Abstract: Composite insulation materials may have small gaseous voids caused by the manufacture process and/or by mechanical stress. The time delay of partial discharge (PD) inception after voltage application in such small voids depends on various parameters including the gaseous volume, and it is subjected to considerable scatter. Average values for this time delay are reported in literature. However, the strong scatter implies a certain probability for the first PD, which has not been explicitly calculated so far. Therefore the probability of the inception of the first PD is calculated in this paper as a function of time and of AC (50 Hz) and DC field strength with the geometry as parameter. At first only one single cylindrical void is considered. Further the calculation is extended to a large number of statistically distributed voids. It is shown that for sufficiently high field values the kind of voltage and the considered pressure range have a minor influence on the PD inception compared to its scatter whereas the geometry of the voids is more important. The calculation allows a better estimation of the delay of PD inception in small voids when performing measurements for material investigations and other test routines.

Introduction

Fibre-reinforced insulation materials are widely used components in the construction of electrical power apparatus. The persistent demand for materials to sustain higher mechanical as well as electrical stresses led to improved fibre-reinforced materials, in which only few gaseous voids occur. Nevertheless bad wetting of the fibres, delamination between fibres and matrix or cracks due to high mechanical stress may cause longish voids in which partial discharges (PD) could develop.

Assuming that the inception of PD initiates dielectric deterioration and the prevailing field strength is high enough, the time lag until a first electron appears which develops into an avalanche is of interest.

Former investigations dealt with the average time delay of PD inception [1, 2] and few experimental data [1, 3] have been published on the increase in time lag with decreasing volume of the void. In [3] it is reported that, in particular, gaseous voids with very clean surfaces e.g. bubbles in epoxy resin, can lead to significant time lags.

Gaseous cavities in fibre-reinforced insulation materials whether produced by the manufacturing process or mechanical stress could lead to a time lag, because the surfaces of the defects could be smooth enough not to provide reservoirs of first electrons.

In contrast to previously investigated voids [1, 3], which mostly had dimensions on the millimetre scale, with fibre-reinforced materials, smaller geometrical dimensions and even a multitude of voids have to be considered.

Therefore the aim of this work is to give a better insight into the probability of the time lag of the appearance of a first PD in very narrow voids and with it the inception of deterioration. Not only on testing insulation materials but also on commissioning of power apparatus the time lag has to be considered in order to subject the test specimen to voltage stress of sufficient duration to ascertain accurate PD test results.

Theoretical basis

The occurrence of an avalanche of charge carriers depends not only on the electric field strength, but also on the availability of an initiating electron. These so-called primary electrons may be generated in various ways. In gaseous voids embedded in insulation materials the ionisation by radiation and the surface emission are the effects to be considered [1].

The field emission from metal electrodes into a vacuum or a solid dielectric can be described by the Richardson-Schottky model. However, this effect is not dominant until field strengths of about 100 kV/mm are attained [5] whereas the field strengths considered here are below 30 kV/mm. Therefore radiation is supposed to be the supplier of first electrons.

A formula for the average time lag in a cylindrical void based on the natural irradiation has already been published [2]:

\[
\tau = \frac{1}{C_{rad} \Phi_{rad} (\rho/p)_0 \rho \pi r^2 (h - l_{inc})}
\]  

(1)

Whereas \( C_{rad} \Phi_{rad} = 2 \cdot 10^6 \text{kg}^{-1}\text{s}^{-1} \) and \( (\rho/p)_0 = 1.2 \cdot 10^{-5} \text{kgm}^{-3}\text{Pa}^{-1} \) regarding an air filled void with pressure \( p \) which is in the range of 0.6 to 1 bar.
lengthened to the growth of an avalanche to the inception length \( l_{inc} \) (cp. below). Voids which are shorter than \( l_{inc} \) do not lead to PD. Therefore the active volume from which primary electrons could develop a critical avalanche is restricted to \( \pi r^2 (h - l_{inc}) \).

To get a better insight into the stochastic of the time lag the probability of the occurrence of a first PD initiated by a single electron for DC-field strengths shall be calculated based on (1). If the probability \( dP \) of the occurrence of a first electron in an interval \( [t, t+dt] \) is constant \( (Idt) \), the probability of a time lag ending in the interval \( [t, t+dt] \) is [6]:

\[
dP [t, t+dt] = e^{-\alpha t} \cdot Idt \tag{2}
\]

with the corresponding probability density function:

\[
f(t) = \frac{dP}{dt} = Ie^{-\alpha t} \tag{3}
\]

Thereby \( I \) is the inverse of the average time lag \( \bar{t} \) \((\bar{t} = 1/\alpha)\). The probability \( P[0 \leq \tau < t] \) that a first electron occurs in the interval \([0, t]\) is then:

\[
P[0 \leq \tau < t] = \int_0^t dP = I \int_0^t e^{-\alpha t} dt = 1 - e^{-\alpha t} \tag{4}
\]

