

POWER FLOW CONTROL BY SENSITIVITY BASED FACTS CONTROLLERS

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Abstract

This paper discusses power flow control by use of controllable series devices. Through a sensitivity based method the control variables, e.g. series capacitor reactances or phase shifter angles, can be calculated quickly and accurately to obtain desired line flows within the load flow sensitivity boundaries. This method is applied to realistic power flow control scenarios in a simple power system.

Keywords

Power flow control, Controllable Series Capacitor (CSC), Sensitivity analysis.

1 INTRODUCTION

With the development of FACTS devices, it becomes possible to enhance power flow controllability considerably. To meet the load and electric market demands, new lines should be added to the system, but due to environmental reasons, the installation of electric power transmission lines must often be restricted. The need to maintain power flow between generations and loads through defined line corridors without affecting other paths or the consumers in the system will be of importance in the future. This paper deals with different aspects of power flow control by controllable series compensation.

New control variables are introduced into power flow calculations when considering series flow control devices. One important component is the Controllable Series Compensator (CSC), which allows rapid and continuous changes of the transmission line impedance, and in that way it is an effective means for power flow control. Active power flows along the compensated transmission line can be maintained at a specified value under various operating conditions. A more exact method for computation of sensitivities that incorporates the actual power system condition is useful for analysis of power flow control with controllable devices. The linearization of the load flow equations around the nominal operating point yields such sensitivities. This sensitivity can be used for calculation of total system active power losses as well,

[1], where the authors have applied the second order sensitivity function to calculate the first order approximations and in the linear programming based optimal power flow. In [2] the authors used the sensitivity for estimation the effect on the transfer capability of variations in parameters such as those describing other transfers, operating conditions or assumed data.

An important question is what line flows regions are obtainable for a set of controllable components with given control range. The authors in [3] have presented the concept of load flow feasibility boundaries to address this question. In this paper it is presented how this concept can be used to calculate possible load and generation increases without violating given constraints on line flows.

The outline of this paper is as follows. The first and the second order sensitivity are presented in Section 2. The quadratic power flow is presented in Section 3. Results on test systems are provided in Section 4.

2 SENSITIVITY ANALYSIS OF LINE FLOWS

The variables in the line flow – line compensation system can be grouped into [1]:

- State variables $\{x_i\}$
- Control variables $\{z_i\}$
- Output (dependent) variables $\{w_i\}$
- Parameters $\{d_i\}$

If n is the number of buses, m is the number of load buses, $n_x = (n + m - 1)$ is the number of state variables, n_z is the number of output variables, variables of above groups can be defined as:

1. State variables $\{x_i\}$, $i = 1, 2, \dots, n_x$, are defined as:

a) Phase angles $\{\theta_i\}$, $i = 1, 2, \dots, n-1$ for all buses except for the slack (reference) bus;

b) Voltage magnitudes of load buses $\{V_i\}$, $i = 1, 2, \dots, m$.

2. Input (control) variables $\{z_i\}$, $i = 1, 2, \dots, n_z$ are for example, series capacitor reactances, x_{c_i} , or phase shifter angles, ϕ_i .
3. Output (dependent) variables $\{w_i\}$, $i = 1, 2, \dots, n_w$ are the active power line flows in the lines of interest.
4. Parameter (uncontrolled) variables $\{d_i\}$ are, line reactances, loads, etc.

2.1 Sensitivity of States to Inputs

A power system in the steady state is modelled by the load flow equations:

$$F(X, Z, D) = 0 \quad (1)$$

where X is the $(n_x \times 1)$ vector of state variables, Z is the $(n_z \times 1)$ vector of control variables, D is the vector of parameters. The first order Taylor expansion of Equation (1) in the neighbourhood of the nominal operating point (X^0, Z^0, D^0) is

$$0 = F(X^0 + \Delta X, Z^0 + \Delta Z, D^0 + \Delta D) \approx F(X^0, Z^0, D^0) + F_x \Delta X + F_z \Delta Z + F_D \Delta D \quad (2)$$

with Jacobian matrices F_x, F_z, F_D that are computed at the nominal operating point (X^0, Z^0, D^0) . From the Equation (2) it follows that

$$F_x \Delta X + F_z \Delta Z + F_D \Delta D = 0 \quad (3)$$

since $F(X^0, Z^0, D^0) = 0$.

