"N-1 Security in Optimal Power Flow Control Applied to Limited Areas"

G. Hug-Glanzmann and G. Andersson, Power Systems Laboratory ETZ Zurich

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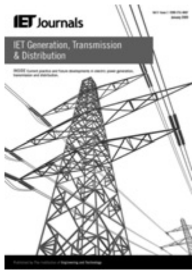

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$N-1$ security in optimal power flow control applied to limited areas

G. Hug-Glanzmann G. Andersson
Power Systems Laboratory, ETH Zurich, 8092 Zurich, Switzerland
E-mail: gabriela.hug@gmail.com

Abstract: Blackouts in recent years have demonstrated that a reliable and secure power system is a key component of an efficient economy. Therefore control devices such as flexible AC transmission system devices (FACTS) are placed in the system and utilised to improve the security of the system. A method to determine appropriate settings for these devices is optimal power flow control. As the area of influence of a FACTS device is usually limited, it is sufficient to include only a reduced area in the optimisation problem. Here, such an optimal power flow problem is formulated where the considered area is defined using sensitivity analysis. To include $N-1$ security as an objective, a current injection method is applied, which facilitates the determination of the system state in the case of a line outage, without having to carry out a full-load flow simulation.

1 Introduction

The overall goal of the power supply industry is to assure continuous access to electrical power of desired quality for all their costumers. In this regard, a reliable power system is indispensable. Consequently, considerable efforts are put into improving the system security of which a fundamental aspect is the consideration of possible outages and the minimisation of their consequences.

To avoid major harmful incidents, preventive and corrective actions are important. Preventive actions are used to keep the system in a state where an outage in the system does not lead to cascading failures possibly resulting in a blackout, and corrective actions come into play when a failure has happened and has to be mitigated. This paper concentrates on preventive actions by formulating an optimal power flow problem [1] to determine the optimal settings of the control devices with the objective to ensure $N-1$ security.

A system is $N-1$ secure if any element in the system may fail without overloading any other element [2]. Here, the focus lies on line outages and the prevention of overloaded lines. When a line fails, the state of the system changes and in order to investigate $N-1$ security, this state has to be determined. One possibility is to carry out power flow simulations for every considered outage. For system studies, this is an adequate procedure but if $N-1$ security should be included as an objective into an optimal power flow problem, this results in a considerable increase in decision variables and constraints. Another option is to apply a current injection method where a line outage is simulated by the introduction of current injections at the buses of the system [3, 4]. For the determination of these current injections, and, consequently, the voltages and currents in case of an outage, only the initial system state before the outage is needed.

An additional aspect which is discussed in this paper is the formulation of the optimal power flow problem for a reduced area of the power system [5]. The motivation for this is that the influence of a control device generally decreases with the distance from the device. Hence, in principle only the area on which the device has significant influence has to be taken into account in the optimisation problem; the rest of the system can be neglected. If $N-1$ security is an objective, also the method applied to determine the state of the system in case of an outage has to be adapted.

The paper is organised as follows: first, the optimal power flow problem for the overall system is formulated. Based on this, Section 3 gives the derivation on how to confine the optimal power flow problem to a limited area. In Section 4, the concept of the applied current injection method as derived in [6] is recapitulated. Section 5 modifies the current injection method to allow for an application in a limited area. Finally, simulation results are given in Section 6 and conclusions are drawn in Section 7.

2 Optimal power flow control

Optimal power flow control is utilised to determine the optimal values of the available control variables with respect to an objective function $f(\boldsymbol{x})$ and subject to equality and inequality constraints $\boldsymbol{g}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ [1, 7]. In principle, it corresponds to formulating and solving the optimisation problem

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) \tag{1}$$

$$\text{s.t. } \boldsymbol{g}(\boldsymbol{x}) = 0 \tag{2}$$

$$\boldsymbol{h}(\boldsymbol{x}) \le 0 \tag{3}$$

The decision variables $\boldsymbol{x}$ are composed of the control variables $\boldsymbol{u}$, the state variables $\boldsymbol{z}$ and the slack variables $\boldsymbol{s}$. In this paper, the state variables are the voltage magnitudes V_i and angles θ_i for all the buses $i \in \{1, \ldots, n_B\}$, where n_B is the number of buses in the system. The control variables are the set values of the FACTS devices placed in the system, for example, a variable susceptance B_SVC shunt-connected to the system for an SVC, a variable reactance X_TCSC in series with the line for a TCSC and a transformer with a phase shift of φ_PST for a TCPST [8, 9].

The equality constraints correspond to the power flow equations [10]. The major objective is to avoid overloads of lines in the base case when all lines are in operation as well as in cases when a line ξ is faulted. This is accomplished by formulating soft inequality constraints [Note 1]

$$|\underline{I}_{ik}| \le I_{ik,\mathrm{lim}} + s_{ik} \tag{4}$$

$$|\underline{I}_{ik}^{\xi}| \le I_{ik,\mathrm{lim}} + s_{ik}^{\xi} \tag{5}$$

$$s_{ik} \ge 0 \tag{6}$$

$$s_{ik}^{\xi} \ge 0 \tag{7}$$

where the slack variables s_{ik} and s_{ik}^{ξ} are heavily penalised in the objective function to give a strong incentive to keep the line currents $\underline{I}_{ik}$ below their limits $I_{ik,\mathrm{lim}}$.

Additional objectives are the minimisation of active power losses and the minimisation of the voltage deviations from specified reference values. The objective function is therefore given by

$$\begin{aligned} f(\boldsymbol{x}) = & \sum_{i=1}^{n_\mathrm{B}} \nu_V \cdot (V_i - V_{i,\mathrm{ref}})^2 + \sum_{(i,k)\in\mathcal{I}} \nu_I \cdot (2 \cdot s_{ik} + s_{ik}^2) \\ & + \sum_{(i,k)\in\mathcal{I},\xi\in\mathcal{O}_\mathrm{crit}} \nu_I \cdot (2 \cdot s_{ik}^{\xi} + (s_{ik}^{\xi})^2) \\ & + \sum_{(i,k)\in\mathcal{I}} \nu_P \cdot P_{ik,\mathrm{loss}}(V_i, \theta_i, V_k, \theta_k) \end{aligned} \tag{8}$$

where the set $\mathcal{I}$ denotes all (i, k), $i \in \{1, \ldots, n_\mathrm{B}\}$, $k \in \{1, \ldots, n_\mathrm{B}\}$ for which there is a line between buses i and k. For each $(i, k) \in \mathcal{I}$, $P_{ik,\mathrm{loss}}(\cdot)$ is the active power loss of the line and $V_{i,\mathrm{ref}}$ is the reference value for the voltage at bus i. The weighting parameters ν_V, ν_I and ν_P are chosen with respect to the desired importance of each term. In this paper, the most important objective is to keep the system $N-1$ secure followed by improving the voltage profile and finally reducing active power losses.

