Load Frequency Control by Aggregations of Thermally Stratified Electric Water Heaters

Evangelos Vrettos, Student Member, IEEE, Stephan Koch, Student Member, IEEE, and Göran Andersson, Fellow, IEEE

Abstract—In this paper, we present a dynamic model of an electric water heater which describes the thermal stratification inside the water tank. The goal of this modeling is to accurately describe the power consumption behavior of a large water heater population and to assess customer comfort loss (lack of hot water) in the presence of external control actions. We present four rule-based control approaches for aggregate power setpoint tracking and compare them by means of time-domain simulations and numerical comparisons. As a control signal, we use a scaled load frequency control (LFC) time series added to a time-varying baseline of the aggregate water heater load.

Index Terms—Demand Response, Demand Side Management, Direct Load Control, Electric Water Heater, Load Frequency Control, Thermal Stratification.

I. INTRODUCTION

A. Motivation and Related Work

The increasing penetration of fluctuating Renewable Energy Sources (RES) in the grid calls for exploitation of the flexibility at the demand side. This approach is commonly referred to as Demand Side Management (DSM) or Demand Response (DR). Due to their inherent thermal energy storage, Thermostatically Controlled Loads (TCLs) are excellent candidates for power system control applications.

In [1], a review of peak shaving and load shifting applications with Electric Water Heaters (EWHs) is provided. In [2], the authors investigate the use of household electricity demand as primary frequency control reserve. References [3], [4], [5], [6] and [7] investigate the ability of TCLs to track a reference power profile using different modeling and control approaches. In all of them, generic models are used for thermal appliances, instead of device-specific ones. In addition, stochastic user interactions with the loads are neglected or simply modeled by a normally distributed noise process.

Most of the papers focusing on reserve power provision by EWHs, e.g. [8] and [9], use simplified “single point” models, which assume uniform temperature within the water tank. Clearly, such models do not capture the thermal stratification phenomena. In [10], a simplified stratified model is used, where only an upper hot water layer and a lower cold water layer are considered. In [11], the authors develop a six-layer model to simulate EWHs subject to off-peak schedules that, however, ignores convection.

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The authors are with the EEH-Power Systems Laboratory, Swiss Federal Institute of Technology (ETH) Zurich, 8092 Zurich, Switzerland (e-mail: vrettos@eeh.ee.ethz.ch; koch@eeh.ee.ethz.ch; andersson@eeh.ee.ethz.ch).

B. Contribution of this Paper

To the best of authors’ knowledge this is the first time that a detailed thermally stratified model is explicitly developed for use in Load Frequency Control (LFC) schemes with populations of EWHs. The model is based on the heat transfer mechanisms inside the storage tank and it demonstrates a good tradeoff between accuracy and complexity. Other contributions of the paper are the definition of State of Charge (SOC) for EWHs as well as the development and evaluation of several load control strategies of various sophistication levels.

The remainder of the paper is organized as follows: in Section II, the developed linear time-varying EWH model for feedback control is presented. In Section III, a model for populations of EWHs is presented and in Section IV the SOC concept is introduced. A number of control strategies for aggregations of EWHs is proposed in Section V and evaluated based on different indicators in Section VI. Last, Section VII summarizes the conclusions of this work.

II. ELECTRIC WATER HEATER MODELING

A. Underlying Partial Differential Equation

The one-dimensional Partial Differential Equation (PDE) governing the laminar heat flow in the storage tank is given by [12]:

\[
\frac{\partial T}{\partial t} + V \frac{\partial T}{\partial x} = \alpha \varepsilon_{\text{eff}} \frac{\partial^2 T}{\partial x^2} - k(T - T_a) + Q(x,t) \quad , \quad (1)
\]

where \(x\) denotes the position along the vertical axis of the tank, \(t\) denotes time, \(V\) is the vertical water velocity in the tank during water draws, \(\alpha\) is the thermal diffusivity, \(k\) is the heat loss coefficient, \(T_a\) is the ambient temperature, \(Q(x,t)\) corresponds to the internal heat generation and \(\varepsilon_{\text{eff}}\) accounts for turbulent mixing at the tank inlet (\(\varepsilon_{\text{eff}} \approx 1\) for laminar flow and \(\varepsilon_{\text{eff}} \gg 1\) for turbulent flow). Including \(\varepsilon_{\text{eff}}\) in (1) models the initial cooling of the whole tank right after a water draw occurs. Note that \(\varepsilon_{\text{eff}}\) is not included in (1) if the simulation time step is greater than a few seconds due to numerical stability issues [13].

