \mathcal{H} \tag{16a}$$

$$\underline{\mathbf{F}}_\alpha \leq \mathbf{F}_\alpha \leq \bar{\mathbf{F}}_\alpha, \ \ \forall \alpha \in \mathcal{E} \tag{16b}$$

The objective function $f(\mathbf{P}_i, \mathbf{F}_\alpha)$ is also dependent on the network flows $\mathbf{F}_\alpha$ as the line utilization causes losses, which have to be covered by additional generation. In compliance with the optimal power flow in electricity networks, the problem defined in (14) to (16b) yields marginal costs of the different energy carriers at the different hubs, i.e. a system of locational marginal prices.

B. An Exergy Maximizing Formulation of the Optimal Power Flow Problem

In section II the principles of exergy and exergy efficiency have been described. In the following we will adopt the above MCOPF framework to optimize exergy efficiency. The idea is to exploit the energy content of different energy carriers as efficiently as possible, where the MCOPF allows the maximization of exergy efficiency not only locally, but for the whole supply chain from production over transmission and distribution to consumption. Considering the above MCOPF problem it appears obvious that also for an maximization of exergy efficiency the constraints of the MCOPF have to be satisfied. Hence, the equations from (15a) to (16b) representing the system constraints remain unchanged, where only the objective function has to be changed. Now the aim is to maximize (ψ), defined in (4) as the quotient of ($\dot{B}_{used}$) and ($\dot{B}_{in}$). As optimization models are commonly formulated as a minimization problem the objective function can be written as follows:

$$\frac{1}{\psi} = \frac{\dot{B}^{in}}{\dot{B}^{used}} \tag{17}$$

In a system as depicted in Fig. 3, incoming exergy is simply the sum of the exergies brought into the system by the various energy carriers, where this inflow may originate from a connected gas network or additional generation outside of the hubs (for further explanation see the application example in section IV). Hence, we additionally define an alternative set of energy carriers $\mathcal{S}$ with $\{\alpha, \beta, ..., N_{SC}\}$ describing the system inflow. Extending the nomenclature with the above considerations, this leads to:

$$\dot{B}^{in} = \sum_{i \in \mathcal{H}} \sum_{\kappa \in \mathcal{S}} \dot{B}^{in}_{i,\kappa} \tag{18}$$

Analogously, used exergy is the sum of the exergies used by the loads at the output sides of all hubs:

$$\dot{B}^{used} = \sum_{i \in \mathcal{H}} \sum_{\kappa \in \mathcal{L}} \dot{B}^{used}_{i,\kappa} \tag{19}$$

For the formulation of the exergy maximizing objective function three steps can be identified: 1) quantify incoming exergy 2) quantify used exergy 3) apply (17). In the end this leads to:

$$\psi = \frac{\dot{B}^{used}}{\dot{B}^{in}} = \frac{\sum_{i \in \mathcal{H}} \sum_{\kappa \in \mathcal{S}} \dot{B}^{in}_{i,\kappa}}{\sum_{i \in \mathcal{H}} \sum_{\kappa \in \mathcal{L}} \dot{B}^{used}_{i,\kappa}} \tag{20}$$

In the application example below we will formulate the exergy maximizing objective function for the system depicted in Fig. 3.

IV. Application example

A. System parameters

In this section, the presented method for exergy efficiency maximization in multi-carrier energy systems is applied to a test system. The results are then compared with the corresponding results of a cost optimization. The chosen system is taken from [17]. Fig. 3 shows the structure of the investigated networks. The system parameters are given in Table II. The system consists of an electricity network (upper part of Fig. 3), a natural gas network (lower part of Fig. 3) and three equally designed energy hubs with the structure shown in Fig. 2. The hubs are equipped with a CHP plant converting gas to electricity and heat, and a gas furnace (F) burning gas to produce heat. The availability of a CHP plant leads to a redundancy both for the electricity and the heat supply of the loads because electricity can alternatively be taken from the network and heat can be produced by using the gas furnace. At the same time, the CHP plants establish a physical connection between the electricity and gas network. Two generators (G_1 and G_2) are connected to the electricity network. Natural gas is obtained from an adjacent network N at node 1 and delivered to the hub inputs at node 2 and 3 via the connections 1-2 and 1-3 which are equipped with the compressors C_{12} and C_{13}. The exergy efficiency of generator 1, Ψ_{G_1}, corresponds to a nuclear power plant and Ψ_{G_2} represents the corresponding value of a coal power plant [21]. The exergy efficiency of the CHP, Ψ_{CHP}, results from assuming flow temperatures of the heating circuit at the three hubs of $T_i = 353\ K$ and a reference temperature of $T_{ref} = 258\ K$ representing the lowest outdoor temperature assumed [20].

