Abstract – During energisation of power transformers, transient inrush currents with peaks up to the short-circuit current may arise. Using controlled switching taking into account the residual flux, the transients can – in theory – be eliminated completely. Real factors such as closing time scatter and residual flux measurement uncertainty have to be considered when checking the suitability of controlled switching in the field. On account of this, a systematic transformer inrush current study depending on closing time- and residual flux measurement deviations for a 400 kVA dry-type distribution transformer will be presented in this paper. To carry out the systematic study for the energisation of the first phase, an interface between MATLAB and EMTP-ATP was implemented to share the strengths of both tools. Finally, recommendations for acceptable closing time- and residual flux measurement deviations depending on the tolerable inrush current peak are given.

Keywords: Controlled switching, transformer energisation, transient inrush current, residual flux, closing time scatter, residual flux measurement uncertainty

1 Introduction

1.1 Adverse effects of transformer inrush currents

Transient transformer inrush currents can exceed the nominal current and may achieve the rated value of the short-circuit current of the power transformer. The amplitude is decaying very slowly and reaches its steady-state magnetising current after some seconds. Transient inrush currents having a high DC-component and being rich in 1st and 2nd harmonics ([1]) affect the power quality and can trip protective relays. Due to these transients, huge current forces arise in the transformer windings ([2]) that possibly reduce the lifecycle of power transformers, which are one of the most expensive components in electric power systems. Observations made at Hydro-Québec might support these statements: The failure rates of power transformers at the 735kV level are much higher than those of lower voltage levels ([3]).
1.2 Methods to reduce transient inrush currents

The phenomenon of transient transformer inrush currents was first published by Fleming in 1892 ([4]). Anyhow, up to 1988 the only method to reduce inrush currents was the installation of pre-insertion resistors. This is however not the best solution because on the one hand they must be included in the circuit breaker design and need a lot of maintenance and on the other hand they just reduce the inrush currents but do not affect the cause of the phenomenon.

In 1988 Moraw et al. introduced the first concept addressing the cause of inrush currents ([5]). With the strategy called “point-on-wave controlled switching” the transformer is energised phase by phase at the corresponding voltage peak. Assuming zero residual flux in the transformer core, the moment of energisation is optimal and no transient inrush current will arise. Even though valuable improvements in the reduction of transient inrush currents can be achieved with this quite simple algorithm, there exists one drawback: The assumption of zero residual flux is solely true, if the transformer will be de-energised under no-load and if there is no current chopping as well as the transformer has no magnetic coupling between the phases (e.g. three single-phase transformers without delta tertiary winding, five-legged and shell-type transformers).

Subsequent studies improved the concept of Moraw et al. so that the drawback could be removed. Finally a much more flexible method called “controlled switching taking into account the residual flux” was presented by Brunke and Fröhlich ([6]). Today, this is the most promising approach because it can be used in every switching case and for any core- and winding-configuration of power transformers. If the residual flux is known exactly, the transient inrush current will be eliminated completely. Despite these results, the algorithm is hardly deployed in substations ([7], using an additional voltage sensor).

In theory and with simulations the concept of Brunke and Fröhlich works perfectly and all the transients are eliminated. By contrast, real substations do not consist of ideal components and suffer real conditions such as closing time scatter, measurement deviations and signal noise whereby the performance of the strategy diminishes.

2 Relation of residual flux and inrush current

Before the transient inrush current can be analysed, the previous de-energisation must be looked at. During this process the residual flux will attend a certain value that is important for the next energisation of the power transformer.