Assuming that F_x is non-singular, we have

$$\Delta X = -F_x^{-1} F_z \Delta Z - F_x^{-1} F_D \Delta D \quad (4)$$

For convenience of notations, Equation (4) is rewritten in terms of sensitivity matrices S_{xz} and S_{xD} as

$$\Delta X = S_{xz} \Delta Z + S_{xD} \Delta D \quad (5)$$

with

$$S_{xz} = -F_x^{-1} F_z \quad (6)$$

$$S_{xD} = -F_x^{-1} F_D \quad (7)$$

2.2 Sensitivity of Output to Input Variables

At the operating point, the load flow vector W^0 is determined by a function H ,

$$W^0 = H(X^0, Z^0, D^0) \quad (8)$$

which with a perturbation ΔZ becomes

$$W^0 + \Delta W = H(X^0 + \Delta X, Z^0 + \Delta Z, D^0 + \Delta D) \quad (9)$$

Linearization yields

$$\Delta W \approx W_x \Delta X + W_z \Delta Z + W_D \Delta D \quad (10)$$

Inserting (4) into (10) we get

$$\Delta W = \left[-W_x F_x^{-1} F_z + W_z \right] \Delta Z + \left[-W_x F_x^{-1} F_D \right] \Delta D \quad (11)$$

and assuming $\Delta D = 0$ we have

$$\Delta W = [W_x S_{xz} + W_z] \Delta Z \quad (12)$$

If S_{wz} is defined as the sensitivity of output variables with respect to control variables, we can define

$$S_{wz} = W_x S_{xz} + W_z \quad (13)$$

In the line flow compensation system, W is the active line flow vector and Z is the vector of control variables, e.g. capacitor reactances or phase shifter angles. F_x is the Jacobian matrix used in standard Newton-Raphson load flow computations. The Jacobian matrices F_x, W_z and W_x are derived in [5].

2.3 Second Order Sensitivity

In this section the second order sensitivity of the line flows is derived with respect to line compensation. It is assumed that the second order derivatives of the functions F and H exist. If the line flow equations $W = H(X, Z, D)$ are expanded in a Taylor series about the nominal operating point to the second order of accuracy, assuming $\Delta D = 0$, we obtain [3]

$$\Delta W = W_x \Delta X + W_z \Delta Z + \frac{1}{2} \Delta X^T W_{xx} \Delta X + \frac{1}{2} \Delta Z^T W_{zz} \Delta Z + \frac{1}{2} \Delta Z^T W_{zx} \Delta X \quad (14)$$

From the first order sensitivity analysis, if the higher order terms are neglected, (This assumption is justified by numerical results.)

$$\Delta X = -F_x^{-1} F_z \Delta Z = S_{xz} \Delta Z \quad (15)$$

If the expression of ΔX from Equation (15) replaces terms in Equation (14), we have the following equation:

$$\Delta W = (W_x S_{xz} + W_z) \Delta Z + \Delta Z^T \left(\frac{1}{2} S_{xz}^T W_{xx} S_{xz} + \frac{1}{2} W_{zz} + W_{zx} S_{xz} \right) \Delta Z \quad (16)$$

Thus, ΔW has the form

$$\Delta W = A \Delta Z + \Delta Z^T B \Delta Z \quad (17)$$

where

$$A = W_x S_{xz} + W_z \quad (18)$$

and

$$B = \left(\frac{1}{2} S_{xz}^T W_{xx} S_{xz} + \frac{1}{2} W_{zz} + W_{zx} S_{xz} \right) \quad (19)$$

are the first and the second order line flow sensitivity matrices, respectively. The matrices for the calculation of the second order sensitivity are derived in [5].

3 POWER FLOW CONTROL

Power flow control aims at controlling active power flows and sometimes reactive power flows, through certain lines at specified levels. Here, a method for power flow control is based on implementation of power flow sensitivities. Controllable series capacitors (CSC's) are the controlled devices. It is assumed that there are n_z control variables and n_w controlled line flows. The controls can be located on the lines with controlled flow or on other lines.

Power flow through a transmission line varies approximately as a quadratic function of series capacitor compensation degree ($0 < k < k_{\max} < 1$)

[3,4]. A general quadratic model of line flows can be proposed for a multi-line control system

$$W = W^0 + AZ + Z^T BZ \quad (20)$$

where

W^0 : vector of line flows without compensation

W : vector of line flows after compensation

A : square matrix ($n_z \times n_z$)

B : three dimensional array ($n_z \times n_z \times n_z$)

Z : control vector.