B. Objective function of the exergy efficiency maximization

In subsection III-B the necessary steps for the definition of the exergy maximizing objective function have been defined as 1) quantify incoming exergy 2) quantify used exergy 3) apply (17). We will now follow this procedure to exemplify the method.

For the hub system depicted in Fig. 3, energy is flowing into the system as electricity and natural gas. As outlined in section II-C, the exergy content of natural gas can be approximated by its energy content. For the case of electricity, it is important to respect the system border. Obviously, electricity entering the hub is 100 % exergy, but the generation of this electricity is associated with exergetic losses, depending on the type of conversion used. These exergetic losses are introduced by exergetic efficiency factors for the two generators, ψ_{G_1} and

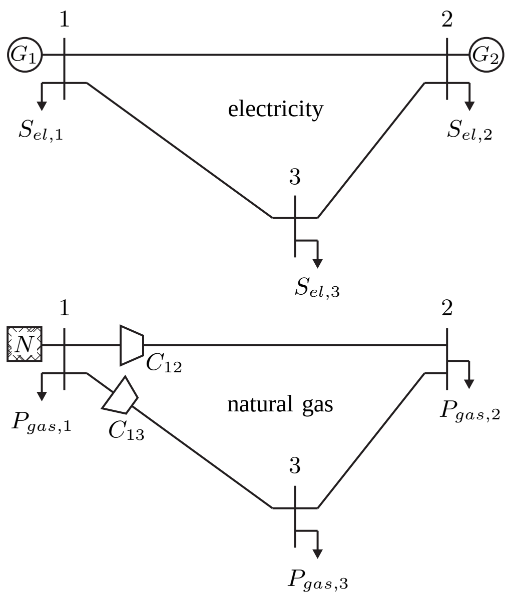


Fig. 3. Analyzed system with one energy hub at each of the three nodes which are interconnected via a natural gas and an electricity network, from [17].

ψ_{G_2}, respectively. Thus, incoming exergy can be formulated as:

$$\dot{B}^{in} = \dot{B}^{in}_{1,el} + \dot{B}^{in}_{2,el} + \dot{B}^{in}_{1,gas} = \frac{P_{G_1}}{\psi_{G_1}} + \frac{P_{G_2}}{\psi_{G_2}} + P_{gas,source} \tag{21}$$

The used exergy can be derived from the electricity and heat at the output of the three hubs ($\mathcal{H} = 3$):

$$\dot{B}^{used} = \sum_{i=1}^{3} \left(\dot{B}^{out}_{el,i} + \dot{B}^{out}_{heat,i} \right) \tag{22a}$$

$$= \sum_{i=1}^{3} \left(L_{el,i} + \left(1 - \frac{T_{ref}}{T_i}\right) \cdot L_{heat,i} \right) \tag{22b}$$

Subsequently, the objective function to be minimized is as follows:

$$\frac{1}{\psi} = \frac{\frac{P_{G_1}}{\psi_{G_1}} + \frac{P_{G_2}}{\psi_{G_2}} + P_{gas,source}}{\sum_{i=1}^{3} \left(L_{el,i} + \left(1 - \frac{T_{ref}}{T_i}\right) \cdot L_{heat,i} \right)} \tag{23}$$

We refrain from presenting the objective function for the cost minimization problem as this problem is well-known and has been covered extensively in the literature (see e.g. [13], [17]).