The formula obtained for the probability shows a strong dependence on the volume of the cylindrical void, which is restricted by the minimal length an avalanche has to traverse to fulfill the Townsend Criterion. Subject to this criterion the inception length is a function of the effective ionisation and the feedback coefficients \( \alpha_{eff} \) and \( \gamma \):

\[
l_{inc} = \frac{1}{\alpha_{eff}} \cdot \ln \left( \frac{1}{\gamma} \right) \tag{5}
\]

\( \gamma \) is reported to be in the range of \( 10^{-4} \) [7] whereas \( \alpha_{eff} \) (cp. fig. 1) is the difference between the ionisation \( \alpha \) and the attachment coefficient \( \eta \). Here \( \alpha \) is based on a parameterisation of the original data of Rao [8] (data taken from [9]) and \( \eta \) on the data of [10]. In the following simulations the density reduced field strength is restricted to the interval \([160 Td < E/N < 2000 Td]\), where \( E \) is the field strength and \( N \) the gas density.

With (5), (1) and the correlation between \( \alpha_{eff} \) and \( E \), the probability \( P[0 \leq \tau < t] \) can be calculated according to (4).

Assuming that the electrical field along the cylindrical void is homogeneous, (4) is easily applicable for DC field strengths. With regard to sinusoidal AC voltages, the absolute value of the field strength exceeds the required field for critical avalanche development only for part of the whole period. This fact can be accommodated by suitably modifying the formula obtained above.

The production of primary electrons by irradiation can be seen as a statistically even distribution and the single event is independent from the other events. Additionally the development of an avalanche from a first electron is a very fast process compared to the change in the 50 Hz field strength. Considering this (4) is still applicable for small time steps \( dt \) [2]:

\[
P[0 \leq \tau < dt] = 1 - e^{-(\bar{t} dt)} \tag{6}
\]

For computer simulations (6) has to be discrete. The probability that no PD occurs during a half period \( T/2 \) is then:

\[
\tilde{P}[0 \leq \tau < T/2] = \prod_{i=1}^n e^{-(\bar{t}, \frac{2\pi}{T})} \tag{7}
\]

with \( n \) as the number of discretisations and \( I_i \) as the inverse of the average time lag for the corresponding field strength. Electrical fields which correspond to inception lengths longer than the voids cause \( I_i = 0 \) and thus have no influence on the value of the partial probabilities. The probability for the occurrence of
a first PD within the time \( t = m \cdot T/2 \) leads then to:

\[
P[0 \leq \tau < m \cdot T/2] = 1 - \left( \prod_{i=1}^{n} e^{-\left(i \cdot \frac{T}{2n}\right)} \right)^m \tag{8}
\]

Here \( m \) is the number of half periods and the inception conditions are the same for both polarities.

**Results and interpretation**

In the following the probability of the PD inception will be calculated depending on time and field strength with the geometry and the gas pressure as parameters.

**Difference between DC and AC**

At first the influence of DC and AC field strengths on the time lag is investigated. Fig. 2 shows the probability (contour lines for four fixed values of \( P \)) for the onset of a PD within time \( t \) and at field strength \( E \). The group of full line curves belongs to the DC field strength and the dashed and dotted lines belong to the peak value of a sinusoidal 50 Hz wave. The calculation is done for a cylindrical air-filled void, for two different pressure values, with a volume equal to that of a sphere of diameter 1 mm to facilitate comparison with former investigations [1, 3].

![Figure 2: \( P = f(t, E) \) with \( r = 0.41 \ mm, h = 1 \ mm, \ p = 0.6 \ bar, \ T = 293 \ K \), contour lines for \( P = 0.2, 0.4, 0.6 \) and 0.8](image)

With regard to the DC contour lines the following general remarks can be made:

- Decreasing the field strength to the inception field strength, where \( t \) approaches a vertical asymptote. The strong increase of the time lag with decreasing field is due to the increase of critical length which restricts the active volume available for avalanche development. A lowering of the field strength below the inception field strength obviously leads to constant \( P = 0 \).

- For \( E > 2 \cdot E_{inc} \) and a given fixed probability \( P \) the change in the time lag \( t \) is only slight since the change of the ratio of the active volume to the real crack volume is marginal.

- The contour lines for a probability of 20 and 80 % show that the PD inception may have a scatter over at least an order of magnitude, which correlates with experimental data [1, 3].

Comparing the AC contour lines with DC ones it can be stated, that for a given probability \( P \) the time lag \( t \) is higher for AC than for DC, when the peak value of the field strength of the 50 Hz field is equal to the DC value. This effect is due to the zero crossing with AC where the field strength sinks below the inception field strength of a certain void. For higher field values this exclusion time becomes relatively small compared to the period and the AC probability converges to that for DC.

Additionally the comparison of the dashed with the dotted lines in fig. 2 shows the influence of the pressure in the void. An increase of the pressure leads to a higher inception field strength as is known from the Paschen relationship. However, for high enough fields shorter time lags \( t \) are obtained for the same probabilities, because a greater absorption of the radiation is associated with the higher pressure.

