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Voltage and Power Stability of HVDC Systems – Emerging Issues and New Analytical Methodologies

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Summary

Traditionally HVDC links have been built as single point-to-point AC/DC interconnections, or single-infeed HVDC systems as they are commonly known. However in recent years, as the use of HVDC transmission continues to develop situations have arisen where multiple HVDC links terminate in close proximity in a common AC system area. These emerging system configurations, generally known as multi-infeed HVDC systems, are particularly inherent in regional power systems predominantly interconnected by HVDC links. Already such configurations are apparent in the Scandinavian and North European power system interconnection. Elsewhere in rapidly industrializing regions such as South America and the ASEAN (Association of South East Asian Nations), multi-infeed HVDC systems potentially arise.

A phenomenon that is of great concern in the planning and operation of HVDC systems is voltage/power stability. In many cases this has been limiting for the operation of a link during weak AC system conditions. Also in AC systems without HVDC links voltage stability has been of concern in the power industry during recent years and a number of methods and tools for studying this phenomenon have been developed. Despite the fact that many aspects of the voltage stability problem are identical for HVDC systems and “pure” AC systems, much of the work has been made focusing on only one of these two system types. This paper provides an attempt to overcome this and a number of methods traditionally mainly applied to AC systems will be applied to various types of HVDC power systems. The paper is a summary of work that has been done by the authors during the last years and much of the work has been reported earlier in refs. [7] – [13] of the paper.

The paper starts with a presentation and discussion of the models used and their limitations. Some of the concepts in modal analysis of voltage stability are reviewed and this method is then applied to multi-infeed HVDC systems. By this method stability margins can be calculated and critical system locations can be identified. In the analysis of single-infeed HVDC systems Maximum Power Curves (MPC) have been extensively used. This concept is extended to multi-infeed HVDC systems and it provides additional insight into the analysis. In the paper the findings gained by the analytical tools are verified by time domain simulations.

The methods described in the previous paragraph are based on static, or rather quasi-static, models. It has been shown that these methods correctly capture and model important aspects of voltage/power stability, but not all. To give a correct description of some important issues a dynamic model is needed. Such a model is given in the paper and a number of new approaches in the analysis of HVDC systems are presented. The traditional MPC is extended to a Dynamic MPC (DMPC) and the validity of the two methods are compared and the discussed.

The last part of the paper contains a study of different instability modes of an HVDC system. By the use of bifurcation theory it is shown that depending on system parameters the voltage/power instability could be of different character. The aperiodic voltage instability that can be predicted by the static methods corresponds to a saddle-node bifurcation in the dynamic analysis. By the dynamic approach oscillatory voltage instability can be identified corresponding to a Hopf bifurcation. Also in this case time domain simulations verify analytical findings.

Keywords: HVDC systems, voltage and power stability, emerging issues, new analytical methodologies.

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Abstract – Voltage and power instability in HVDC systems have traditionally been studied for simplified system models using classical analytical methods based on quasi steady-state assumptions. However as the usage of HVDC transmission increases, new system configurations, the corresponding need for new analytical tools or adaptation of classical ones, and new concerns about the adequacy of system modeling have emerged. In this paper these emerging issues are addressed and new methodologies that could potentially meet the new needs of the HVDC industry are presented.

1. INTRODUCTION

Traditionally HVDC links have been built as single point-to-point AC/DC interconnections, or single-infeed HVDC systems as they are commonly known. However in recent years, as the use of HVDC transmission continues to develop situations have arisen where multiple HVDC links terminate in close proximity in a common AC system area [1]. These emerging system configurations, generally known as multi-infeed HVDC systems, are particularly inherent in regional power systems predominantly interconnected by HVDC links. Already such configurations are apparent in the Scandinavian and North European power system interconnection. Elsewhere in rapidly industrializing regions such as South America and the ASEAN (Association of South East Asian Nations), multi-infeed HVDC systems potentially arise.

In the past, voltage and power instability have been experienced in electrically weak single-infeed HVDC systems, i.e. when the HVDC link is terminated at an AC system location of low short-circuit capacity relative to the power rating of the HVDC link. These problems have traditionally been studied using analytical methods based on quasi steady-state assumptions [2]-[4]. For multi-infeed HVDC systems, adverse AC/DC interactions resulting in voltage and power instability are also expected to arise when one or more of the constituent AC/DC interconnections are electrically weak, similar to the situation for single-infeed HVDC systems. Since the multi-infeed HVDC system is an outgrowth of the single-infeed case, many similarities exist between them. Thus the existing analytical concepts and tools developed for single-infeed HVDC systems may be adapted for the analysis of multi-infeed HVDC systems. Nevertheless, multi-infeed HVDC systems are of recent origin and distinct from their single-infeed counterpart, new analytical concepts and tools are therefore also needed to investigate the associated voltage and power instability problems. This paper presents these new analytical concepts and a comprehensive tool for analysis of these newly emerging HVDC system configurations.

To date these analytical concepts and tools, irrespective for single or multi-infeed HVDC system configurations, have only been based on quasi steady-state assumptions. This paper also addresses voltage and power stability in HVDC systems from a dynamic approach in line with recent keen power industry interest in this context. In particular the impact of dynamic system modeling on common industry assumptions based on quasi steady-state conditions are examined. Also the newly emerging issue of nonlinear dynamics in power systems are addressed in the context of HVDC systems particularly where power system devices such as nonlinear loads and long HVDC cables are increasingly becoming common. Using a nonlinear dynamical theoretic methodology, the role of these devices in the aperiodic and oscillatory voltage collapse mechanisms of HVDC systems are also demonstrated in this paper.