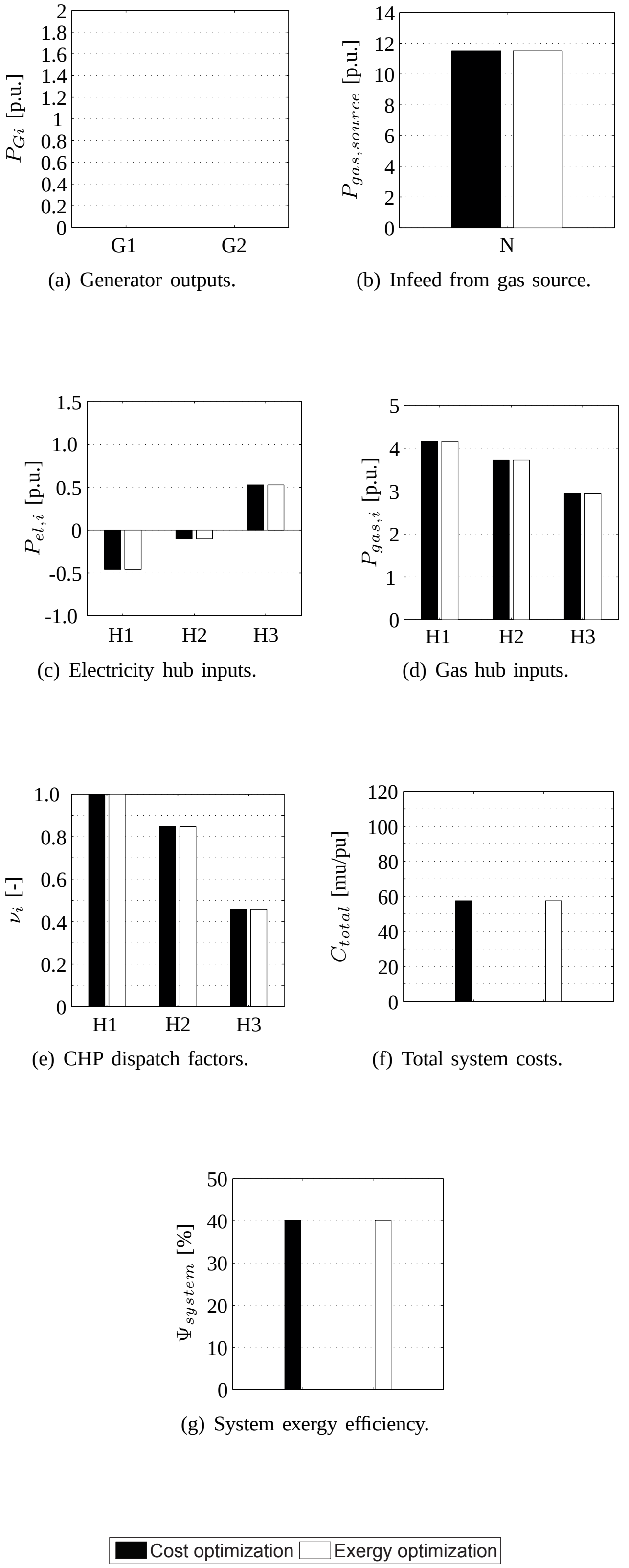
In the following, the results of exergy and cost optimization for the above described example system are compared. In a first step, the data from Table II are used to generate a reference result. Based on this reference case, two sensitivity analyses are carried out to determine the influence of relevant system parameters. The optimal power flow problems are solved using the fmincon solver available in the Matlab optimization toolbox. Due to the non-convex character of the problem, it can not be guaranteed that the solution point is indeed the global optimum.

TABLE II
EXAMPLE SYSTEM PARAMETERS FOR THE REFERENCE CASE.