The set of outages which are taken into account is denoted by $\mathcal{O}_\mathrm{crit}$. These are the critical line outages, that is, outages for which overloading of certain other lines have to be expected [11]. A systematic way to determine the critical line outages is to carry out system studies for all possible line failures. If for a certain line failure any other line is overloaded, this line failure, consequently, is a critical outage. However, as the power system operators typically are able to list the critical outages because of their long-term experience with the system, extensive system studies can often be omitted.

3 Limited area control

Control devices in power systems have often only significant influence on a certain area around the device. For the rest of the system, the influence is negligible. The conclusion is that not the entire system has to be included in the formulation of the optimal power flow problem, but it is sufficient to include the area on which the device has significant influence [5].

The first step is to find the area of influence of the considered device. This is accomplished by sensitivity analysis [12, 13], where the sensitivity K_{y-u} of a system variable y with respect to a control variable u corresponds to

$$\begin{aligned} K_{y-u} &= \frac{\mathrm{d}y(\boldsymbol{z}, \boldsymbol{u})}{\mathrm{d}u} = \frac{\partial y}{\partial u} + \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial u} \\ &\simeq \frac{\partial y}{\partial u} - \frac{\partial y}{\partial z} \cdot \left(\frac{\partial \boldsymbol{g}}{\partial z}\right)^{-1} \cdot \frac{\partial \boldsymbol{g}}{\partial u} \end{aligned} \tag{9}$$

As this is a measure for the influence of u on y, it can be used to determine the area of influence of a device: if the sensitivity of the voltage at a bus with respect to the control variable is larger than a certain pre-specified limit, then this bus is included in the area. Similarly, if the sensitivity of the

Note 1: In this paper, the underlining of a variable indicates that it is a complex variable.

power flow on a line exceeds a certain limit, it is part of the area.

The sensitivities and, consequently, the resulting area of influence are to some extent dependent on the working conditions. Most often the areas differ by a few buses and lines. A practical approach to determine a reasonable area of influence for all situations is to identify typical loading situations and choose the area as a combination of the areas of influence derived from these loading situations. Generally, the area of influence includes the buses and lines that are close to the location of the control device.

Having defined the area, the optimal power flow problem confined to this area is formulated. The decision variables, the equality and inequality constraints are reduced to variables and constraints associated with buses and lines within the area, and only terms are taken into account in the objective function, which are associated with these buses and lines.

However, further consideration is needed for the buses at the border of the area. The formulation of the power balances at these buses will result in equations which are dependent on voltages at buses outside of the area, that is, variables that are not state variables any more. Therefore the border buses are turned into a kind of slack buses, where the influence of the FACTS device on the voltage magnitude V_i and angle θ_i is determined using sensitivity values. Hence, the adapted power flow equations for such a bus i are given by

$$V_i = V_{0_i} + \tilde{K}_{V_i-u} \cdot u \tag{10}$$

$$\theta_i = \theta_{0_i} + \tilde{K}_{\theta_i-u} \cdot u \tag{11}$$

where V_{0_i} and θ_{0_i} are the voltage magnitude and angle with the set point of the FACTS device equal to zero. For an accurate determination of V_i and θ_i, it has to be taken into account that the sensitivities (9) are not constant over the range of the FACTS device but dependent on its actual setting. Hence, second-order polynomials

$$K_{y-u} \simeq a_{y-u} \cdot u^2 + b_{y-u} \cdot u + c_{y-u} \tag{12}$$

are used to incorporate the dependency of K_{y-u} on u, and $\tilde{K}_{V_i-u}$ and $\tilde{K}_{\theta_i-u}$ are equal to the average value of the sensitivities K_{V_i-u} and K_{θ_i-u} at the initial point where the FACTS device is set to zero and the considered set point for u [14]. The same approximations will be used in Section 5 for the determination of the load flow in the limited area in case of a line failure.

4 Current injection method

The idea of current injection methods is to simulate a change in the system state by introducing current injections at the buses of the system [3, 4]. Equations are formulated and solved to find the values of the current injections enabling the direct derivation of the new system state.

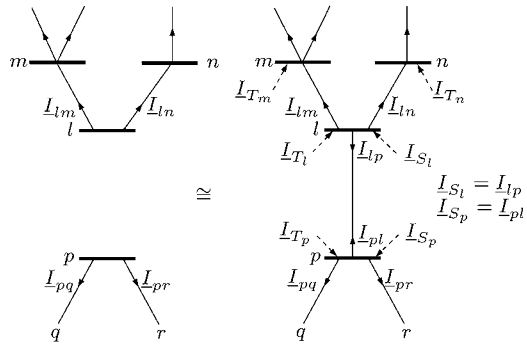

Figure 1 *Concept of the current injection method*

A possible application is the determination of the system state in case of a line outage. The current injection method described in this paper is an extension of a basic method presented in [3]. To simulate the outage with the state before the outage as the initial situation, two types of current injections are introduced: $\underline{\boldsymbol{I}}_\mathrm{S}$, only non-zero at the ends of the faulted line, compensates the current on the actually faulted line [3] and $\underline{\boldsymbol{I}}_\mathrm{T}$ is used to adapt the bus currents such that the power flow equations are fulfilled [6, 11].

Fig. 1 visualises the method. On the left-hand side, the situation with an outage of line (l, p) is shown, whereas the right-hand side shows the situation with the grid topology before the fault but with additional injection currents. The currents $\underline{\boldsymbol{I}}_\mathrm{S}$ cancel out the virtual current on line (l, p), whereas $\underline{\boldsymbol{I}}_\mathrm{T}$ correspond to the changes in the currents injected by the generators or drawn by the loads to fulfill the power flow equations.