2.1 Residual flux

Looking at a single-phase transformer and neglecting leakage and other magnetic air fluxes as well as the coil resistance, the magnetic core flux $\Phi_{\text{core}}$ is related to the coil voltage $u_{\text{coil}}$ by Equation (1).

\[
u_{\text{coil}}(t) = N_{\text{coil}} \frac{d\Phi_{\text{core}}(t)}{dt}
\]

(1)

When de-energising the transformer out of no-load steady-state, the current will be
interrupted at time $t_{\text{open}}$ and the residual flux $\Phi_{\text{Res}}$ is calculated using Equation (1):

$$\Phi_{\text{Res}} = \frac{1}{N_{\text{coil}}} \int_{0}^{t_{\text{open}}} u_{\text{coil}}(t)dt$$

(2)

With

$$u_{\text{coil}}(t) = U_0 \sin(\omega_0 t)$$

(3)

and assuming steady-state, Equation (2) becomes

$$\Phi_{\text{Res}} = -\Phi_0 \cos(\omega_0 t_{\text{open}}).$$

(4)

Because the magnetising current of transformers is often smaller than the chopping current of the circuit breaker, the current will be interrupted prior to its natural zero crossing and the opening time $t_{\text{open}}$ of Equation (4) can take any value. As a consequence of this, the residual flux can reach any value between -1 p.u. and 1 p.u.. Because no magnetising curve is able to exceed the maximum magnetising characteristic given by the properties of the core material, the residual flux margin will shrink to the range between the two points of the maximal residual flux $\pm \Phi_{\text{Res,max}}$ (Figure 1). In real substations the maximal accessible residual flux is further reduced to a value of approximately 0.9 p.u. due to transients during de-energisation (micro hysteresis loop, [8]) as can be seen in Figure 1.

Figure 1: Maximum magnetising characteristic and range of accessible residual fluxes

2.2 Formation of inrush current

If a transformer is energised at a random instant, it is possible that no transient inrush current will occur; but mostly transient inrush currents will arise. This happens because transient inrush currents depend not only on the instant of energisation, but also on the residual flux of the previous de-energisation.

Using equations (1) and (3), the magnetic flux during the first period of energisation can be calculated analytically neglecting damping effects (core losses, winding resistance):
Later the influence of damping becomes more significant and decreases $\Phi_{\text{offset}}$ towards zero. When $\Phi_{\text{offset}}$ reaches zero, the transient phenomenon has finished and the steady-state magnetising current will flow. Looking at Equation (5), it is easy to see that an energisation at the positive voltage zero crossing with a residual flux of 0.9 p.u. respectively at the negative voltage zero crossing with a residual flux of -0.9 p.u. will result in the highest value of $\Phi_{\text{offset}}$. A value of 2.9 p.u. respectively -2.9 p.u. is reached which is far above the saturation point of the maximum magnetising characteristic. Thus, the transformer core is driven into saturation and can be considered as an air-core inductance that leads to high inrush currents (Figure 2).

$$\Phi_{\text{core}}(t) = \frac{1}{N_{\text{coil}}} \int_{t_{\text{close}}}^{t} u_{\text{coil}}(t) \, dt + \Phi_{\text{Res}} = -\Phi_0 \cos(\omega_0 t) + \Phi_0 \cos(\omega_0 t_{\text{close}}) + \Phi_{\text{Res}}$$

(5)

$\Phi_{\text{Res}}$ and $\Phi_{\text{offset}}$

---

**Figure 2**: Magnetic flux $\Phi$ and inrush current $I$ ($\Phi_{\text{Res}} = 0.1$ p.u. and $t_{\text{close}} = 3$ ms)

### 3 Controlled switching taking into account the residual flux – Single phase strategy

As mentioned above, controlled switching taking into account the residual flux can be used for many different transformer configurations. There is not a single “multi purpose” strategy; there exist many strategies for different transformer configurations, measurement systems and circuit breaker configurations ([9]). Except for the simultaneous closing strategy, the method for the first phase to be energised is always the same and will be shortly discussed in this section.