B. Numerical Solution Scheme

Equation (1) is a standard parabolic PDE with an additional convective term. To eliminate numerical diffusion and inaccuracy that are inherent in first-order upwind differencing schemes, the second-order, three level finite difference Crank-Nicolson scheme proposed in [14] is used to numerically solve (1). The mesh in time is denoted by \(\Delta t\), the mesh in space by \(\Delta x\), the number of grid points in \(x\) direction by \(n\) and the
temperature of \( i \)th layer at time \( m \) by \( T_i^m \). Equation (1) can be written in the more general form:

\[
\frac{\partial T}{\partial t} = a \varepsilon_{\text{eff}} \frac{\partial^2 T}{\partial x^2} + f(T, \frac{\partial T}{\partial x}) , \tag{2}
\]

\[
f(T, \frac{\partial T}{\partial x}) = -V \frac{\partial T}{\partial x} - k(T - T_a) + Q(x, t) . \tag{3}
\]

Now, the Crank-Nicolson scheme can be applied to (2), which is defined by the following substitutions:

\[
\frac{\partial T}{\partial t} \rightarrow \frac{T_i^{m+1} - T_i^{m-1}}{2\Delta t} , \tag{4}
\]

\[
\frac{\partial^2 T}{\partial x^2} \rightarrow \frac{1}{2} \left( \frac{T_i^{m+1} - 2T_i^m + T_i^{m-1}}{\Delta x^2} + \frac{T_{i-1}^{m-1} - 2T_i^{m-1} + T_{i+1}^{m-1}}{\Delta x^2} \right) , \tag{5}
\]

\[
f(T, \frac{\partial T}{\partial x}) \rightarrow f \left( \frac{T_i^m + T_i^{m-1}}{2}, D \left( \frac{T_i^m + T_i^{m-1}}{2} \right) \right) , \tag{6}
\]

\[
D \left( \frac{T_i^m + T_i^{m-1}}{2} \right) = \left[ \frac{T_{i+1}^{m+1} + T_{i-1}^{m+1}}{2} - \frac{T_{i+1}^{m-1} + T_{i-1}^{m-1}}{2} \right] . \tag{7}
\]

Applying this scheme results in the following expressions for all the tank layers apart from the two boundaries:

\[

- q_1 T_{i-1}^m + q_2 T_i^m + q_3 T_{i+1}^m = q_1 T_{i-1}^{m-1} + q_4 T_i^{m-1} + k T_{i+1} + Q(x, t) , \tag{8}

\]

where: \( q_1 = \frac{e}{2} + d, \quad q_2 = \frac{1}{2} \Delta t + e + \frac{k}{2} \quad q_3 = d - \frac{e}{2} \), \quad q_4 = \frac{1}{2} \Delta t - e - \frac{k}{2} \quad d = \frac{V}{4 \Delta x} \quad e = \frac{a \varepsilon_{\text{eff}}}{\Delta x^2} . \tag{9}

To solve (1) numerically, two boundary conditions are needed, namely Ordinary Differential Equations (ODEs) for the top and the bottom layer. Note that two artificial layers are considered to represent the incoming water temperature \( T_1 = T_{cw} \) and the ambient temperature \( T_{n+2} = T_a \). This is done to facilitate the problem formulation in matrix form. Therefore, the temperature of the bottom layer is denoted by \( T_2 \) and the temperature of the top layer by \( T_{n+1} \):

\[
\frac{d T_2}{d t} = -\frac{a \varepsilon_{\text{eff}}}{\Delta x^2} (T_2 - T_3) + \frac{V}{\Delta x} (T_{cw} - T_2) - k' (T_2 - T_3) , \tag{10}
\]

\[
\frac{d T_{n+1}}{d t} = \frac{a \varepsilon_{\text{eff}}}{\Delta x^2} (T_n - T_{n+1}) + \frac{V}{\Delta x} (T_n - T_{n+1}) - k' (T_{n+1} - T_n) . \tag{11}
\]