2. SYSTEM MODELING ISSUES

2.1 Modeling Emerging System Configurations

A typical system model to represent HVDC systems for quasi steady-state stability analysis is as shown in Figure 1. This is a simplified single-infeed HVDC system model that can capture many of the important system phenomena in physical AC/DC interconnections.

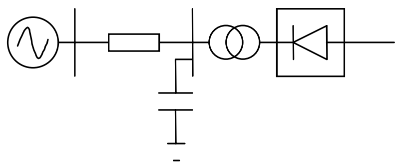

Figure 1: Classical simplified model of a single-infeed HVDC configuration

Based on this system model, analytical concepts such as Maximum Available Power (*MAP*), Voltage Sensitivity Factor (*VSF*) and Control Sensitivity Index (*CSI*) have been proposed [2]-[4]. However, recently emerging multi-infeed HVDC systems worldwide as explained in the previous section have motivated newer system models to be proposed for use in voltage and power stability analysis [7]-[9], [12]-[13]. One possible type of multi-infeed situation is the *ring-type* configuration where a multi-terminal HVDC system rings a large city to inject power at a number of strategic locations. This motivates a system model as shown in Figure 2. Another possible multi-infeed situation is the *chain-type* configuration where two AC power systems with HVDC links existing in the vicinity are interconnected. This motivates a system model as shown in Figure 3. Many variants of Figure 2 and 3 are possible but the work as presented in this paper considers mostly the chain-type configuration. Nevertheless the methodologies as proposed in this paper are generic and system independent.

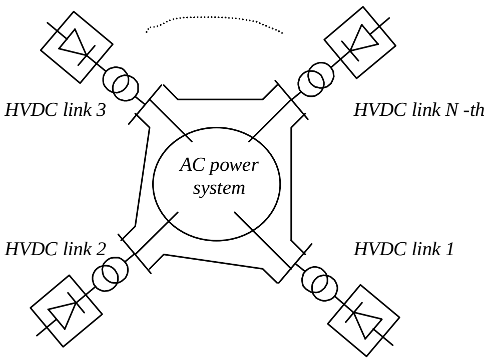

Figure 2: Ring-type multi-infeed HVDC configuration

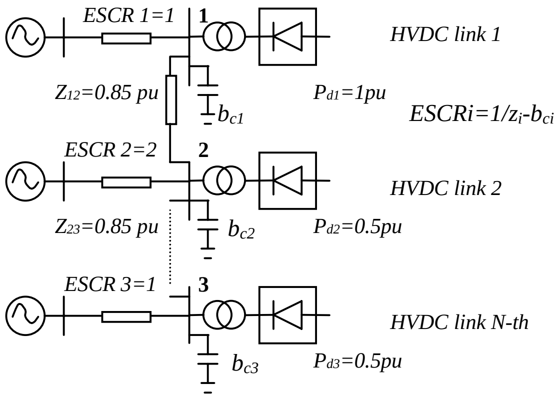

Figure 3: Chain-type multi-infeed HVDC configuration

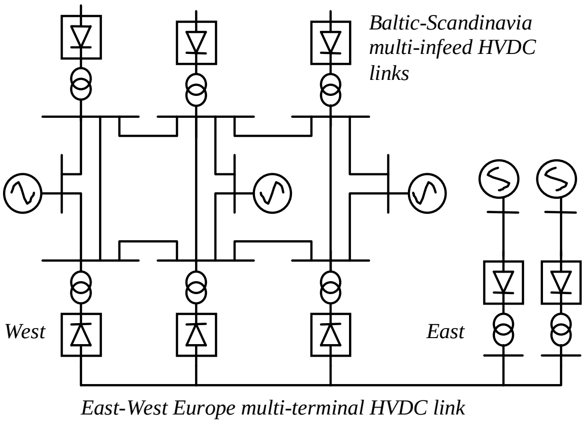

Figure 4: A potentially complex multi-infeed HVDC system

Perhaps a most general multi-infeed HVDC system configuration would ultimately be one constituted by point-to-point as well as multi-terminal HVDC links. This scenario could possibly come about with the realization of the proposed Baltic Ring and East-West Europe HVDC projects and the existing southern Scandinavia HVDC links , as shown in Figure 4.

2.2 Adequacy of System Models and Assumptions

Generally for preliminary investigations on fundamental-frequency voltage/power stability of HVDC systems, it is adequate to represent the AC system as a constant Thevenin voltage source with an equivalent short-circuit impedance. This is deemed justified due to the assumed slower voltage response in the AC system (related to the excitation control of synchronous machines) compared with that of the primary DC current controls. Moreover for such a quasi steady-state system model as shown in Figure 1, the dynamics associated with the outer-loop DC controls, DC transmission system are assumed not to impact significantly on voltage/power stability limits and therefore also typically neglected. Nevertheless recent efforts have been made to investigate the impact of machine dynamics on stability limits [5], [6]. Subsequently a dynamic model was proposed in [10] as shown in Figure 5, and systematically used to investigate the impact of various system component dynamics (which the quasi-static model typically neglects) on power stability limits. As would be seen in section 4 the validity of these assumptions are contingent upon the dynamic model of the synchronous machine and excitation system.