Element data	
el. line 1-2	$Z_{12} = 0.3 + j0.9$ pu, $Y_{12} = j1.5 \cdot 10^{-6}$ pu
el. line 1-3	$Z_{13} = 0.2 + j0.6$ pu, $Y_{13} = j2.5 \cdot 10^{-6}$ pu
el. line 2-3	$Z_{23} = 0.1 + j0.4$ pu, $Y_{23} = j3.5 \cdot 10^{-6}$ pu
G_1	slack type, $V_1 = 1\angle 0°$ pu, $a_{G_1} = 0$ mu, $b_{G_1} = 10$ mu/pu, $c_{G_1} = 0.0010$ mu/pu^2, $\Psi_{G_1} = 0.37$
G_2	PQ type, $a_{G_2} = 0$ mu, $b_{G_2} = 12$ mu/pu, $c_{G_2} = 0.0012$ mu/pu^2, $\Psi_{G_2} = 0.335$
pipe 1-2	$GHV \cdot k_{12} = 4.5$
pipe 1-3	$GHV \cdot k_{13} = 3.0$
pipe 2-3	$GHV \cdot k_{23} = 2.0$
C_{12}, C_{13}	$GHV \cdot k_{C_{12}} = GHV \cdot k_{C_{13}} = 0.5$ pu^{-1}
N	slack type, $p_1 = 1$ pu, $a_N = 0$ mu, $b_N = 5$ mu/pu, $c_N = 0$ mu/pu^2
CHP	$\eta_{CHPgas,el} = 0.35$, $\eta_{CHPgas,heat} = 0.48$, $\Psi_{CHP} = 0.45$
F	$\eta_F = 0.85$, $\Psi_F = 0.1$
loads	$L_{el,i} = 1 + j0.1$ pu, $L_{heat,i} = 2$ pu
Limitations	
nodes $m = 1, 2, 3$	$0.9 \leq \lvert V_m \rvert \leq 1.1$ pu, $0.8 \leq p_m \leq 1.2$ pu
G_2	$0 \leq P_{G_2} \leq 4$ pu, $0 \leq \lvert Q_{G_2} \rvert \leq 4$ pu, $0 \leq \lvert P_{G_2} + jQ_{G_2} \rvert \leq 5$ pu
C_{12}, C_{13}	$1.2 \leq \frac{p_m}{p_k} \leq 1.8$

C. Reference case

Fig. 4 shows the simulation results for the reference case. The black bars indicate the results of the cost optimization, i.e. for the objective of minimizing total system costs, whereas the white bars represent the results for the objective of maximizing exergy efficiency of the whole system. For the reference case, cost and exergy optimization give relatively similar results. None of the generators is used to meet the electricity load (see Fig. 4(a)) because they are too costly and their exergy efficiency is too low. Instead, natural gas is taken from the adjacent network N (see Fig. 4(b)) to be converted to electricity in the CHPs of the three hubs. As the gas line from node 1 to node 3 features higher losses than the one from node 1 to node 2, hub 1, which is directly connected to the gas network N, and hub 2 both consume more gas than hub 3 (see Fig. 4(d)). The hubs 1 and 2 produce more electricity than their own loads consume. Therefore, they export their excess electricity to hub 3 (see Fig. 4(c)). Fig. 4(e) shows the dispatch factors of the three CHPs, i.e. the percentage of gas that is used in the CHPs. In hub 1, all gas is converted by the CHP; the gas furnace is not used at all. Hub 2 uses only a small part of the total gas input for generating heat by means of the furnace. As hub 3 imports electricity from the network, its CHP usage is relatively low and the remaining heat load is covered by the gas furnace. With respect to total system costs (see Fig. 4(f)) and system exergy efficiency (see Fig. 4(g)), both cost and exergy optimization give almost identical results.



(a) Generator outputs. (b) Infeed from gas source.

(c) Electricity hub inputs. (d) Gas hub inputs.