By applying an explicit scheme on these ODEs we obtain:

\[
\frac{1}{\Delta t} T_2^m - 4d T_{cw} - k' T_2 = q_5 T_3^m + e T_2^{m-1} , \tag{12}
\]

\[
\frac{1}{\Delta t} T_{n+1}^m - k' T_n = q_5 T_{n+1}^{m-1} + q_6 T_{n-1}^{m-1} , \tag{13}
\]

where: \( q_5 = \frac{1}{\Delta t} - e - 4d - k' \quad q_6 = e + 4d . \tag{14}

In (10)–(14), \( k' \) is the heat loss coefficient of the top and bottom layers (\( k' > k \) due to larger heat loss area). Equations (8),(12),(13) can be expressed in matrix form:

\[
Z_1 T_{m+1} = Z_2 T_m + Z_3 u_m , \tag{15}
\]

where \( T_m \in \mathbb{R}^{n^2} \) is the temperature vector, \( u_m \in \{0,1\} \) is the binary variable of heating operation, \( m \) denotes time and:

\[
Z_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & -k' \ 0 & -q_1 & q_2 & 0 & \cdots & 0 & 0 & 0 & 0 & k & 0 & \cdots & 0 & 0 & 0 & 0 \ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \ 0 & 0 & 0 & 0 & \cdots & -q_1 & q_2 & q_3 & -k \ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1/\Delta t & -k' \ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \ \end{pmatrix} , \tag{16}
\]

\[
Z_2 = \begin{pmatrix} 0 & \cdots & 0 & \rho_{el} & \cdots & 0 & \cdots & 0 \ \end{pmatrix} , \tag{17}
\]

\[
Z_3 = \begin{pmatrix} 0 & \cdots & 0 & \rho_{el} & \cdots & 0 & \cdots & 0 \ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 \ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \ 0 & 0 & 0 & 0 & \cdots & q_1 & q_2 & -q_3 \ 0 & 0 & 0 & 0 & \cdots & 0 & q_4 & -q_3 \ 0 & 0 & 0 & 0 & \cdots & 0 & q_4 & 0 \ 0 & 0 & 0 & 0 & \cdots & 0 & q_4 & 0 \ \end{pmatrix} , \tag{18}
\]

where \( P_{el} \) is the nominal electrical power of the heating element, \( \eta \) is the electrical efficiency, \( m_i \) is the water mass of the \( i \)th layer and \( c \) is the specific heat capacity of water. Equation (18) signifies that the heating zone includes only one layer, however, larger zones can also be considered.

It is easy to show that matrix \( Z_1 \) is nonsingular by construction. Thus, (15) can be rewritten in the more elegant form:

\[
T_{m+1} = A_m T_m + B_m u_m , \tag{19}
\]

where:

\[
A_m = Z_1^{-1} Z_2 \quad \text{and} \quad B_m = Z_1^{-1} Z_3 . \tag{20}
\]

C. Modeling Natural Convection

Natural convection is the combined effect of two opposite forces: buoyancy that accelerates warmer water layers and viscous friction that opposes the fluid motion. To account for natural convection, the density of each water layer \( (\rho_i) \) is calculated and buoyant forces are obtained according to:

\[
F_i^b = \min\left[ 0, \frac{g m_i \rho_{\text{ref}} - \rho_i}{\rho_{\text{ref}} + \rho_i} \right] , \tag{21}
\]

where \( \rho_{\text{ref}} \) is the minimum between \( \rho_{i+1} \) and the average density of the water layers above layer \( i \). Using the natural convection velocities for each layer from the previous time step \( (U_i) \), the viscous forces can be calculated according to:

\[
F_i^v = -b U_i . \tag{22}
\]

Parameter \( b \) depends both on water viscosity and tank geometry. With both forces known, application of Newton’s law of motion gives the displacement \( \Delta t \) of each water layer, which determines the degree of mixing with upper layers:

\[
T_j = r T_{j+1} + (1 - r) T_j , \quad \text{for} \quad j \in [i, i + \Delta i] , \tag{23}
\]

where \( r \in [0,1] \) is the mixing ratio. Both \( b \) and \( r \) can be identified from experimental results. Modeling buoyancy in such a way captures the initial cooling of the upper part of the tank, while the lower part is heated up, which is observed in the experimental results (see Fig. 1). Equations (21), (22) and (23) are applied after (1) is numerically solved at each time step.
D. Model Tuning and Experimental Validation

The model has been tuned and validated using ten temperature measurements from a cylindrical tank from the Lawrence Berkeley National Laboratory (LBNL) [15]. The tank was subject to six water draws of 11.4 l/min, lasting for 3.5 minutes each, which occurred at the beginning of each hour. Six of the thermometers were placed in the center of six zones with equal volume within the tank. The position of the other four thermometers could not be exactly specified, however, a preliminary analysis suggested that they were clustered around the lowest thermometer.