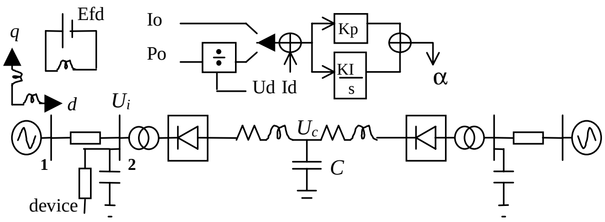

Figure 5: Dynamic model of a single-infeed HVDC configuration

3. NEW ANALYTICAL METHODOLOGIES: QUASI STEADY-STATE METHODS

3.1 Voltage Stability

The concept of voltage stability of an AC/DC interconnection was first proposed by Hammad in [3]. For the single-infeed HVDC system model of Figure 1, the *Voltage Stability Factor* (*VSF*) is defined as

$$VSF = \frac{\Delta U}{\Delta Q} \qquad .....(1)$$

In essence this *VSF* is the voltage sensitivity of the AC/DC interconnection measured as a ratio of the incremental change in the converter AC bus voltage, ΔU , to the incremental change in the injected reactive power, ΔQ , at the same bus. The *VSF* indicates the relative stability of the system – an increasing positive *VSF* indicates a corresponding increase in system voltage sensitivity to a small change in injected reactive power. Ultimately at the system stability limit, the *VSF* becomes infinite and the system is unstable for negative *VSF*.

3.1.1 Minimum Eigenvalue

The concept of voltage stability or sensitivity can be similarly extended to the multi-infeed situation using a mathematical technique known as modal analysis [7], [13]. Starting from the well known Jacobian of the Newton-Raphson power flow solution, the reduced

Jacobian J_R for the multi-infeed HVDC system can be obtained by assuming that there is no active power injection at all converter AC buses as well as neglecting all non-AC converter buses, and given by;

$$\left[\Delta Q\right] = J_R \left[\frac{\Delta U}{U}\right] \qquad \text{..... (2)}$$

where

$J_R = J_{QU} - J_{Q\theta} J_{P\theta}^{-1} J_{PU}$ and $J_{P\theta}, J_{PU}, J_{Q\theta}, J_{QU}$ are the sub-matrices of the power flow Jacobian. ΔQ , $\Delta U / U$ are column vectors comprising the reactive power mismatches and incremental bus voltage magnitudes, respectively, of the converter AC buses. Assuming that the eigenvalues of J_R are distinct, then it is diagonalizable to $J_R = \phi.\Lambda.\eta$ where ϕ is the right column and η the left row eigenvector matrix, and Λ is the diagonal eigenvalue matrix. Thus (2) may be decomposed and reduced to;

$$\Delta \tilde{Q}_i = \frac{\Delta \tilde{U}}{\lambda_i} \qquad \text{..... (3)}$$

where $\Delta \tilde{Q}_i = \eta_i [\Delta Q]$ and $\Delta \tilde{U}_i = \eta_i [\Delta U/U]$ are the incremental modal reactive power and voltage, respectively, and λ_i is the eigenvalue of the i-th eigenmode. From (3), it is seen that the incremental modal voltage response to an incremental modal reactive power is determined by the eigenvalue of the mode. When the eigenvalue of a voltage variation mode is small or zero, an infinitesimally small change in the modal reactive power causes large or infinite modal voltage magnitude change, hence instability of that mode. Consequently, the bus voltage vector which is a linear combination of the modal voltage vector would be correspondingly unstable. The minimum eigenvalue which is near or equals zero, or negative is thus the critical eigenvalue, and it indicates how close a system operating point is to voltage instability. It is also seen intuitively that (3) is akin to the *VSF* of the single-infeed case and can thus be interpreted as the *Modal Voltage Sensitivity Factor* (*MVSF*).

3.1.2 Participation Factors

Using the left and right eigenvectors corresponding to an eigenvalue, the participation factor P_{ki} of converter AC bus k in the i-th eigenmode may be defined as;

$$P_{ki} = \eta_{ik} \cdot \phi_{ki} \qquad \text{..... (5)}$$

where ϕ_{ki}, η_{ik} are the k-th element of the right column and left row eigenvector, respectively, of the i-th eigenmode. Physically, η_{ik} is a measure of the activity of the converter AC bus k in the i-th voltage variation mode. ϕ_{ki} is the weighting of the contribution of this activity, and so their product is a measure of the net participation of the converter AC bus k in the i-th voltage variation eigenmode.

The bus participation factors computed from the critical mode provide information on the critical system location of voltage instability. The converter AC bus with the largest participation factor is the critical bus, meaning that it has the largest involvement in the voltage instability. Consequently it is also the most effective location for implementing remedial measures [8], [13].

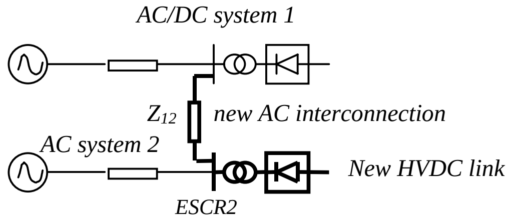

Figure 6: Example case of a planning scenario

3.1.3 Practical Examples

Modal analysis as described in the preceding sections can be applied as a comprehensive planning tool as illustrated in the practical examples below.