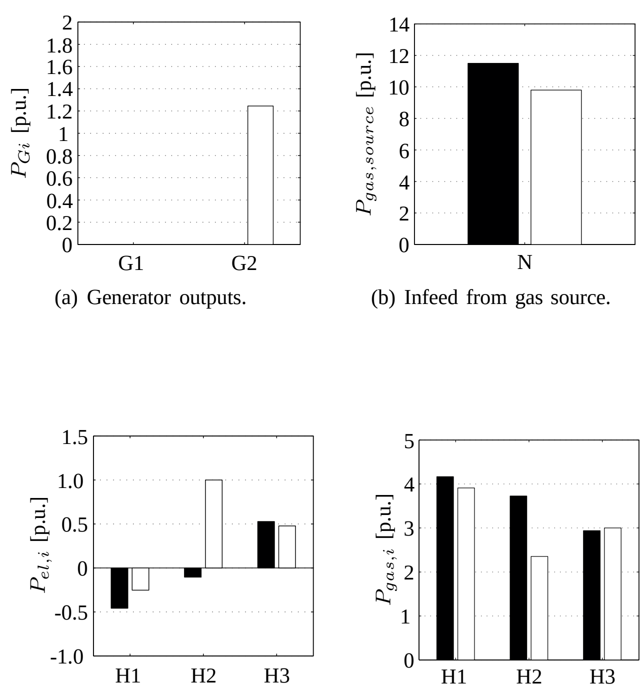
(e) CHP dispatch factors. (f) Total system costs.

(g) System exergy efficiency.

Fig. 4. Simulation results for the reference case.

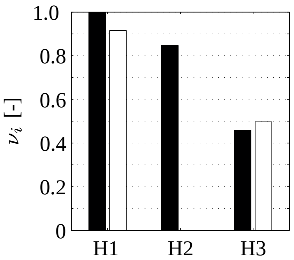
D. Sensitivity analysis with respect to the exergy efficiency of generator G2

In a next step, the sensitivity of the simulation results to the exergy efficiency of generator G2 is analyzed. All other things being equal with respect to the reference case, the exergy efficiency of generator G2 is changed to $\Psi_{G_2} = 0.78$ in order to study the impacts of a generator with a high exergy efficiency on the system behavior. This does, of course, not affect the results of the cost optimization because in this case only costs are considered in the objective function. The results of the exergy optimization, however, change significantly. Maximizing exergy efficiency leads to an electricity production of generator G2 (see Fig. 5(a)) and therefore to a reduced gas infeed from the gas network (see Fig. 5(b)). As the electrical load at node 2 is supplied directly by the generator G2, the CHP of hub 2 is completely switched off, i.e. $\nu_2 = 0$ (see Fig. 5(e)). It may seem counterintuitive at the first glance that the exergy optimization hardly leads to a higher system exergy efficiency than the cost optimization (see Fig. 5(g)). This can be explained by the fact that the electricity output of G2 replaces some part of the electricity that would have otherwise been generated by the CHPs. At the same time, the heat output of the CHPs decreases and therefore, the overall heat production of the gas furnaces has to be increased. As the gas furnaces have a very low exergy efficiency, this almost completely compensates the increase in exergy efficiency from the use of the generator G2. The total system costs resulting from the exergy optimization, however, are about 10% higher than the costs obtained by the cost optimization (see Fig. 5(f)). Hence, in this case the trade-off between costs and efficiency would be in favor of the cost optimization.

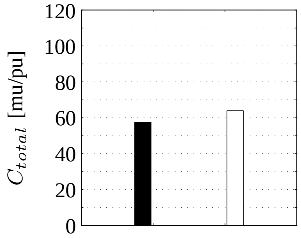


(a) Generator outputs. (b) Infeed from gas source.

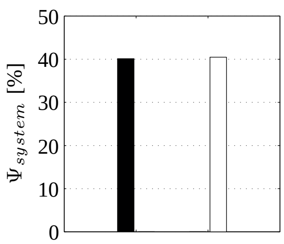
(c) Electricity hub inputs. (d) Gas hub inputs.



(e) CHP dispatch factors.



(f) Total system costs.

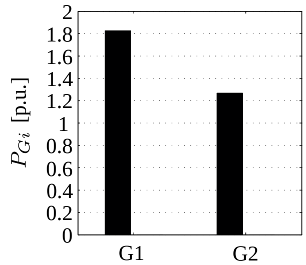


(g) System exergy efficiency.

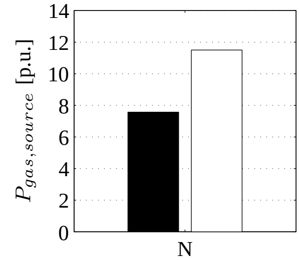
Fig. 5. Simulation results for the sensitivity analysis with respect to the exergy efficiency of generator 2.