From the quadratic model (20), if we assume $Z^0 = 0$, it can be obtained

$$\Delta W = A \Delta Z + \Delta Z^T B \Delta Z \quad (21)$$

As seen, A is the first order line flow sensitivity matrix, and B can be obtained by either multiple load flow solution or by second order line flow sensitivity matrix. From Equations (20) and (21) is straightforward to predict the line flows through desired lines, for a set of controllable components with given control ranges.

4 RESULTS

The simple 5-bus test system, Figure 1, is investigated. The data for the system can be found in the Appendix A. It is assumed that the CSC's are located in lines 3 and 4, with maximum degree of compensation, $k_{\max} = 0.7$. (The line reactance of a controlled line is $x_l^c = x_l^0 - z_l = x_l^0 - k_l x_l^0$.) The line flows without compensation when $z_1 = 0$ and $z_2 = 0$ are $w_3^0 = 0.2414$ p.u. and $w_4^0 = 0.2728$ p.u.

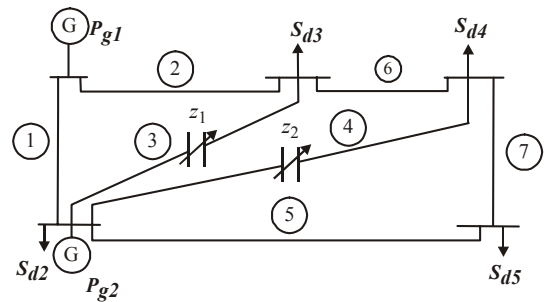


Figure 1: Test system

The quadratic models of the line flows w_3 and w_4 obtained by the first and the second order sensitivity matrices are:

$$\begin{aligned} w_3 &= 0.2414 + 0.82z_1 - 0.41z_2 + 2.98z_1^2 \\ &\quad - 1.97z_2^2 - 1.01z_1z_2 \\ w_4 &= 0.2728 - 0.36z_1 + 0.93z_2 - 1.71z_1^2 \\ &\quad + 3.36z_2^2 + 0.92z_1z_2 \end{aligned}$$

The error is 1.04% for line 3 and 1.56% for line 4 as compared with the exact solution at k_{\max} . The accuracy of the first order sensitivity method is less than of the second order sensitivity method: The maximum error is around 5-6% for $k_{\max} < 0.7$. The above quadratic models can be used for representing the line flow feasibility boundaries for different levels of compensation degree. The boundaries are not exactly straight lines, but they could with very good accuracy be approximated by straight lines, see Figure 2. Thus it suffices to calculate the line flows in the corner points. Suppose that the power flows in lines 3 and 4 are going to be controlled according to the power flow control data in Table 1.

Table 1: Power flow control data

Control line	k_{\max}	k_{\min}	Initial power [p.u.]	Specified power [p.u.]
3	0.7	0	0.2414	0.2864
4	0.7	0	0.2728	0.3128

To achieve the specified power, it is possible to use the sensitivity matrix to calculate the control variables, according to relation (12). The start point is the base case. Using the sensitivity relation (12) and substituting numerical values, a solution is obtained after 3 iterations. As seen from Figure 2, the first iteration gives a solution that is very close to the desired value. The improvements of the second and third iterations are consequently very small.

The desired point is reached for control variables:

$$\begin{bmatrix} z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0.0450 & p.u. \\ 0.0428 & p.u. \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 0.2864 & p.u. \\ 0.3128 & p.u. \end{bmatrix}$$

The compensation of 47% and 40% of the lines 2 and 3 respectively is needed in order to achieve the desired operational point. The amount of additional active power through the lines 3 and 4 is

$\Delta w_3 + \Delta w_4 = 0.0850$ p.u. The influence on the active power of the other lines due to this transfer is presented in Table 2, where different amounts of the active power flow is delivered through the lines 3 and 4, keeping the sum $\Delta w_3 + \Delta w_4 = 0.085$ p.u. constant.

As seen the influences on other lines are different for different combinations of Δw_3 and Δw_4 . This influence could be minimised due to given requirements, but that is outside the scope of this paper.

Now another scenario would be studied. Assume that from a base case the loads at buses 3 and 4 are increased and this load increase is compensated by generator 2.