All the parameters used in the EWH model are shown in Table I. Fig. 1 shows the temperature distribution inside the water tank, the temperature evolution at the top of the tank and around the heating element, as well as the Root Mean Squared Error (RMSE) and the Mean Average Percentage Error (MAPE) to assess model accuracy. With the exception of the lowest water layer, the temperature errors are relatively small, particularly for upper tank layers (less than 3%). The increased model mismatch at the bottom of the tank is due to the uncertainty related with the lowest thermometers’ position. In addition, the heater operating intervals predicted by the model closely match the experimental results.

III. WATER HEATER POPULATION MODEL

In this section, we will develop a representation of a population of EWH models by assembling statistical distributions for the governing parameters. The main purpose of this model is the investigation of the aggregate power consumption of a large EWH population with and without external control actions from a central coordination entity.

A. Modeling of Population Parameters

In the following, we describe a routine that creates a heterogeneous set of parameters, such as tank volume, $P_{el}$ and daily water consumption, for any desired number of water heaters. We consider a certain amount of correlation between these parameters, which means e.g. that a small EWH is more likely to have a small heating element and a low daily water draw volume. The methodology includes the following steps:

1) First, the EWH category vector $m_{cat}$, containing the possible tank sizes, is defined. The individual EWH sizes are drawn from $m_{cat}$ with a probability according to a predefined vector of percentages $p_{cat}$.

2) A matrix $P_{el,cat}^{rated}$ is defined, the column $j$ of which contains possible power ratings of EWHs of category $j$. The rated power $P_{el,i}^{rated}$ of EWH $i$ of category $j$ is drawn from column $j$ of $P_{el,cat}^{rated}$ based on a uniform discrete distribution.

3) For each EWH category, an interval $[m_{\text{min, daily, cat}}, m_{\text{max, daily, cat}}]$ for the total water consumption per day is defined. The actual daily draw volume $m_{\text{daily}}$ of EWH $i$ is determined by a uniform distribution between these bounds.

4) The center and width of the thermostat’s deadband of EWH $i$, $T_{set,i}$ and $T_{dead,i}$ respectively, are drawn from uniform distributions in the intervals $[T_{set}^{\text{min}}, T_{set}^{\text{max}}]$ and $[T_{dead}^{\text{min}}, T_{dead}^{\text{max}}]$.

5) The thermal loss coefficient $U$ is varied (based on a uniform distribution) in the interval $[U_{\text{min}}, U_{\text{max}}]$.

Table II shows an exemplary parametrization for creating the water heater population.

B. Modeling of Water Draws

In order to model random water draw events induced by customer behavior, we propose to generate a draw scenario for a single water heater by taking random values for draw starting time, draw duration, and flow rate from predefined probability distributions. In what follows, we describe how

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**Table I.** EWH Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>$V_{T}$</td>
<td>190 l</td>
</tr>
<tr>
<td>Height</td>
<td>$H_{T}$</td>
<td>1.19 m</td>
</tr>
<tr>
<td>Power</td>
<td>$P_{el}$</td>
<td>4.1 kW</td>
</tr>
<tr>
<td>Efficiency</td>
<td>$\eta$</td>
<td>95%</td>
</tr>
<tr>
<td>Heater position</td>
<td>$H_{el}$</td>
<td>0.24 m</td>
</tr>
<tr>
<td>Thermodynamic Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water diffusion</td>
<td>$\alpha$</td>
<td>$0.1434 \times 10^{-6}$ m²/s</td>
</tr>
<tr>
<td>Water specific heat capacity</td>
<td>$c_{w}$</td>
<td>$4186.5$ J/(kg K)</td>
</tr>
<tr>
<td>Tank heat loss coeff. 1</td>
<td>$k_1$</td>
<td>$6.3588 \times 10^{-7}$ l/s</td>
</tr>
<tr>
<td>Tank heat loss coeff. 2</td>
<td>$k_2$</td>
<td>$1.2382 \times 10^{-15}$ l/s</td>
</tr>
<tr>
<td>Simulation Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mesh in time</td>
<td>$\Delta t$</td>
<td>10 s</td>
</tr>
<tr>
<td>Mesh in space</td>
<td>$\Delta x$</td>
<td>0.119 m</td>
</tr>
<tr>
<td>Number of layers</td>
<td>$n$</td>
<td>10</td>
</tr>
<tr>
<td>Inlet water temperature</td>
<td>$T_{in}$</td>
<td>14.3°C</td>
</tr>
<tr>
<td>Ambient temperature</td>
<td>$T_{a}$</td>
<td>19.7°C</td>
</tr>
</tbody>
</table>