The starting point for the derivation of the equation system to find the values of the injection currents is the relation between the bus voltages $\underline{\boldsymbol{V}}_0$ and bus currents $\underline{\boldsymbol{I}}_0$ in the initial system state given by

$$\underline{\boldsymbol{V}}_0 = \underline{\boldsymbol{Z}}_0 \cdot \underline{\boldsymbol{I}}_0 \tag{13}$$

where $\underline{\boldsymbol{Z}}_0$ is the bus impedance matrix for the non-faulted grid topology. The according relation for the case with a line outage is

$$\underline{\boldsymbol{V}}_\mathrm{F} = \underline{\boldsymbol{Z}}_\mathrm{F} \cdot \underline{\boldsymbol{I}}_\mathrm{F} \tag{14}$$

where $\underline{\boldsymbol{V}}_\mathrm{F}$ and $\underline{\boldsymbol{I}}_\mathrm{F}$ are the bus voltages and currents in the faulted situation and $\underline{\boldsymbol{Z}}_\mathrm{F}$ is the adapted bus impedance matrix.

The injection currents $\underline{\boldsymbol{I}}_\mathrm{T}$ correspond to the changes in the bus currents caused by the line outage, and thus

$$\underline{\boldsymbol{I}}_\mathrm{F} = \underline{\boldsymbol{I}}_0 + \underline{\boldsymbol{I}}_\mathrm{T} \tag{15}$$

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On the other hand, the currents $\underline{\boldsymbol{I}}_{\mathrm{S}}$ are used to compensate for the currents flowing on the line that is actually faulted. This results in the equation

$$\underline{\boldsymbol{V}}_{\mathrm{F}} = \underline{\boldsymbol{Z}}_0 \cdot \left(\underline{\boldsymbol{I}}_{\mathrm{F}} + \underline{\boldsymbol{I}}_{\mathrm{S}}\right) \tag{16}$$

Consequently, the changes in the bus voltages caused by the line outage are

$$\Delta\underline{\boldsymbol{V}} = \underline{\boldsymbol{V}}_0 - \underline{\boldsymbol{V}}_{\mathrm{F}} = -\underline{\boldsymbol{Z}}_0 \cdot (\underline{\boldsymbol{I}}_{\mathrm{T}} + \underline{\boldsymbol{I}}_{\mathrm{S}}) \tag{17}$$

and the changes in the line currents are

$$\begin{aligned}\Delta\underline{\boldsymbol{I}}_{\mathrm{line}} &= \underline{\boldsymbol{Y}}_{\mathrm{L}} \cdot \Delta\underline{\boldsymbol{V}} = \underbrace{-\underline{\boldsymbol{Y}}_{\mathrm{L}} \cdot \underline{\boldsymbol{Z}}_0}_{\underline{\boldsymbol{D}}} \cdot (\underline{\boldsymbol{I}}_{\mathrm{S}} + \underline{\boldsymbol{I}}_{\mathrm{T}}) \\ &= \underline{\boldsymbol{D}} \cdot (\underline{\boldsymbol{I}}_{\mathrm{S}} + \underline{\boldsymbol{I}}_{\mathrm{T}}) \end{aligned} \tag{18}$$

where $\underline{\boldsymbol{Y}}_{\mathrm{L}}$ is the line admittance matrix of the situation without fault and $\underline{\boldsymbol{D}}$ is the matrix of distribution factors reflecting the influence of the injection currents on line currents.

Because the injection currents $\underline{\boldsymbol{I}}_{\mathrm{S}}$ are used to cancel out the virtual current on the actually faulted line (l, p), the following equations have to hold

$$\begin{aligned}\underline{I}_{\mathrm{S}_l} &= \underline{I}_{lp,0} - \Delta\underline{I}_{lp} \\ &= \underline{I}_{lp,0} - \underline{D}_{(lp,l)} \cdot \underline{I}_{\mathrm{S}_l} - \underline{D}_{(lp,p)} \cdot \underline{I}_{\mathrm{S}_p} - \underline{\boldsymbol{D}}_{lp} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}} \end{aligned} \tag{19}$$

$$\begin{aligned}\underline{I}_{\mathrm{S}_p} &= \underline{I}_{pl,0} - \Delta\underline{I}_{lp} \\ &= \underline{I}_{pl,0} - \underline{D}_{(pl,l)} \cdot \underline{I}_{\mathrm{S}_l} - \underline{D}_{(pl,p)} \cdot \underline{I}_{\mathrm{S}_p} - \underline{\boldsymbol{D}}_{pl} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}} \end{aligned} \tag{20}$$

where $\Delta\underline{I}_{lp}$ corresponds to the element in $\Delta\underline{\boldsymbol{I}}_{\mathrm{line}}$ associated with line (l, p) and $\underline{I}_{lp,0}$ is the initial current before the fault. The notation $\underline{D}_{(lp,p)}$ indicates the element in the row associated with line (l, p) of column p in matrix $\underline{\boldsymbol{D}}$ and $\underline{I}_{\mathrm{S}_l}$ the lth element in the current injection vector. These equations basically state that the currents $\underline{\boldsymbol{I}}_{\mathrm{S}}$ injected at the buses adjacent to the faulted line have to be equal to the virtual currents on this line, also taking into account the changes in the line currents introduced by the injection currents $\underline{\boldsymbol{I}}_{\mathrm{S}}$ themselves and $\underline{\boldsymbol{I}}_{\mathrm{T}}$.

Additional equalities result from the power flow equations, which are dependent on the type of the considered bus. In the following derivations, a single index added to a matrix corresponds to the row in the matrix given by this index, and an index added to a vector indicates the element at this index, that is, $\underline{\boldsymbol{Z}}_{0_i}$ is the ith row of matrix $\underline{\boldsymbol{Z}}_0$ and $\underline{V}_{\mathrm{F}_i}$ is the ith element in vector $\underline{\boldsymbol{V}}_{\mathrm{F}}$.

The equations for bus i are given as follows.