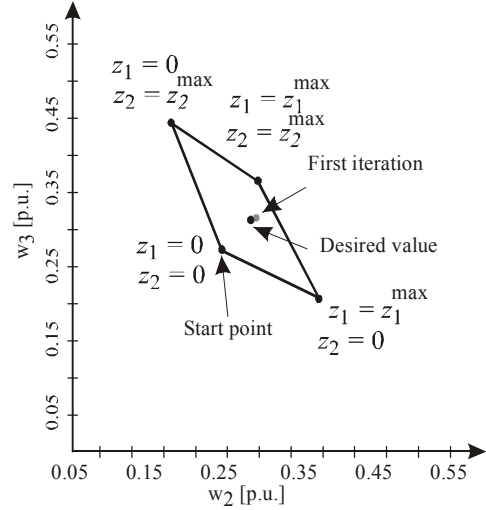


Figure 2: Line flow feasibility boundaries

Furthermore, the additional power should flow on lines 3 and 4, where the CSC's are located. A relevant question is then how large this load increase can be and still controlling the power increase to flow on lines 3 and 4. For this study the impedances of some lines were modified in order to get illustrative results.

A base case is chosen according to Figure 3. All operating points with the line flow feasibility boundaries on the straight line will give the same sum of $w_3 + w_4$. The scenarios for the increase of loads and the generation are presented in Table 3.

Table 2: Power line flows with different level of line compensation and the influence on the active power flow of the other lines

Line	w_{basecase} [p.u.]	$\Delta w_{l3} = 0.02$ $\Delta w_{l4} = 0.065$	Δw_{l7} [p.u.]	$\Delta w_{l3} = 0.065$ $\Delta w_{l4} = 0.02$	Δw_{II} [p.u.]	$\Delta w_{l3} = 0.04$ $\Delta w_{l4} = 0.045$	Δw_{III} [p.u.]
w_1	0.8701	0.9182	0.0481	0.9203	0.0502	0.9191	0.049
w_2	0.4016	0.3553	-0.0463	0.3533	-0.0483	0.3544	-0.0472
w_3	0.2414	0.2614	0.02	0.3065	0.065	0.2815	0.04
w_4	0.2714	0.3378	0.065	0.2928	0.02	0.3178	0.045
w_5	0.2728	0.4968	-0.0383	0.4988	-0.0363	0.4976	-0.0375
w_6	0.5351	0.1663	-0.0262	0.2093	0.0168	0.1855	-0.070
w_7	0.1925	0.1032	0.0383	0.1012	0.0363	0.1024	0.0375
$\sum \Delta w$			0.2822		0.2730		0.3262

Table 3: Three scenarios for test system

	Base case [p.u.]	Scenario I [p.u.]	Scenario II [p.u.]	Scenario III [p.u.]
P_{L3}	0.45	0.45 +0.07	0.45 +0.15	0.45 +0.20
P_{L4}	0.40	0.40 +0.08	0.40 +0.10	0.40 +0.15
G_2	0.40	0.40 +0.15	0.40 +0.25	0.40 +0.35
$w_3 + w_4$	$w^0 = 0.5142$	$w^0 + 0.15$	$w^0 + 0.25$	$w^0 + 0.35$
	Figure 3	Figure 4	Figure 5	Figure 6