No transient inrush current will appear if the optimal moment of energisation is met. Assuming steady-state and a virtually closed circuit breaker, the virtual magnetic core flux corresponds to the integral of the source voltage virtually applied to the transformer. This virtual magnetic flux is called prospective flux ($\Phi_{\text{prosp}}$). At the optimal instant of energisation $t_{\text{opt}}$, the prospective flux has to be equal to the residual flux (see Figure 3) and Equation (5) can be rewritten as:
\[
\Phi_{opt}(t_{opt}) = \Phi_0 \cos(\omega_0 t) + \Phi_{Res} = 0
\]

The optimal instant of energisation without considering prestrike of real circuit breakers can be calculated as follows:

if \( \Phi_{Res} < 0 \):
\[
t_{opt} = -\frac{1}{\omega_0} \arccos\left(\frac{\Phi_{Res}}{\Phi_0}\right)
\]

else
\[
t_{opt} = \frac{1}{\omega_0} \left[ \arccos\left(\frac{\Phi_{Res}}{\Phi_0}\right) + 1 \right]
\]

**Figure 3: Optimal closing times for single phase strategy**

4 System representation in EMTP-ATP

4.1 Overview

For the systematic study of closing time- and residual flux measurement-deviations, the simplest model possible shown in Figure 4 was implemented in ATPDraw. The transformer represented with the linear BCTRAN-model and external pseudo-nonlinear hysteretic inductors (type 96) is fed by a three-phase voltage source. The switches used to energise the transformer are independent single phase time controlled switches. In the present study only the switch of the centre phase will be closed and the others will be left open. Since the transformer is energised without any load connected to the secondary side, the load resistance has a value of \(10^{12}\) \(\Omega\).
4.2 Calculation of transformer parameters

Typical test data of Table 1 are used to configure the linear BCTRAN-model and extended test data or construction data are required for the parameterisation of the nonlinear inductors.

<table>
<thead>
<tr>
<th>Rating</th>
<th>S_N</th>
<th>400</th>
<th>kVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rated voltage</td>
<td>U_N</td>
<td>16.8/0.42</td>
<td>kV</td>
</tr>
<tr>
<td>Rated current</td>
<td>I_N</td>
<td>13.75/550</td>
<td>A</td>
</tr>
<tr>
<td>Vector group</td>
<td></td>
<td>Dyn5</td>
<td></td>
</tr>
<tr>
<td>Core type</td>
<td></td>
<td>3-legged dry-type</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No-load test data (performed at low voltage side)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
</tr>
<tr>
<td>Losses</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Short circuit test data (performed at high voltage side)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short circuit impedance</td>
</tr>
<tr>
<td>Losses</td>
</tr>
</tbody>
</table>

Table 1: Nominal and test data of transformer

In the present case construction data was provided by the manufacturer so that accurate core magnetising characteristics (also called current–flux characteristics) can be calculated. They are evaluated based on the magnetic flux density–magnetic field strength characteristic (B–H characteristic) using construction data and basic electromagnetic laws. The current I of the current–flux characteristic is computed with the formula for the magnetic field strength of a cylindrical coil:

\[ I = \frac{l_{\text{core}}}{N_{\text{core}}} H \]  

with the core length \( l_{\text{core}} \) and the number of turns per phase \( N_{\text{core}} \). Additionally, the magnetic flux is calculated using

\[ \Phi = BA_{\text{core/coil}} \]  

with the cross section \( A_{\text{core/coil}} \) flown through by the magnetic flux. The value of the cross section depends on the position of the magnetic flux density relative to the saturation point of the B–H characteristic:
if $|B| < B_{sat}$ : $A_{core/coil} = A_{core}$  
else : $A_{core/coil} = A_{coil}$  

where $A_{core}$ is the cross section of the iron core and $A_{coil}$ is the inner cross section of the energised coil. This is based on the assumption that all of the magnetic flux will flow through the iron core and none in the air if the magnetic flux density is smaller than the saturation flux density and otherwise the core acts like air, hence the magnetic flux will flow in the iron core as well as in the air bordered by the coil (Figure 5).