- *Slack bus*: Voltage angle as well as voltage magnitude stay unchanged and hence

$$\Delta\underline{V}_i = -\underline{\boldsymbol{Z}}_{0_i} \cdot (\underline{\boldsymbol{I}}_{\mathrm{S}} + \underline{\boldsymbol{I}}_{\mathrm{T}}) = 0 \tag{21}$$

- *PV bus*: Voltage magnitude and active power injection are fixed and hence the same before and during the outage. For the voltage, this yields

$$|\underline{V}_{\mathrm{F}_i}| = |\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot (\underline{\boldsymbol{I}}_0 + \underline{\boldsymbol{I}}_{\mathrm{T}})| = |\underline{V}_{0_i}| \tag{22}$$

and taking the square on both sides

$$\begin{aligned}|\underline{V}_{0_i}|^2 &= (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot (\underline{\boldsymbol{I}}_0 + \underline{\boldsymbol{I}}_{\mathrm{T}})) \cdot (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot (\underline{\boldsymbol{I}}_0 + \underline{\boldsymbol{I}}_{\mathrm{T}}))^* \\ &= |\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_0|^2 + (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_0) \cdot (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}})^* \\ &\quad + (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_0)^* \cdot (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}}) + |\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}}|^2 \end{aligned} \tag{23}$$

For the active power injection, the equation is

$$\begin{aligned}\Re\left\{\underline{S}_{\mathrm{F}_i}\right\} &= \Re\left\{\underline{V}_{\mathrm{F}_i} \cdot (\underline{I}_{0_i} + \underline{I}_{\mathrm{T}_i})^*\right\} \\ &= \Re\left\{(\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_{\mathrm{F}}) \cdot (\underline{I}_{0_i} + \underline{I}_{\mathrm{T}_i})^*\right\} \\ &= \Re\left\{(\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot (\underline{\boldsymbol{I}}_0 + \underline{\boldsymbol{I}}_{\mathrm{T}})) \cdot (\underline{I}_{0_i} + \underline{I}_{\mathrm{T}_i})^*\right\} \\ &= \Re\left\{(\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_0) \cdot \underline{I}^*_{0_i} + (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_0) \cdot \underline{I}^*_{\mathrm{T}_i} \right. \\ &\quad \left. + (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}}) \cdot \underline{I}^*_{0_i} + (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}}) \cdot \underline{I}^*_{\mathrm{T}_i}\right\} \\ &= \Re\left\{\underline{V}_{0_i} \cdot \underline{I}^*_{0_i}\right\} \end{aligned} \tag{24}$$

- *PQ bus with load*: Here, the active and the reactive power injections are pre-defined which is expressed by

$$\begin{aligned}\underline{S}_{\mathrm{F}_i} &= \underline{V}_{\mathrm{F}_i} \cdot (\underline{I}_{0_i} + \underline{I}_{\mathrm{T}_i})^* \\ &= (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot (\underline{\boldsymbol{I}}_0 + \underline{\boldsymbol{I}}_{\mathrm{T}})) \cdot (\underline{I}_{0_i} + \underline{I}_{\mathrm{T}_i})^* \\ &= (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_0) \cdot \underline{I}^*_{0_i} + (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_0) \cdot \underline{I}^*_{\mathrm{T}_i} \\ &\quad + (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}}) \cdot \underline{I}^*_{0_i} + (\underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot \underline{\boldsymbol{I}}_{\mathrm{T}}) \cdot \underline{I}^*_{\mathrm{T}_i} \\ &= \underline{V}_{0_i} \cdot \underline{I}^*_{0_i} \end{aligned} \tag{25}$$

- *PQ bus without load*: The injection currents for these buses are zero and, hence

$$\underline{I}_{\mathrm{T}_i} = 0 \tag{26}$$

As the quadratic terms of $\underline{\boldsymbol{I}}_{\mathrm{T}}$ in (23)–(25) are assumed to be comparably low, they are omitted. The resulting equation system is a linear, complex system of equations in $\underline{\boldsymbol{I}}_{\mathrm{T}}$ and $\underline{\boldsymbol{I}}_{\mathrm{S}}$. Splitting the real and imaginary parts and writing it in matrix form, yields

$$\boldsymbol{A}_{\mathrm{ic}} \cdot \underbrace{\begin{pmatrix} \Re\{\underline{\boldsymbol{I}}_{\mathrm{T}}\} \\ \Im\{\underline{\boldsymbol{I}}_{\mathrm{T}}\} \\ \Re\{\underline{\boldsymbol{I}}_{\mathrm{S}}\} \\ \Im\{\underline{\boldsymbol{I}}_{\mathrm{S}}\} \end{pmatrix}}_{\boldsymbol{I}_{\mathrm{ic}}} = \boldsymbol{b}_{\mathrm{ic}} \tag{27}$$

Hence, the elements in $\boldsymbol{A}_{\mathrm{ic}}$ are equal to the factors with which $\underline{\boldsymbol{I}}_{\mathrm{T}}$ and $\underline{\boldsymbol{I}}_{\mathrm{S}}$ are multiplied in (19)–(26) and $\boldsymbol{b}_{\mathrm{ic}}$ corresponds to the constant terms. Both are functions of the elements in $\underline{\boldsymbol{V}}_0$, $\underline{\boldsymbol{I}}_0$, $\underline{\boldsymbol{Z}}_0$ and $\underline{\boldsymbol{Z}}_{\mathrm{F}}$.

To minimise the deviations from the exact values, the injection currents obtained by solving (27) can be adapted by applying an additional correction step [11]. The equation system taking into account also the quadratic terms is defined by

$$\boldsymbol{A}_{\mathrm{ic}} \cdot \boldsymbol{I}_{\mathrm{ic}} + \underbrace{\boldsymbol{A}_{\mathrm{ic},2} \cdot \boldsymbol{I}_{\mathrm{ic}}^2}_{\varsigma} = \boldsymbol{b}_{\mathrm{ic}} \tag{28}$$

An improved solution $\boldsymbol{I}_{\mathrm{ic,cor}}$ for the injection currents results if first $\boldsymbol{I}_{\mathrm{ic}}$ is determined from (27), and then the term ς is evaluated and used as a correction term in

$$\boldsymbol{A}_{\mathrm{ic}} \cdot \boldsymbol{I}_{\mathrm{ic,cor}} = \boldsymbol{b}_{\mathrm{ic}} - \varsigma \tag{29}$$

to solve for $\boldsymbol{I}_{\mathrm{ic,cor}}$.