(a) **Determining voltage stability margins**

Figure 6 depicts an example system-planning scenario where an AC system 2 is existing in the neighborhood of AC/DC system 1. A new HVDC link is planned for AC system 2 and with that an interconnection with AC/DC system 1 is also envisaged, perhaps to export the then excess power of AC/DC system 2 to 1. In this situation the system planner is interested to know the voltage stability of the integrated AC/DC system, as affected by the coupling impedance, z_{12}, and the *ESCR* of the newly established AC/DC system 2. Using modal analysis, the voltage stability boundary for the system model of Figure 6 can be determined in the *ESCR*-z_{12} parameter space as shown in Figure 7. Thus the system would be voltage stable at an operating point P1 and voltage unstable at P2 to a small system disturbance at any one of the AC converter buses, as verified by nonlinear time-domain simulations shown in Figure 8 and 9, respectively.

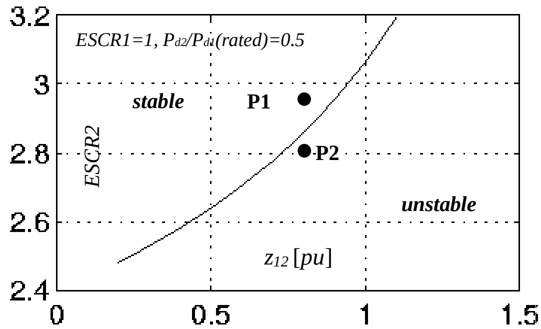

Figure 7: ESCR-z_{12} voltage stability graph

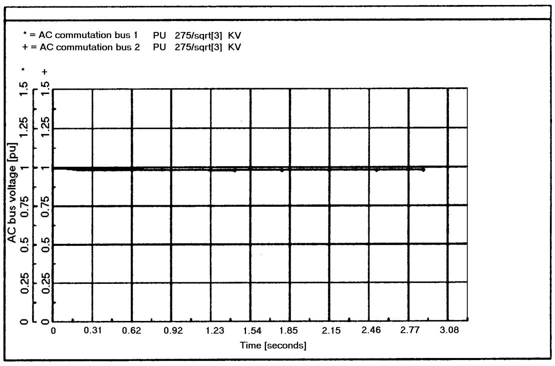

Figure 8:Operating point P1, voltage stable system

(b) Determining Critical system locations

Figure 3 depicts another example where the multi-infeed HVDC system has three HVDC links. If more than one of the constituent HVDC links are electrically weak and reactive power compensation equipment, for example a static var compensator (SVC), is required to provide voltage stability support, a question arises as to where the equipment should be installed. With the use of participation factors it is possible to identify the most effective or critical converter AC bus to install the SVC. In this example, with the given sample system parameters (see Figure 3) and Effective Short Circuit Ratios (*ESCR*), bus 1 has the largest bus participation factor (see Figure 11) and thus the most effective system location.

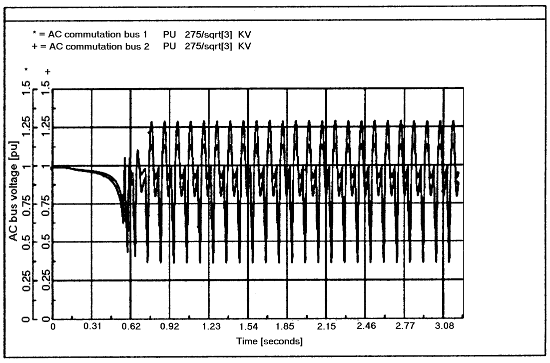

Figure 9: Operating point P2, voltage unstable system

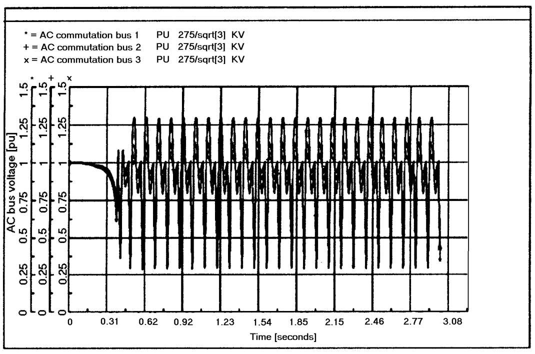

Figure 10: SVC installed at bus 3, voltage unstable system

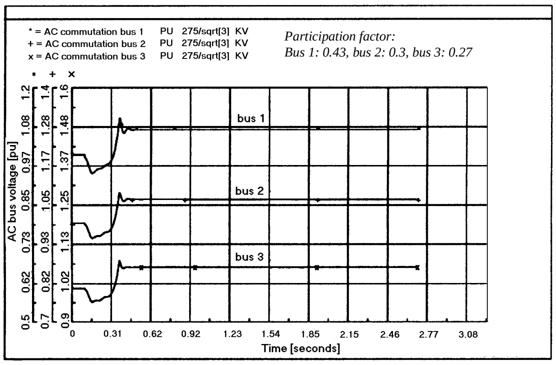

Figure 11: SVC installed at bus 1, voltage stable system

This is seen in Figure 11 from the stable voltage response to a system disturbance when the SVC is installed at bus 1. In comparison, the system is voltage unstable to the same disturbance when the same SVC is installed at bus 3 as seen from the voltage response in Figure 10.