Comparing the calculated probabilities for PD inception after time \( t \) with experimental data shows that the average time lag of measurements [1, 3] can be over one decade higher. However, the measurements themselves vary within a decade. The determination of the relevant parameters is difficult and their number too high to expect an accurate result of the simulation. In contrast to the differences found in the measurements the simulation seem to indicate that the influence of the pressure and the kind of the applied voltage is small when the field strength is high enough \((E > 2 \cdot E_{inc})\). Nevertheless it can be seen that a void with dimensions in the order of a millimetre can lead to time lags in the range of a few minutes.
Influence of geometry

In visual assessment of mechanically strongly stressed fibre-reinforced insulation materials, cracks with diameters of some micrometres and lengths of some 100 µm were observed (compare also [11]).

To investigate the likelihood of the occurrence of a first PD after time \( t \) in such thin cracks, the probability is calculated as a function of the height \( h \) of the void for two different radii (5 µm, 410 µm). Here the peak value of the 50 Hz field strength is held constant at twice the inception field strength of each void geometry, whereas the pressure was assumed to be constant at 0.6 bar.

![Figure 3](image_url)

**Figure 3:** \( P = f(t, h) \) with \( E_p = 2 \cdot E_{inc}, r_1 = 5 \, \mu m, r_2 = 0.41 \, mm, p = 0.6 \, bar, T = 293 \, K \), contour lines for \( P = 0.2, 0.4, 0.6 \) and 0.8

Considering lines of constant PD probability (cp. fig. 3), a reduction of cavity height by an order of magnitude leads to an increase of the time lag of the same order. A variation of the cross section has the same effect as the primary electron generation is proportional to the volume.

Fig. 3 reveals that a very thin crack leads to only high probabilities at very long times \( t \). It is to be mentioned that the stated model does not regard the diffusion of the charge carriers in an avalanche and their interaction with the walls of the void. Such effects could restrict the development of the avalanche and would therefore lead to even greater time lags. On the other hand field inhomogeneity to be expected in the real case in longish gaseous voids could bring about an enhanced avalanche development.

Therefore the calculated probabilities of the time lag of thin cracks shall give us more a feeling for the time lags to be expected and their scatter rather than their absolute values.

Influence of a multiplicity of voids

Normally such capillary cracks can be numerous in mechanically stressed fibre-reinforced materials. The probability of PD inception shall therefore be calculated for a distribution of cracks.

Based on the findings with the polyester fabric reinforced epoxy tubes previously investigated [11] which had been mechanically stressed up to a relative elongation of 8 %, a crack density of 5 cracks/mm\(^3\) is assumed. This would lead to about 30'000 cracks in the sample. Furthermore a stochastic exponential distribution of these crack lengths with an average crack length of 100 µm and classification in 100 bins are taken into account, whereas cracks shorter than 30 µm and longer than a millimetre were ignored. The average radius \( \bar{r} \) was set to 5 µm for all these cylindrical voids.

In fig. 4 the probability of a PD inception in any one of these cracks is depicted as a function of the time \( t \) to inception and of the peak value of a sinusoidal 50 Hz field (calculation based on (8)). The pressure for each crack is assumed to be 0.6 bar.

![Figure 4](image_url)

**Figure 4:** \( P = f(t, E_p), h \) exponentially distributed, \( \bar{r} = 5 \, \mu m, p = 0.6 \, bar, T = 293 \, K \), contour lines for \( P = 0.2, 0.4, 0.6 \) and 0.8

Observing fig. 4 the probability of a distribution of voids shows the same basic shape as the one for a single cavity (cp. fig. 2). However, here the field strength plays an even more important role,
since with its increase more and more cracks fulfil the Townsend Criterion and this leads to a strong decrease of the time lag. The overall inception field strength lies at low values because of the favourable inception conditions with long cracks which nevertheless could be associated with high time lags due to their small volume.

Compared to the values previously measured with fibre-reinforced materials [11] where the time lag seems to be of the order of some minutes, the values obtained here lie higher. For example, for 150 kV/cm a probability of about 0.8 could be reached within 3 hours approximately. The reason for the difference may be sought in the simple cylindrical geometry assumed for the doubtless more complicated cracks.

Conclusions

The probabilities of the occurrence of a first PD in a small cylindrical void or even a population of voids subjected to high enough AC and DC fields were calculated.

Although pressure, gas mixture, geometry of the void and local field strengths are often difficult to determine, insight into their influence on the time lag and its scatter is given.

With the voids considered here and sufficiently high field values \(E > 2 \cdot E_{inc}\) the pressure and the kind of voltage have a minor influence on the time lag compared to the inherent scatter. The calculated spread of the probability due to the scatter of the time lag is similar to that found with measurements [1, 3].

Parameters such as radius and field strength (resp. \(l_{inc}\)) which affect the active volume have a major influence on the time lag and smaller active voids lead to greater time lags. However, the influence of the walls on the probability in connection with the diffusion of the avalanche has not been considered here.

A population of narrow cracks causes a smaller time lag than a single one.

For high voltage equipment, where small voids have to be expected, tests of sufficient duration of a high enough voltage level have to be conducted in order not to miss PD inception. However, one has to be aware that for field values higher than \(E_{inc}\) PD inception and thus the beginning of deterioration can take place at any time. Thus materials sensitive to PD with high requirements with respect to lifetime must not be stressed above the inception field.

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