Either way, carrying out the correction step or not, having finally determined the values for the injection currents, the changes in bus voltages and line currents can easily be calculated by (17) and (18).

When the presented current injection method is included into the optimal power flow problem, where the elements in the matrices $\boldsymbol{A}_{\mathrm{ic}}$, $\boldsymbol{A}_{\mathrm{ic},2}$ and vector $\boldsymbol{b}_{\mathrm{ic}}$ are functions of the state variables, the additional computation effort should not be underestimated. A way to reduce the computational effort is to set the elements in $\underline{\boldsymbol{I}}_{\mathrm{T}}$ associated with buses, where the change in the bus current induced by the line outage is expected to be small to zero. This principally corresponds to decreasing the dimension of the equation in (27). The identification of the buses for which the change in the bus current is negligible can be done offline.

An advantage of using the current injection method to include $N-1$ security in an optimal power flow problem is that the current injections and therefore also the voltage and bus currents, in case of a line outage, can explicitly be given as a function of the decision variables in the base case.

5 Reduction to a limited area

In the limited area control introduced in Section 3, the optimal power flow problem is defined for a reduced area. The influences of the FACTS devices on the voltage magnitudes and angles at the border buses are approximated using sensitivity values, yielding a new type of bus in addition to the commonly used slack, PV and PQ buses. For the determination of the injection currents $\underline{\boldsymbol{I}}_{\mathrm{T}}$, equations in the line with (21)–(26) have to be formulated for this type of bus.

Assuming a single device and applying (10) and (11), the voltage magnitude and angle at the border bus i for the faulted situation are given by

$$V_{\mathrm{F}_i} = V_{\mathrm{F}_i,0} + \tilde{K}^{\xi}_{V_{\mathrm{F}_i}-u} \cdot u \tag{30}$$

$$\theta_{\mathrm{F}_i} = \theta_{\mathrm{F}_i,0} + \tilde{K}^{\xi}_{\theta_{\mathrm{F}_i}-u} \cdot u \tag{31}$$

where $V_{\mathrm{F}_i,0}$ and $\theta_{\mathrm{F}_i,0}$ are the voltage magnitude and angle in case of the considered outage ξ but with FACTS device setting equal to zero, and $\tilde{K}^{\xi}_{V_{\mathrm{F}_i}-u}$ and $\tilde{K}^{\xi}_{\theta_{\mathrm{F}_i}-u}$ are the corresponding sensitivity polynomials. The identification of $\boldsymbol{V}_{\mathrm{F},0}$ and $\theta_{\mathrm{F},0}$ and the parameters for the sensitivity functions can be done offline, that is, independent of the optimisation. Therefore the voltages for all considered outages and also the required sensitivity values are determined beforehand. The equation for a border bus i in the current injection method then follows as

$$\begin{aligned} \underline{V}_{\mathrm{F}_i} &= \underline{\boldsymbol{Z}}_{\mathrm{F}_i} \cdot (\underline{\boldsymbol{I}}_0 + \underline{\boldsymbol{I}}_{\mathrm{T}}) \\ &= (V_{\mathrm{F}_i,0} + \tilde{K}^{\xi}_{V_{\mathrm{F}_i}-u} \cdot u) \cdot \mathrm{e}^{\mathrm{j}\left(\theta_{\mathrm{F}_i,0} + \tilde{K}^{\xi}_{\theta_{\mathrm{F}_i}-u} \cdot u\right)} \end{aligned} \tag{32}$$

The accuracy of the results obtained from the current injection method applied to a reduced area depends on the combination of the considered line outage and the chosen area. If the accuracy is unsatisfactory, a remedy is to adapt the area. Such situations arise most often when the reduced grid is radial and an outage yields two disconnected grids. As realistic grids are generally highly meshed, this should not be an issue. However, if necessary, lines and buses can be added, which close the ring.

A modification of the current injection method for a limited area, which possibly improves accuracy, is to assume that the faulted line is not included in the area and to turn both buses at the ends of this line into border buses. Consequently, there is no need for the injection currents $\underline{\boldsymbol{I}}_{\mathrm{S}}$ and (19) and (20). As the buses at the ends of the faulted line are now border buses, (32) is formulated for these buses instead of the according equations in (21)–(26).

This implies that the sensitivity values $\tilde{K}^{\xi}_{V_{\mathrm{F}_i}-u}$ and $\tilde{K}^{\xi}_{\theta_{\mathrm{F}_i}-u}$ are known for the buses at the ends of the faulted line. A disadvantage is that the higher the influence of the FACTS device on these buses, the higher are the errors introduced by inaccurate sensitivity values.

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For a clear distinction between the method where injection currents $\underline{I}_S$ are used and the modified version where these currents are omitted, the first option is denoted as procedure A and the latter as procedure B.

6 Simulation results

In this section, simulation results for the derived methods applied to the IEEE 57 bus system (with an additional generator at bus 30) are presented. First, the accuracy of the current injection method applied to reduced areas is evaluated, and then the method is used to guarantee $N-1$ security employing optimal power flow control. The implementation is done in MATLAB.

6.1 Application to reduced area

In the following, results obtained by applying the current injection method to a reduced area are discussed. Both approaches A and B described in Section 5 are applied for different types of FACTS devices. The areas are determined by sensitivity analysis and extended such that islanding only considering the limited area is avoided in case of any line outage. The considered devices, their location and the buses (B.) and lines (L.) within the area are given in Table 1. The chosen set value for the SVCs is 0.3 p.u., for the TCSCs 80% capacitive compensation of the line where it is placed, and for the TCPSTs 8° phase shift.

For each of these scenarios, the currents for all possible line outages, inside as well as outside of the area, are determined and the results are compared with the exact values obtained from a full load flow calculation. The interesting values per outage are the average deviation over all lines and the maximal deviation on a single line.