E. Sensitivity analysis with respect to the gas price

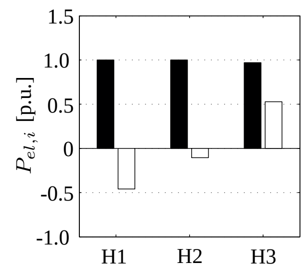
A second sensitivity analysis is carried out with respect to the gas price. Based on the parameters from the reference case shown in Table II, ceteris paribus, the gas price is doubled to $b_N = 10$ mu/pu. The results for both cost and exergy optimization are illustrated in Fig. 6. As the changed gas price does not affect the objective function of the exergy optimization, the respective results are the same as in the reference case. Due to the high costs for gas, the cost optimization results in a substantially lower gas input from the adjacent gas network N (see Fig. 6(b)). The relatively small gas input to the hubs (see Fig. 6(d)) is mainly used for heat generation in the furnaces. Only a very small amount of gas is used in the CHP of hub 3 (see Fig. 6(e)). The electrical loads are exclusively covered by taking electricity from the network (see Fig. 6(c)). This electricity is supplied by the generators G1 and G2 (see Fig. 6(a)). Fig. 6(f) shows that the costs resulting from exergy optimization are only slightly higher than those from cost optimization. This is due to the fact that the higher amount of gas that is consumed in the case of exergy optimization is used at a high efficiency in the CHPs (see Fig. 6(e)). Therefore, the increase in natural gas prices is mitigated. At the same time, the usage of the generators G1 and G2 in combination with the gas furnaces in the case of cost optimization leads to a substantial drop of exergy efficiency compared with the exergy optimization. In this case, unlike in the previous sensitivity analysis, the trade-off between costs and exergy efficiency would rather be in favor of the exergy optimization.



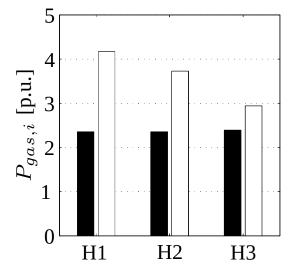
(a) Generator outputs.



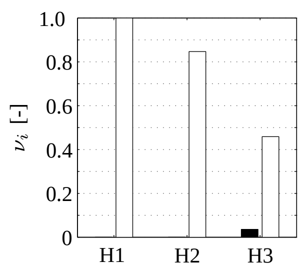
(b) Infeed from gas source.



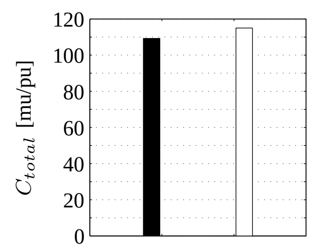
(c) Electricity hub inputs.



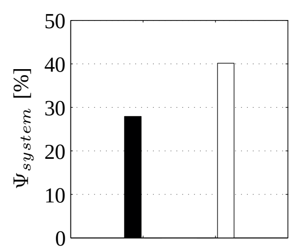
(d) Gas hub inputs.



(e) CHP dispatch factors.



(f) Total system costs.



(g) System exergy efficiency.

Fig. 6. Simulation results for for the sensitivity analysis with respect to the gas price.

V. CONCLUSIONS

The exergy approach applied to multi-carrier energy systems that has been presented in this paper can be used for the planning of energy systems involving several energy carriers. Using this approach, energy system planning is not only performed with an integrated perspective on all relevant energy carriers and potentially resulting synergies, but the quality of the different forms of energy and the efficiency of the different conversion processes is also taken into account explicitly. As illustrated in the application example, the presented method allows to assess exergy efficiency on a system level, i.e. the implications of changes of individual system parameters for the exergy efficiency of the whole system can be analyzed. Furthermore, a comparison of cost minimization and maximization of exergy efficiency gives insight regarding the trade-off between these two quantities. In this respect, future work may deal with multi-criteria optimization aiming at developing solutions that offer the best possible trade-off between costs and efficiency.

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