**Table II.** Parameters Used for Creating the Water Heater Population

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{cat}$</td>
<td>Water mass categories</td>
<td>[190, 250, 300, 400]</td>
<td>(kg)</td>
</tr>
<tr>
<td>$P_{cat}$</td>
<td>Category shares</td>
<td>[10%, 25%, 25%, 40%]</td>
<td>[-]</td>
</tr>
<tr>
<td>$P_{el,cat}^{rated}$</td>
<td>Rated power per cat.</td>
<td>[2, 3, 4, 4]</td>
<td>[kW]</td>
</tr>
<tr>
<td></td>
<td>Daily draw bounds per cat.</td>
<td>[2.5, 3.5, 4.5, 5]</td>
<td>[kW]</td>
</tr>
<tr>
<td></td>
<td>Temp. setpoint bounds</td>
<td>[3, 4, 5, 6]</td>
<td>[K]</td>
</tr>
<tr>
<td></td>
<td>Deadband size bounds</td>
<td>[55, 65]</td>
<td>[°C]</td>
</tr>
<tr>
<td></td>
<td>Heat loss coefficient bounds</td>
<td>[0.2, 1]</td>
<td>[\frac{W}{K}]</td>
</tr>
<tr>
<td>$T_{0}$</td>
<td>Initial temperature vector</td>
<td>[10, 60, \ldots, 60, 20]^T</td>
<td>[°C]</td>
</tr>
</tbody>
</table>
we assemble the probability distributions and demonstrate a sample draw event time series.

1) For determining the probability of a draw event, we utilize a profile from [16] depicted in Fig. 2 and further denoted by \( p_{\text{draw}} \). Furthermore, we determine the total number of water draws per day by drawing from a uniform distribution:

\[
 n_{\text{daily}} \sim U(n_{\text{min}}^{\text{daily}}, n_{\text{max}}^{\text{daily}}),
\]

(24)

where \( U \) denotes the uniform distribution between a minimum and a maximum value. We multiply the hourly draw probability with the total number of draws of the day to determine the hourly average water draws:

\[
 n_{\text{hourly}} = p_{\text{draw}} \cdot n_{\text{daily}}. \tag{25}
\]

2) We determine the vector of time intervals between water draws in the course of the day by:

\[
 \Delta t_{\text{draw}} \sim EXP\left(\frac{1}{n_{\text{hourly}}}\right), \tag{26}
\]

where \( EXP \) is the exponential distribution with the expected value as a parameter. By calculating a cumulative sum of \( \Delta t_{\text{draw}} \), we assemble the vector of time instants \( t_{\text{draw}} \) when a draw takes place.

3) We distinguish between two types of water draws: long draws that correspond to e.g. a shower or bath, and shorter draws related to activities such as washing hands or cooking. We assume that a water draw is more likely to be a long one during times of the day when a lot of water is used. The draw duration times, \( T_{\text{long}} \) and \( T_{\text{short}} \), are random variables drawn from normal distributions. Then, the draw duration times are assembled in the vector \( \Delta t_{\text{duration}} \).

4) In order to induce a variation of flow rates, we draw a normalized flow rate from a normal distribution for every draw event:

\[
 m_{\text{draw}}^{\text{norm}} \sim N(1, \sigma_{m_{\text{draw}}}^\text{norm}). \tag{27}
\]

5) The overall resulting normalized mass flow time series \( m_{\text{norm}}^{\text{draw}} \) is assembled with the information contained in the vectors \( t_{\text{draw}}, m_{\text{draw}}^{\text{norm}}, \) and \( \Delta t_{\text{duration}} \). The draw time series is then scaled so that the total daily consumption equals \( n_{\text{daily}} \). An exemplary water draw scenario is shown in Fig. 3.

**TABLE III**

**ASSUMED PROBABILISTIC PARAMETERS FOR WATER DRAW SCENARIOS**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Distr.</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{\text{daily}} )</td>
<td>Daily water usage</td>
<td>normal</td>
<td>( \mu = 200 \text{l}, \sigma = 20 \text{l} )</td>
</tr>
<tr>
<td>( n_{\text{daily}} )</td>
<td>Number of draws per day</td>
<td>uniform</td>
<td>( n_{\text{min}}^{\text{daily}} = 30, n_{\text{max}}^{\text{daily}} = 50 )</td>
</tr>
<tr>
<td>( T_{\text{long}} )</td>
<td>Long water draw duration</td>
<td>normal</td>
<td>( \mu = 10 \text{min}, \sigma = 1 \text{min} )</td>
</tr>
<tr>
<td>( T_{\text{short}} )</td>
<td>Short water draw duration</td>
<td>normal</td>
<td>( \mu = 1 \text{min}, \sigma = 6 \text{sec} )</td>
</tr>
<tr>
<td>( m_{\text{draw}} )</td>
<td>Hot water flow rate</td>
<td>normal</td>
<td>( \mu = 1 \text{l/min}, \sigma = 0.2 \text{l/min} )</td>
</tr>
</tbody>
</table>