Table 2 contains an overview of the results. The notations have the following meanings:

- *Approach A/B:* States if approach A or B is used according to the derivations in Section 5 (if the faulted line is not included within the area, both approaches are the same).

- *w/o/with correction*: Indication if the correction step is applied.

- *Maximum*: Gives the three outages (out) with the largest maximal errors (value) and the line on which this error occurs (line). If the number of the faulted line is written in italics, this indicates that this line is not included in the reduced area.

- *Aver.:* Gives the average error over all included lines for the considered outage (out).

The application of the current injection method for reduced areas yields in most of the cases very accurate values for the line currents. In addition, the correction step decreases the maximal as well as the average deviations

Table 1 Buses (B.) and lines (L.) for the cases where the current injection method for reduced areas is applied

SVC B.14	B.	1, 3, 9, 11, 12, 13, 14, 15, 20, 21, 22, 23, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 56, 57
	L.	10, 11, 12, 13, 14, 15, 18, 24, 25, 28, 31, 32, 33, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 66, 72, 73, 74, 75, 76, 77, 78, 79
SVC B.36	B.	21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 44, 48, 56, 57
	L.	32, 33, 34, 35, 36, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 57, 73, 75, 76, 77, 79
TCSC L.23	B.	1, 2, 3, 4, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 38, 46, 47, 48, 49, 50, 51, 55
	L.	1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24 25, 26, 27, 28, 59, 60, 61, 62, 63, 64, 65, 66, 78, 79, 80
TCSC L.28	B.	1, 3, 4, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 29, 38, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55
	L.	3, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 22, 25, 26, 27, 28, 41, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 72, 78, 79, 80
PST L.44	B.	3, 4, 6, 7, 8, 9, 11, 13, 15, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57
	L.	3, 5, 6, 7, 8, 10, 12, 14, 18, 22, 24, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 67, 68, 69, 70, 73, 74, 76, 77, 80
PST L.70	B.	3, 4, 6, 7, 8, 9, 11, 13, 15, 22, 23, 24, 26, 27, 28, 29, 38, 44, 45, 52, 53, 54, 55
	L.	3, 5, 6, 7, 8, 10, 12, 14, 18, 22, 24, 33, 34, 37, 38, 39, 40, 41, 53, 57, 58, 67, 68, 69, 70, 72, 80

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Table 2 Maximum and averaged deviations for the application of the current injection method to reduced areas

	Approach A, w/o correct.			Approach A, with correct.			Approach B, w/o correct.			Approach B, with correct.		
	Maximum		Aver. $\times 10^{-3}$	Maximum		Aver. $\times 10^{-3}$	Maximum		Aver. $\times 10^{-3}$	Maximum		Aver. $\times 10^{-3}$
	out	value/line		out	value/line		out	value/line		out	value/line	
SVC	48	0.0303/47	1.8848	48	0.0297/47	1.7837	15	0.0046/33	0.7959	*42*	0.0014/45	0.0371
bus	15	0.0046/33	0.7958	46	0.0018/45	0.0508	50	0.0027/48	0.3605	50	0.0008/49	0.0735
14	50	0.0030/55	0.4316	47	0.0018/45	0.0503	48	0.0025/45	0.0646	48	0.0005/45	0.0132
SVC	34	0.0204/48	5.0907	50	0.0184/42	4.0584	49	0.0187/73	4.9274	49	0.0187/73	4.7033
bus	50	0.0166/53	5.4861	34	0.0134/38	3.6472	50	0.0167/49	5.2959	50	0.0153/49	5.0759
36	53	0.0137/48	3.6537	53	0.0075/42	2.1180	42	0.0083/48	1.6229	42	0.0104/43	0.7788
TCSC	2	0.1928/1	13.9276	1	0.0242/25	3.5959	1	0.0493/2	3.8443	65	0.0157/23	1.1091
line	1	0.1616/25	29.3077	2	0.0179/1	1.3756	15	0.0340/25	8.4756	1	0.0121/3	0.9775
23	15	0.0947/25	20.8725	15	0.0134/3	2.5463	8	0.0244/25	3.0468	15	0.0110/3	1.5433
TCSC	15	0.0514/8	4.9261	15	0.0126/8	1.0613	15	0.0338/8	3.5286	15	0.0108/8	1.3506
line	8	0.0239/22	2.5532	8	0.0077/25	1.3620	3	0.0108/8	1.5274	18	0.0102/13	1.0876
28	*1*	0.0100/8	1.5871	3	0.0054/18	0.7802	18	0.0102/13	1.0862	58	0.0102/13	1.2705
PST	40	0.0339/34	6.2551	40	0.0217/42	3.2548	*15*	0.0294/8	2.4062	42	0.0094/43	0.4117
line	*15*	0.0294/8	2.4062	8	0.0122/22	1.3683	3	0.0133/8	0.8278	*15*	0.0092/8	0.5202
44	42	0.0242/50	4.4336	*15*	0.0092/8	0.5202	*1*	0.0098/8	0.7419	3	0.0024/8	0.1285
PST	8	0.0339/6	4.9846	*15*	0.0091/8	0.7431	*15*	0.0303/8	3.6186	*15*	0.0091/8	0.7431
line	*15*	0.0303/8	3.6186	8	0.0058/40	1.2297	3	0.0152/8	1.3859	67	0.0025/68	0.1526
70	3	0.0161/8	1.7005	67	0.0030/8	0.3029	41	0.0114/67	0.8898	3	0.0024/8	0.2186

from the exact values significantly. In most of the considered scenarios, the maximal errors do not exceed 0.035 p.u. without the correction step and 0.02 p.u. with the correction step. The decrease with the correction step is often more than 50%. The reason for the comparably large deviations for the case with the TCSC in line 23 with approach A can be explained by the fact that the slack generator is placed at bus 1 and a failure of line 1 or 2 results in large injection currents at the adjacent buses. This leads to larger quadratic terms that are neglected if no correction step is applied. The situation for the other considered cases and generally with approach B is different as lines 1 and 2 are not included in the area of influences, and the influence of a possible outage is taken into account by border buses.