![Cut view of transformer leg:](image)

**Figure 5**: Cut view of transformer leg and distribution of magnetic flux density

## 5 Controller Module

With ATPDraw it is possible to carry out systematic switching studies using the systematic switch but it is not possible to change the residual flux value of the type 96 elements automatically. In addition the post-processor PlotXWin is not able to handle the results of systematic studies. On the other hand MATLAB is a very powerful tool for programming, numerical simulations as well as analysis of numerical data. Although MATLAB provides the SimPowerSystems library that is intended to model electric power systems and looks quite similar to EMTP, computing times are huge and the variety of elements is little compared to EMTP-ATP. Thus it was decided to combine the advantages of both tools: The handling of numerical data, the process control as well as the analysis of results is done by the controller module running in MATLAB and the simulation of the model will be executed in EMTP-ATP. ATPDraw is just used once to generate the netlist (ATP-file) out of the graphical representation of the network.
The controller module serves as master that controls the progress of the study as well as the simulations with EMTP-ATP (Figure 6). To systematically pass all possible combinations of residual flux and closing time—respectively residual flux measurement-deviations, two nested loops were implemented. At the beginning of the module it is checked whether all residual fluxes have already been passed. If not, the first loop is entered, a residual flux value is set according to a list and its optimal closing time is calculated using Equation (7). For the study of closing time-deviations, the closing time vector is generated. At this point all the data needed for the second loop are available. Within this loop the closing time vector is passed through systematically. If this vector is run through completely, the second loop will be interrupted. The program flow returns to the first loop and a new residual flux will be set or the script will be terminated after displaying the results. Otherwise, the algorithm is still in the second loop and therein a new simulation must be prepared. First, the actual closing time and all residual fluxes are written directly to the ATP-file. Subsequently, the ATP-solver is called via a DOS-command. The controller module waits until the solver has finished and the output file (*.pl4) has been generated. The pl4 file will then be imported to MATLAB, the absolute inrush current peak value will be determined and stored in a matrix.

6 Results

In real substations, circuit breakers always have closing time scatter and the measurement of residual flux is never ideal. Therefore, it is important to know the acceptable scatter respectively measurement deviation for an effective application of controlled switching in real substations.

For an easier analysis of the results, the deviations are examined separately. So, in Chapter 6.1 the circuit breaker is implemented like in a real substation and the residual flux measurement is ideal. Chapter 6.2 focuses on real residual flux measurement using an ideal circuit breaker.
6.1 Closing time-deviation

Because the closing time scatter varies for different circuit breakers and to date a lot of transformers are randomly energised, the closing time deviations are set within one period of the voltage wave, e.g. [-10 … 10] ms. Nevertheless this study focuses on controlled switching and thus closing time-deviations relative to the optimal moment of energisation are used.

\[ \Delta t_{\text{opt}} \]

\[ |I_{\text{peak}}| \ [A] \]

\[ \Phi_{\text{Res}} = [0.1 ... 0.9] \text{ p.u.} \]

\[ \Phi_{\text{Res}} = [0.0 ... -0.9] \text{ p.u.} \]

Figure 7: Inrush current peak depending on closing time-deviations and residual flux

The results shown in Figure 7 can be divided into two groups of curves based on the sign of the residual flux. If both closing time-deviation \( \Delta t \) and residual flux are positive, the real closing moment happens later relative to the optimal one. Because a part of the magnetic flux necessary to compensate the residual flux is missing (cyan area of Figure 8), \( \Phi_{\text{offset}} \) of Equation (5) is positive and therefore the peak current will be positive too. If closing time-deviation \( \Delta t \) is negative and \( \Phi_{\text{Res}} > 0 \), the real closing moment happens earlier relative to the optimal one. This leads to a negative \( \Phi_{\text{offset}} \) because an additional magnetic flux (magenta area of Figure 8) is added to the magnetic flux necessary to compensate the residual flux. Thus, the peak current will be negative too. For one particular residual flux, the global maximum of the absolute peak current value for positive closing time-deviations is always bigger than the one for negative ones. This can be explained looking at the lower graph of Figure 8: The magenta area corresponding to negative closing time-deviations is always smaller than the cyan one corresponding to positive closing time-deviations. The same observations can be made analogue for residual fluxes smaller or equal to zero.