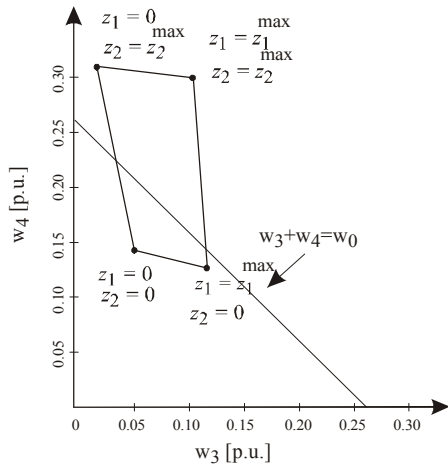


Figure 3: Load flow feasibility boundaries for the test system

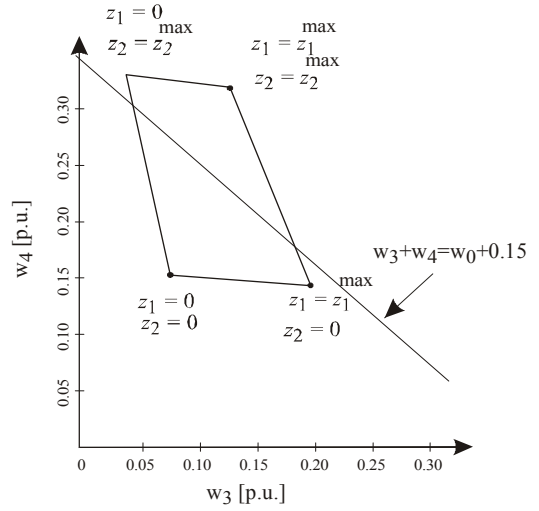


Figure 4: Load flow feasibility boundaries with increase of 0.07 p.u. and 0.08 p.u. of the loads 3 and 4

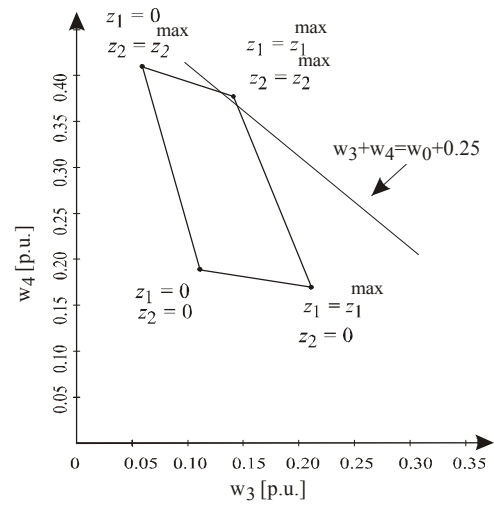


Figure 5: Load flow feasibility boundaries with increase of 0.15 p.u. and 0.10 p.u. of the loads 3 and 4

Figure 4, Figure 5 and Figure 6 present the load flow feasibility boundaries for above scenarios as well as the lines giving the desired values of $w_3 + w_4$. As can be seen, on Figure 4, the controllability is still flexible, but on Figure 5, the available controllability is almost lost. On Figure 6 it is not possible to increase active power flow through the lines 3 and 4 to the specified amount. Of course it is possible to keep power through that lines 3 and 4 on the point (z_1^{\max}, z_2^{\max}) and to let that the rest flow through other lines.

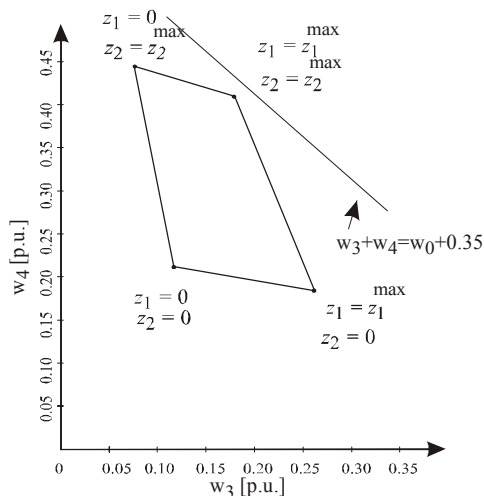


Figure 6: Load flow feasibility boundaries with increase of 0.20 p.u. and 0.15 p.u. of the loads 3 and 4

5 CONCLUSION

It has been shown how power flow can be controlled by use of controllable series devices by using a sensitivity analysis. The sensitivity analysis together with quadratic model of power flow is used for establishing the line flow feasibility boundaries and it has been shown what regions of line flows that would be obtainable for a set of controllable components with given control range. It was presented how this concept can be used to calculate possible load and generation increases without violating given constraints on line flows for different scenarios.

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APPENDIX A

HALE SYSTEM – SYSTEM DATA

A.1 Bus data

Table A.1: Bus data for Hale System

Bus No.	Bus Type	V [p.u.]	P _G	Q _G	P _L	Q _L
1	Slack	1.06			0	0
2	PV	1.00	0.4		0.2	0.10
3	PQ		0	0	0.45	0.15
4	PQ		0	0	0.40	0.05
5	PQ		0	0	0.60	0.10

A.2 Branch data

Table A.2: Branch data for Hale System

Line No.	From	To	R [p.u.]	X [p.u.]
1	1	2	0.0200	0.0599
2	1	3	0.0799	0.2397
3	2	3	0.0599	0.1798
4	2	4	0.0599	0.1798
5	2	5	0.0399	0.1198
6	3	4	0.0100	0.0300
7	4	5	0.0799	0.2397