For most of the cases, the incorporation of the correction term yields significantly improved results. However, especially approach B sometimes suffers from the inaccuracies introduced by the approximation of the voltage magnitudes and angles by sensitivity values at the border of the area or at the ends of the faulted line. In such cases, the correction step is not of much use. For example, the reason why the errors in the scenario with the SVC at bus 36 cannot be decreased any further applying the correction step is that for an outage of line 49 or 50, the SVC has extremely large influence on one of the buses at the end of the faulted line and the error in the approximation using sensitivities cannot be compensated by the correction step.

The general conclusion is that the current injection method yields satisfactory results even applied to a reduced area. However, it is important that after the determination of the limited area by sensitivity analysis, an additional analysis is carried out to adapt the area such that satisfying results for all considered outages are obtained.

6.2 Optimal power flow control

Incorporated into an optimal power flow problem, the current injection method is able to ensure $N-1$ security. However, before applying the optimal power flow control, the following steps should be carried out

1. Determine the critical outages $\mathcal{O}_{\text{crit}}$ for which other lines are expected to be unacceptably highly loaded.

2. For each of the outages ξ in $\mathcal{O}_{\text{crit}}$, identify the lines $\mathcal{I}_{\xi}$ which are overloaded or at risk to be overloaded.

3. Check the accuracy of the extended current injection method for the critical outages $\mathcal{O}_{\text{crit}}$ and the according lines $\mathcal{I}_{\xi}$.

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Table 3 Critical outages and lines at risk

Outage	Lines overloaded	Lines at risk
8	14, 28	13, 18, 41
60	66	–
66	59	–

4. In case of the limited area control, if necessary and possible, adapt the area for improved accuracy.

The critical outages for the test system and chosen line limits are outages of lines 8, 60 and 66. In Table 3, the lines that are overloaded or at risk to be overloaded as a consequence of these outages are listed. An outage of the line where the FACTS device is located or generally an outage of the control devices is not considered here as this leads to a non-controllable system.

A situation is considered where a TCSC is placed in line 72 and approach A for the reduction of the current injection method to a limited area is used. The resulting area for this case is given in Fig. 2. The critical outages are highlighted in the figure.

The next step is to check the accuracy of the current injection method for the reduced area. For this purpose, the TCSC is set to 70% capacitive compensation, and the values for the line currents obtained from the reduced current injection method with a correction step are compared with the exact values. The maximal absolute error over all three outages is 2.4×10^{-4} p.u., which indicates a high accuracy.

To find the solution of the optimisation problem, the solver fmincon in MATLAB is used and the results are double-checked with snopt in TOMLAB [15]. Equations (17), (18), (27) and (29) are directly implemented in the constraint file for each critical outage but only the final inequality constraints (5) are included in the constraint set $\boldsymbol{h}(\boldsymbol{x})$. The weighting parameters in (8) are set to $\nu_V = 1$, $\nu_I = 5$ and $\nu_P = 0.2$.

The set value for the TCSC obtained from the optimal power flow control is -0.1015 p.u. As the TCSC mainly