The yellow box of Figure 7 marks the region for which no transient inrush current occurs. The width of this area is directly proportional to the spare flux of the core:

\[ \Delta t_{\text{CS max}} \sim \Phi_{\text{sat}} - \Phi_{\text{Res max}} \]  

(11)

Thus a circuit breaker suitable for controlled switching of this transformer must have at most a closing time-deviation of 1.15 ms if no transient inrush currents should occur. If the conditions are less stringent and the transient inrush current must not exceed the rated current, then a circuit breaker with a maximum closing time deviation of 3.87 ms can be used.
6.2 Residual flux measurement-deviation

To clarify the accuracy needed for a new measurement device, a transient inrush current study depending on residual flux measurement-deviations was carried out. The ideal closing times are calculated with Equation (7) and the residual flux of Figure 9. The residual flux in the transformer core will then be changed according to the residual flux-deviations.

The result of this study is shown in Figure 9. The curves begin respectively end at different residual flux measurement-deviations because the residual flux of the core is limited by the maximal residual flux $\Phi_{Res,max}$ of Figure 1. The width of the flat region around zero residual flux measurement-deviation is independent of the residual flux and the difference of the curves in the other regions is quite small. Theoretically, all curves should overlap; the
differences arise from the fact that the behaviour of a real iron core is slightly different to the one used for modelling with the type 96 element. The shape of the curves in Figure 9 can be explained regarding Figure 10 and recalling the maximum magnetising characteristic: The switch will always close at the corresponding moment for $\Phi_{\text{Res}}$, but the real value in the transformer core is different ($\Phi_{\text{Res,High}}$, $\Phi_{\text{Res,Low}}$). Therefore $\Phi_{\text{offset}}$ of Equation (5) will have a value different to zero and this yields to asymmetric dynamic fluxes. Neglecting damping effects, the maximal absolute value of dynamic flux $|\Phi_{\text{dyn,max}}|$ will be reached once per period and has a value of

$$|\Phi_{\text{dyn,max}}| = |\Delta \Phi_{\text{Res}}| + \Phi_0$$  \hspace{1cm} (12)

with residual flux measurement-deviation $\Delta \Phi_{\text{Res}}$ and steady-state amplitude of the magnetic flux $\Phi_0$. As a rough simplification, it can be said that transient inrush currents will only occur if the dynamic core flux exceeds the saturation flux $\Phi_{\text{sat}}$.

![Figure 10: Effect of high and low residual flux on the magnetic flux when the closing time is calculated based on the nominal value $\Phi_{\text{Res}}$](image)

The region with no transient inrush current is marked again with a yellow box in Figure 9. Half of the width of this box corresponds exactly to the spare flux of the core:

$$\Delta \Phi_{CS,\text{max}} = \Phi_{\text{sat}} - \Phi_{\text{Res,max}}$$  \hspace{1cm} (13)

As a conclusion of this, the residual flux measurement device must have a minimum accuracy of 0.29 p.u. if an ideal circuit breaker is used. If there are less stringent conditions and the transient inrush current must not exceed the rated current, then a measurement device with a minimum accuracy of 1.0 p.u. can be used. In general, the tighter the design of the core (maximal residual flux close to saturation flux) the more accurate the residual flux measurement device has to be.

7 Conclusion

This paper presented a simple EMTP-ATP model for systematic transformer inrush current studies. The parameters of the transformer model are based on typical test data and
construction data for correct core representation. For the systematic studies, a controller module was implemented in MATLAB. This module controls the workflow of the systematic studies, communicates with EMTP-ATP wherein the simulations run and is responsible for data analysis.

The effect of non-idealities such as closing time scatter and residual flux measurement deviations on transformer inrush currents were investigated for controlled switching taking into account the residual flux. Only the centre phase of the 400 kVA three-leg dry-type distribution transformer was energised and two studies were carried out to separate the effects of the two deviations.

The results of these studies show that the acceptable deviations depend directly on the limitation of the tolerable inrush current peak. If no inrush currents are allowed, the circuit breaker must have a closing time scatter less than 1.15 ms respectively a measurement uncertainty of 0.29 p.u. is admitted. The constraints can be widened by permitting larger inrush current peaks. If nominal inrush current is admitted, the tolerable closing time scatter increases to 3.87 ms respectively 1.0 p.u. of residual flux measurement uncertainty is allowed.

8 References