3.2 Power Stability

The concept of *Maximum Available Power* (*MAP*) was first introduced by Ainsworth et. al. in [2] using a system model as shown in Figure 1 with the converter operating in constant extinction angle (CEA) control mode. This is the maximum DC power the converter is capable of delivering to the AC system corresponding to a DC current I_{MAP}. A further increase in the DC current I_d beyond I_{MAP} actually results in a decrease in the DC power P_d. This is due to the larger percentage decrease in the converter AC bus voltage as compared with the increase in I_d, resulting in a net decrease in P_d. Such a phenomenon corresponds with unstable system behavior, thus the *MAP* condition determines the power stability limit of the AC/DC interconnection. Mathematically this point corresponds to the condition that;

$$\frac{dP_d}{dI_d} = 0 \qquad \text{..... (6)}$$

3.2.1 Maximum Available Power

The concept of maximum available power applied to single-infeed HVDC systems can be extended to the multi-infeed case by using a similar approach to the one described in section 3.1.1 [9], [13]. In this case the power flow equations for the converter AC buses are augmented with those for the associated DC buses, neglecting all other non-converter AC buses. Applying the well-known Newton-Raphson power flow solution for these equations the AC/DC Jacobian is thus obtained. This is reduced to a DC Jacobian J_{dcR} for the multi-infeed HVDC system by eliminating all the converter AC buses based on the assumption that there are no active/reactive power injections at those buses. Thus J_{dcR} is given by;

$$\Delta P_d = J_{dcR} \Delta I_d \qquad \text{..... (7)}$$

where $J_{dcR} = J_{DI} - J_{DU} J_{R1}^{-1} J_{R2}$, $J_{R1} = J_{QU} - J_{Q\theta} J_{P\theta}^{-1} J_{PU}$, and $J_{R2} = J_{QI} - J_{Q\theta} J_{P\theta}^{-1} J_{PI}$. Here, ($J_{P\theta}$, J_{PU}, J_{QU}, $J_{Q\theta}$)

and (J_{DU} , J_{DI}) are the Jacobian submatrices for the AC and DC power flow equations, respectively. The expressions for these submatrices are given in [9], [13]. ΔP_d, ΔI_d are column vectors of incremental DC powers and currents, respectively, of the multi-infeed converter DC buses.

From (7) two approaches to analyze power stability of the multi-infeed HVDC system may be derived. The first approach is to focus on the DC current variation of only one HVDC link of interest and constraining those of the other HVDC links in the multi-infeed configuration to be constant. If the *i*-th HVDC link of the multi-infeed HVDC system is of interest, then from (7) this can be expressed as;

$$\left.\frac{\Delta P_{di}}{\Delta I_{di}}\right|_{\Delta I_{dj}=0} = J_{dcRii} \qquad j \neq i \qquad \text{..... (8)}$$

where J_{dcRii} is the *i*-th diagonal element of J_{dcR} and *j* represents the off-diagonal entry.

It is recognized that this approach gives a system-specific or local assessment of power stability, by virtue of allowing only a single degree of freedom for DC current variation. A second approach is to apply the eigenvalue decomposition technique to (7), similar to that in section 3.1.1. This conceivably provides a more system-wide or global assessment of power stability since this technique treats J_{dcR} in its entirety and no constraints on the degrees of freedom for DC current variation are imposed. Thus using $J_{dcR} = \phi.\Lambda.\eta$ in (7) and simplifying, the *i*-th eigenmode element is given by;

$$\frac{\Delta p_{dc_i}}{\Delta i_{dc_i}} = \lambda_i \qquad \text{..... (9)}$$

where $\Delta p_{dci} = \eta_i \Delta P_d$, $\Delta i_{dci} = \eta_i \Delta I_d$ are *i*-th element of the column vectors of the incremental modal DC power and DC current, respectively.

When J_{dcRii} or λ_i of (8) and (9), respectively, becomes zero it is seen that the multi-infeed HVDC system reaches a system condition akin to the *MAP* condition defined in (6) for the single-infeed case. Thus (8) and (9) may also be used to define the power stability limit of the multi-infeed HVDC system. Equation (8) defines a direct though constrained incremental DC power-current relationship for a single DC link in the multi-infeed HVDC system, much alike the single-infeed case such that this approach is known as *Standard Maximum Power Curve* (*SMPC*) in [9],[13], whereas equation (9) defines an unconstrained modal relationship and is known as the *Modal Maximum Power Curve* (*MMPC*) in [9], [13].

The two approaches yield power stability boundaries of the multi-infeed HVDC system that are non-equivalent since they are derived under different system conditions, i.e. constrained and unconstrained DC current variations for the *SMPC* and *MMPC* method, respectively. In a practical context, the *SMPC* power stability boundary applies to a situation where one constituent HVDC link of the multi-infeed HVDC system operates in constant-power control mode while the others are in constant-DC current control, whereas the *MMPC* power stability boundary applies to a situation where all the constituent HVDC links operate in constant-power control mode. The non-coincident power stability boundaries is illustrated by a practical example shown in Figure 12 using sample system parameters for the system model of Figure 6. It is noted that the *MMPC* power stability boundary gives a more conservative estimate of the system stability region as compared with the *SMPC* boundary. This is physically intuitive since the *MMPC* method allows more degrees of freedom for DC current variation to cause the system to become unstable.