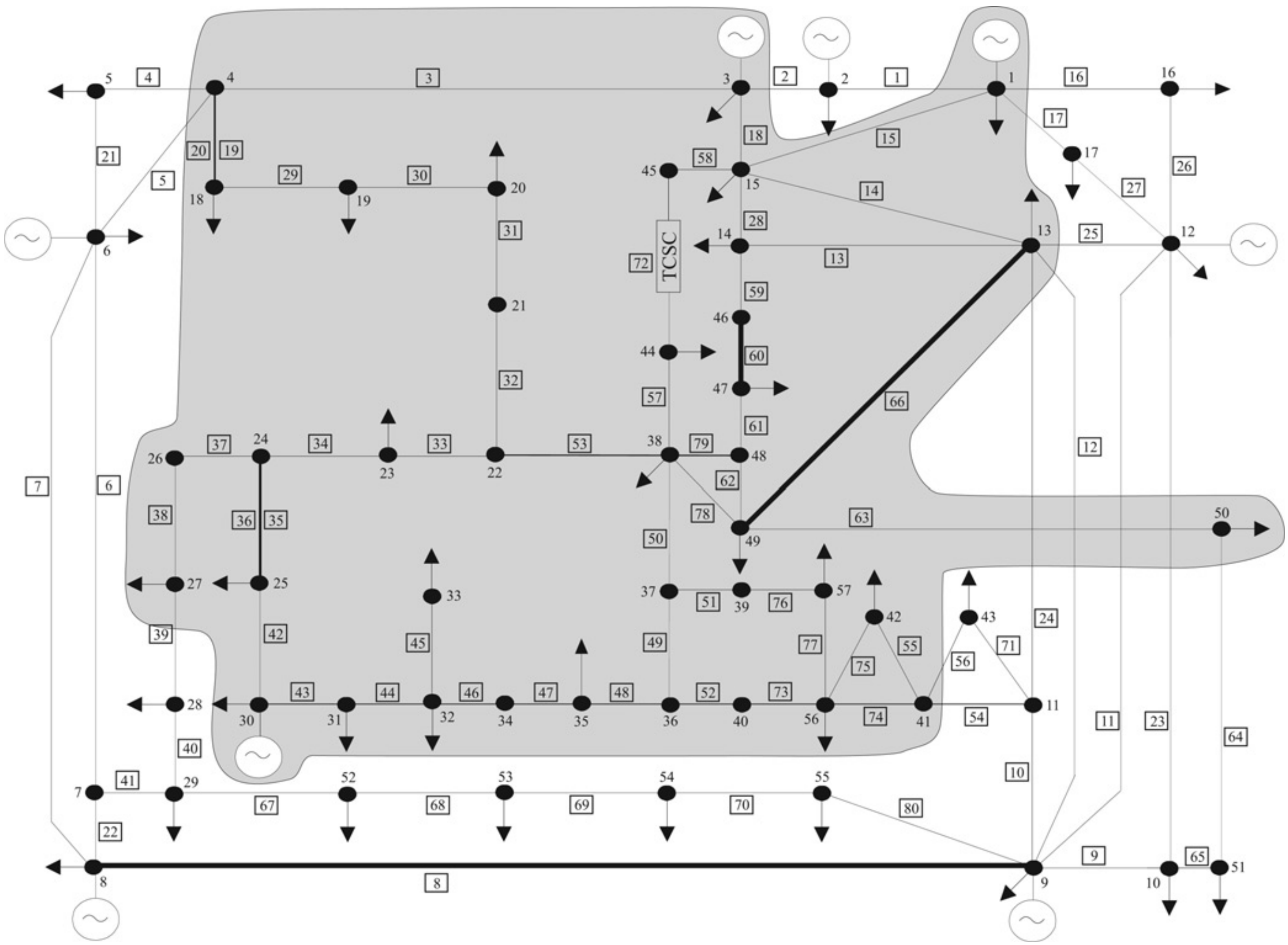

Figure 2 *Area of influence for TCSC in line 72 and critical lines (bold)*

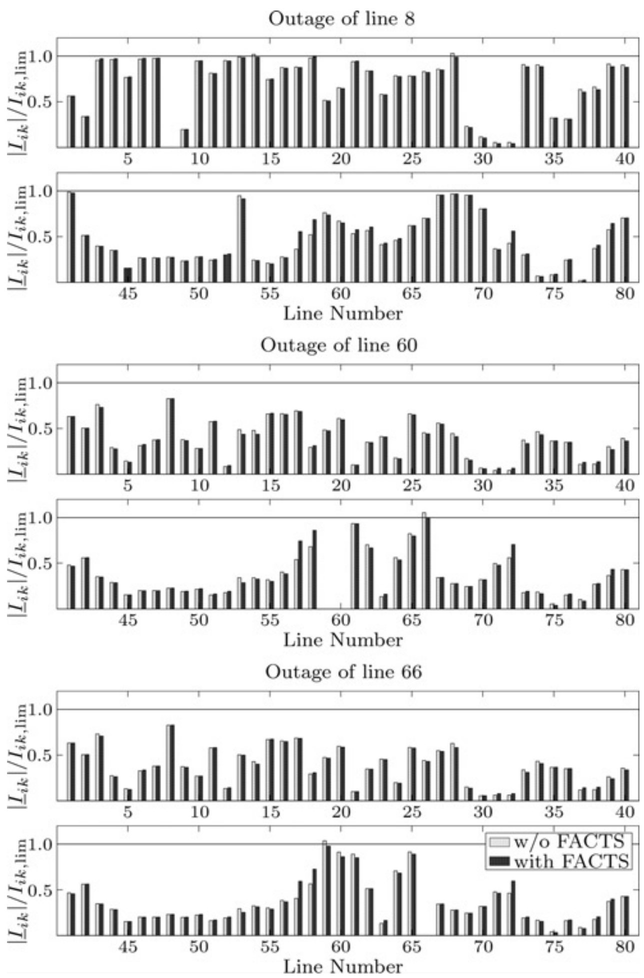

Figure 3 *Line currents in case of outages of lines 8, 60 and 66 with TCSC in line 72*

influences line flows and voltage angles, the changes in the voltage profile are negligible and the active power losses are slightly increased from 0.2554 to 0.2575 p.u. The line currents in case of the critical outages are shown in Fig. 3. Hence, the usage of the FACTS device yields an $N-1$ secure system.

7 Conclusion

In this paper, a control based on optimal power flow to ensure $N-1$ security with respect to line faults is presented. To avoid an increase of the number of decision variables, a current injection method is used to determine the system state in a faulted situation. In addition, it is taken into account that the influence of a control device is limited to the area in its vicinity by confining the optimal power flow problem to the area of influence determined by sensitivity analysis. The simulation results for the IEEE 57 bus grid show that the current injection method reduced to a limited area gives accurate results and that it can be used to improve the security of the system.

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9 References

[1] CARPENTIER J.: 'Optimal power flows', *Int. J. Electr. Power Energy Syst.*, 1979, **1**, (1), pp. 3–15

[2] UCTE: 'Operations handbook' (Website: www.ucte.org), 2004

[3] SCHNYDER G., GLAVITSCH H.: 'Integrated security control using an optimal power flow and switching concepts', *IEEE Trans. Power Syst.*, 1988, **3**, (2), p. 782

[4] BACHER R., GLAVITSCH H.: 'Network topology optimization with security constraints', *IEEE Trans. Power Syst.*, 1986, **PWRS-1**, (4), pp. 103–111

[5] GLANZMANN G., ANDERSSON G.: 'FACTS control for large power systems incorporating security aspects'. Proc. X SEPOPE, IP012, Florianopolis, Brazil, 2006

[6] HUG-GLANZMANN G., ANDERSSON G.: 'An accurate and efficient current injection method for the determination of the system state during line outages'. Proc. 16th Power Systems Computation Conf., Glasgow, Scotland, 2008

[7] CARPENTIER J., MERLIN A.: 'Optimization methods in planning and operation', *Int. J. Electr. Power Energy Syst.*, 1982, **4**, (1), pp. 11–18

[8] HINGORANI N.G., GYUGYI L.: 'Understanding FACTS concepts and technology of flexible AC transmission systems' (IEEE Press, 2000)

[9] MOHAN M.R., VARMA R.K.: 'Thyristor-based FACTS controllers for electrical transmission systems' (IEEE Press, 2002)

[10] BERGEN A.R., VITTAL V.: 'Power systems analysis' (Prentice Hall, 2000, 2nd edn.)

[11] GLANZMANN G., ANDERSSON G.: 'Incorporation of N-1 security into optimal power flow for FACTS control'. Proc. Power Systems Conf. and Exposition, Atlanta, USA, 2006, pp. 683–688

[12] KARAKATSANIS T.S., HATZIARGYRIOU N.D.: 'Probabilistic constrained load flow based on sensitivity analysis', *IEEE Trans. Power Syst.*, 1994, **9**, (4), pp. 1853–1860

doi: 10.1049/iet-gtd:20080112

[13] ELGERD O.I.: 'Control systems theory' (McGraw-Hill Kogakusha, 1967)

[14] HUG-GLANZMANN G., ANDERSSON G.: 'Coordinated control of FACTS devices in power systems for security enhancement'. Proc. of IREP Symp. – bulk power system dynamics and control, Charleston, USA, 2007

[15] TOMLAB, http://www.tomopt.com, accessed September 2007

doi: 10.1049/iet-gtd:20080112