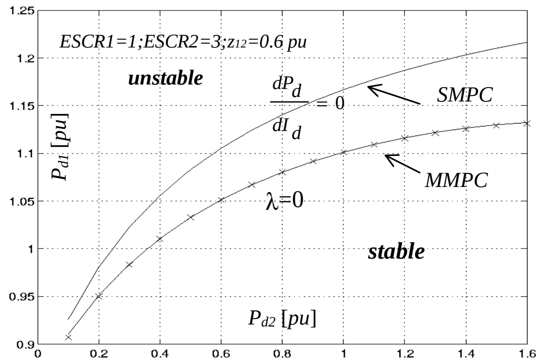

Figure 12: MMPC and SMPC power stability graphs

4. NEW ANALYTICAL METHODOLOGIES: DYNAMIC METHODS

As explained in section 2.2, the quasi-static methods for voltage/power stability analysis of single or multi-infeed HVDC systems neglect the various dynamics present in a real power system. However there had been recent industry concerns regarding the impact of system dynamics on the stability limits of HVDC systems [5], [6]. In line with these concerns, recent works [10], [11], [13], had proposed dynamic system models and methods to address these concerns systematically.

These dynamic methods are based on a mathematical representation of the power system by a set of differential-algebraic equations (DAE) given by;

$$\frac{dy}{dt} = f(y, x, \mu) \qquad \text{..... (10)}$$

$$0 = g(y, x, \mu) \qquad \text{..... (11)}$$

where $y \in \Re^n$, $x \in \Re^m$ are the differential and algebraic states of the system, respectively. $\mu \in \Re^p$ are the system parameters. Nonlinear functions *f* and *g* describe the dynamic behavior of the power system and relationship between the differential-algebraic states, respectively.

Applying the mathematical model given by (10)-(11) to the single or multi-infeed HVDC system, the dynamics of power and voltage stability in these system configurations are investigated, as described in the following.

4.1 Dynamic Maximum Available Power

The impact of system dynamics on the power stability of HVDC systems can be investigated with the dynamic system model of Figure 5 whose mathematical representation is of the form given by (10)-(11) (describing equations are given in [10], [13]). Based on this dynamic system model, the DC power-current relationship is thus derived. This is done using time-domain simulation with a transient stability program as distinguished from using only steady-state equations to derive the maximum power curve for the quasi-static system model. The resulting relationship is referred to as the *Dynamic Maximum Power Curve (DMPC)* and the limiting DC power deliverable is called the *Dynamic Maximum Available Power* (*DMAP*).

Two approaches to derive the *DMPC* were proposed in [10], [13]. One approach is to operate the system initially in steady-state under nominal conditions and subsequently ramping the DC current up and down to obtain the corresponding dynamic response of the DC power delivered by the converter. The resulting DC power-current relationship is known as the *nominal-one DMPC.* Another approach is similar to the nominal-one but with the converter initially unloaded. The DC current is then similarly ramped up dynamically to a higher value. This resulting relationship is referred to as the *nominal-zero DMPC* .

Since the *DMPC* is derived under dynamic conditions, a consideration is the DC current ramp up or down rate. Two categories are generally defined, i.e. *slow* ramp rate corresponding to the gradual and manual operator action to change the DC power order, and *fast* ramp rate corresponding to the spontaneous and immediate DC power modulation action.

Using the methodology as described above, examples of nominal-one and zero slow *DMPC*'s and fast *DMPC*'s are derived as shown in Figure 13 and 14, respectively. In Figure 13, for the same sample system parameters it can be seen that the slow *DMPC*'s (plot *a*, *b*) are lower than the corresponding Quasi-static Maximum Power Curve (*QMPC*) (plot *c*) derived from using steady-state assumptions. This implies that system dynamics impact negatively on the power stability of the system model such that the maximum power achievable is less than the quasi-static case. Figure 14 shows fast *DMPC*'s derived for exciters with fast and slow response. It is seen that the *DMPC*'s initially closely follow plot *h* which is the equivalent *QMPC* for a dynamic situation where the excitation system has not yet responded, i.e. the *QMPC* derived with system strength specified using aggregate of the synchronous machine reactance and AC system impedance. This implies that the DC power-current relationship derived under quasi-static assumptions is valid for the dynamic conditions during this initial time period. However, beyond this initial period the fast *DMPC*'s depart from plot *h* and migrate towards plot *g*. This implies that the excitation system has started to respond since plot *g* represents an equivalent *QMPC* for a dynamic situation where the Thevenin AC bus voltage is ideally maintained constant, i.e. the *QMPC* derived with the system strength specified using only the AC system impedance and excluding the synchronous machine reactance. Thus in practical situations, the basic quasi-static assumptions crucially depend on the excitation system response speed and synchronous machine model, and the associated *DMPC* is effective in the intervening region between the two equivalent *QMPC*'s, i.e. plots *g* and *h*.

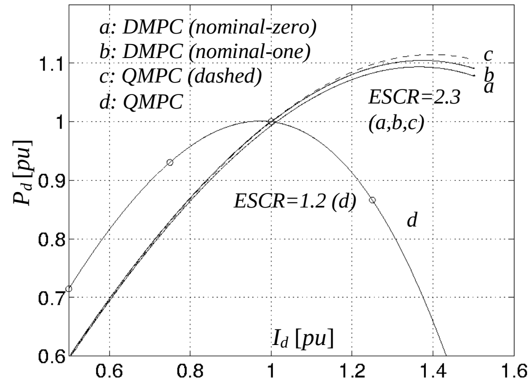

Figure 13: Nominal-one and zero slow DMPC

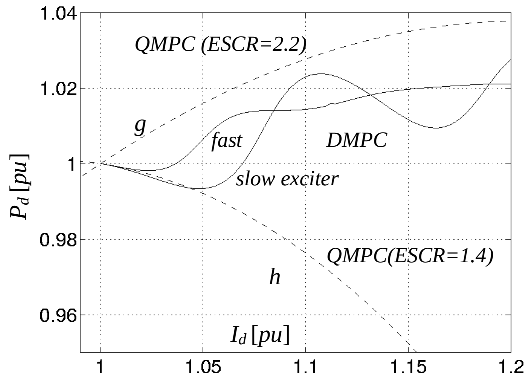

Figure 14: Fast DMPC for fast/slow exciter

4.2 Nonlinearity-caused Voltage Instability

Recent works [11], [13], have investigated the effects of nonlinear dynamics on the voltage stability of HVDC systems. There the HVDC system is treated as a parameter dependent system of DAE of the form given by (10)-(11) which defines its underlying qualitative dynamical structure. When this dynamical structure changes qualitatively under parameter variation, the system is said to undergo *bifurcation* and loss of system stability may be associated with this phenomenon. Local bifurcations in which the dynamical structure changes qualitatively in the neighborhood of an operating point, are of particular interest in this work. In this respect the DAE of the HVDC system are linearized and reduced to a form given by;

$$\frac{dy}{dt} = A\Delta y \qquad \text{….. (12)}$$

where $A = f_y - f_x\, g_x^{-1}\, g_y$ is the dynamic state matrix. f_y , f_x , g_y , g_x are the Jacobian submatrices comprising partial derivatives with respect to *y, x* as indicated by the subscripts. Though *A* is linearized its eigenvalues can detect the types of nonlinear bifurcation as described below;

- *Saddle-node bifurcation* : an eigenvalue of *A* crossing the origin axis under parameter variation. Loss of system stability is of aperiodic nature.
- *Hopf bifurcation* : A complex eigenvalue pair of *A* crossing the imaginary axis as system parameters vary. A Hopf bifurcation is characterized by periodic orbits (limit cycles) emanating from equilibra at the bifurcation point. The periodic orbits can be *supercritical* or *subcritical.* A subcritical Hopf bifurcation occurs when an unstable limit cycle coalesces with a stable equilibrium point. A supercritical Hopf bifurcation occurs when a stable limit cycle coalesces with an unstable equilibrium point.

Based on the nonlinear dynamical methodology described above, analytical expressions for the existence of saddle-node and Hopf bifurcations may be derived for the HVDC system model of Figure 5 with various system components and devices incorporated and also assuming typical constant Thevenin AC voltage sources (analytical expressions can be found in [11], [13]). These system components and devices are such as nonlinear loads or an SVC connected at the inverter AC bus, or a long HVDC cable.

From these analytical expressions it can be shown [11], [13] that aperiodic voltage collapse through saddle-node bifurcation under quasi-static conditions is equivalent to that under dynamic conditions. Oscillatory voltage instability through nonlinear behavior of Hopf bifurcation is also a feasible voltage collapse mechanism for HVDC systems. To illustrate this, the system model of Figure 5 with a long DC cable is used as an example. From the analytical expressions, an admissible solution of the Hopf bifurcation boundary C_{root1} may be derived as shown in Figure 15 for the *C-ESCR* parameter space. A system operating point is chosen to be ($ESCR_b$,C_b) which is on the Hopf bifurcation boundary. The nature of system behavior around this operating point cannot be determined by eigenvalue analysis since the phenomena is essentially nonlinear. It can however be demonstrated using time-domain simulations with a transient stability program.

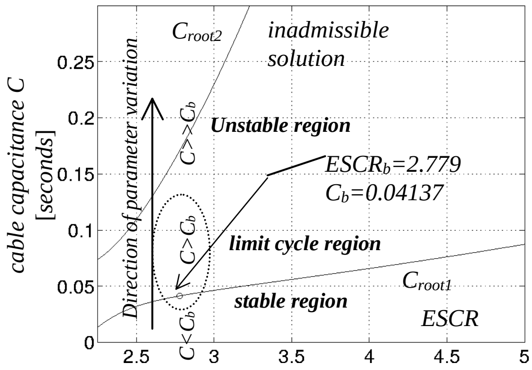

Figure 15: C-ESCR Hopf bifurcation boundary

Using the DC cable capacitance as the bifurcation parameter Figures 18, 19 show the inverter voltage time response to a DC power order step increase ΔPo for *C*=0.04203 seconds which is a little greater than the bifurcation value C_b, i.e. $C>C_b$. For this value of *C*, a large (ΔPo=0.015 pu) or small (ΔPo=0.002 pu) disturbance causes the system to develop sustained oscillations as shown in Figure 18 and 19, respectively. These imply that a stable limit cycle exists around the original stable operating point which has turned unstable under parameter variation such that the system is attracted to the stable limit cycle when bumped outside of (see phase portrait of Figure 16) or within it (Figure 17). Thus the Hopf bifurcation is supercritical. These nonlinear phenomena cannot be predicted by the linearized equations given by the dynamic state matrix *A* in (12). As seen in Figure 20, the response of the linearized system is still unstable to the same disturbance (ΔPo=0.002 pu) corresponding to the case in Figure 17 and 19. For *C*=0.0432 seconds which is much larger than C_b, i.e. $C>> C_b$ the system response is unstable as shown in Figure 21. Note that the unit of *C* is in seconds when all other quantities except time are expressed in per unit, see [11], [13].

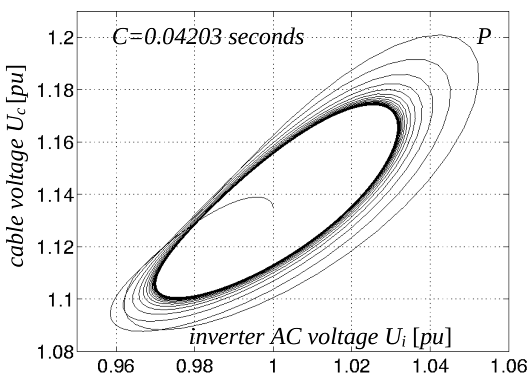

Figure 16: Nonlinear limit cycle for $C>C_b$ ΔPo=0.015pu

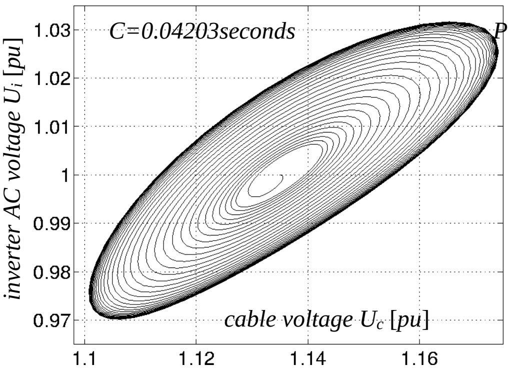

Figure 17: Nonlinear limit cycle for C>C_b ΔPo=0.002pu

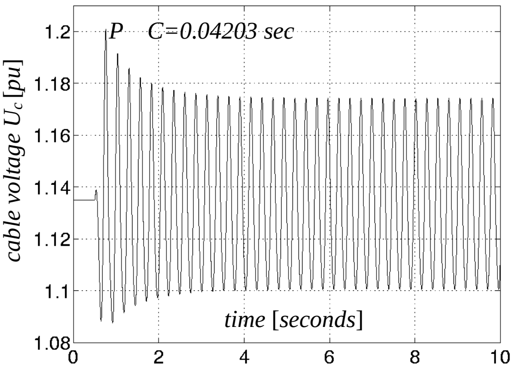

Figure 18: Supercritical region for C>C_b ΔPo=0.015 pu

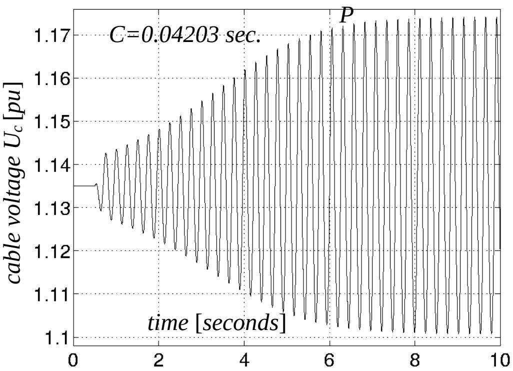

Figure 19: Supercritical region for C>C_b ΔPo=0.002pu

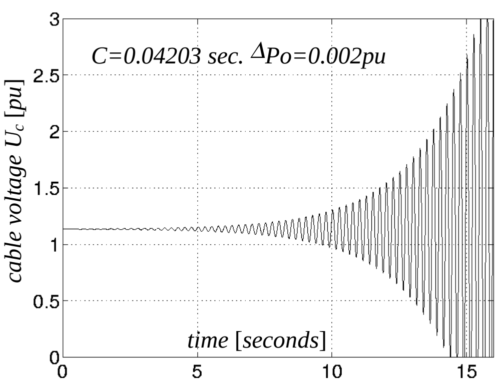

Figure 20: Linearized DAE dynamics for C>C_b

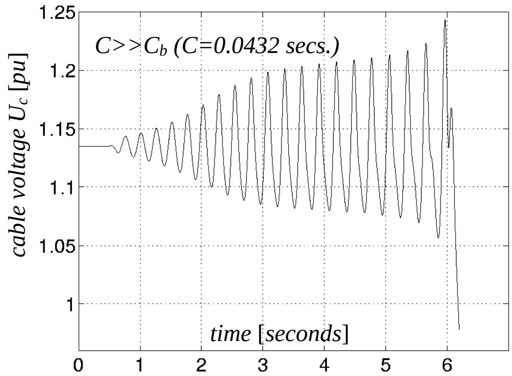

Figure 21: Unstable region for C>>C_b

5. CONCLUSION

This paper provided an overview of recently developed quasi-static and dynamic voltage/power stability analysis methodologies appropriate for newly emerging HVDC system configurations. It was shown how the quasi-static concepts and methods suited for the single-infeed system model could be extended to the higher dimension HVDC system models. Dynamic methods presented have also been shown to be able to address some of the power industry concerns regarding system modelling adequacy and the impact of system dynamics on stabilty limits, as well as facilitating fundamental understanding on the lesser established oscillatory voltage collapse mechanism in HVDC systems.

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