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**IV. SOC DEFINITIONS**

TCLs save energy in their thermal mass, therefore, the concept of SOC, which is typically used for conventional storage devices such as batteries, can also be defined for TCLs. In this section, three different SOC formulations for EWHs will be proposed for use in LFC schemes.

A. SOC based on User Perception or Heating Element Sensor

If the water temperature perceived by the user during a draw \( T_{\text{draw}(n+1)} \) or the temperature around the heating element \( T_{\text{el}} \) is used, then SOC can be defined as:

\[
SOC = \frac{T_{\text{el}} - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}}, \tag{28}
\]

where \( T_{\text{el}} \) is either \( T_{\text{draw}(n+1)} \) or \( T_{\text{el}} \), \( T_{\text{max}} \) and \( T_{\text{min}} \) are the upper and lower temperature setpoints respectively. Note that if \( T_{\text{el}} \) is used in (28), SOC boils down to the relative thermal energy concept introduced in [3] and used in [17].

B. SOC based on Temperature Distribution

In this SOC formulation, the whole temperature distribution inside the water tank is used to calculate its energy content. The reference energy content, \( E_{\text{ref}} \), is defined as the thermal energy stored when the temperature of all EWH layers is equal to the upper setpoint \( T_{\text{max}} \). Using the lower temperature setpoint \( T_{\text{min}} \) as a reference temperature, \( E_{\text{ref}} \) and SOC are given by:

\[
E_{\text{ref}} = n m_{\text{e}} c (T_{\text{max}} - T_{\text{min}}), \tag{29}
\]

\[
SOC = \frac{\sum_{i=2}^{n+1} w_i m_{\text{e}} c \max([T_i - T_{\text{min}}], 0)}{E_{\text{ref}} \sum_{i=2}^{n+1} w_i}, \tag{30}
\]

Note that water layers with \( T_i < T_{\text{max}} \) have zero thermal energy content and do not contribute to the total SOC. Weighting factors are used in summation of (30) to reflect the dependency of the value of thermal energy on layer position along the tank vertical axis. In the simpler case, this dependency can be neglected and all \( w_i \) can be set equal to 1. Not surprisingly, the lowest SOC is 0, however, (30) allows SOC > 1, which corresponds to average water temperatures greater than \( T_{\text{max}} \).

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**V. CONTROL STRATEGIES**

The models and definitions introduced in Sections II, III and IV were used to develop a number of different strategies for controlling the aggregate power demand of a population of EWHs. The control algorithms differ in two main aspects: the degree of information feedback from an individual device to the central coordination unit and the degree to which the central controller can interact with the device’s internal duty cycle.


**A. Stochastic Blocking Control (C1)**

In this strategy, only the desired setpoint \(P_{\text{set}}\), the measured aggregate load \(P_{\text{agg}}\) and the total maximum power demand of the population \(P_{\text{max}}\) are available to the central controller. The controller can only block, or release from blocking individual EWHs.

This approach leans on the work presented in [17], [18] and is based on the control parameter \(\gamma\), which is broadcasted to all EWHs and indicates the fraction of appliances that shall be blocked (or released from blocking) in each time step:

\[
\gamma = \frac{P_{\text{agg}} - P_{\text{set}}}{P_{\text{max}}}
\]  

(31)

If \(0 < \gamma \leq 1\) blocking action is required, whereas if \(-1 \leq \gamma < 0\) EWHs that are blocked should be released. The decision about blocking or releasing devices is made by generating a uniformly distributed random number \(n_i\) for each device \(i\), where \(0 \leq n_i \leq 1\). At time \(t\), a binary block signal \(b_i^t\) is assigned to each EWH according to the rule:

\[
b_i^t = \begin{cases} 
1 & \text{if } b_i^{t-1} = 0 \land \gamma > 0 \land n_i \leq \gamma \\
0 & \text{if } b_i^{t-1} = 1 \land \gamma < 0 \land -n_i \geq \gamma \\
b_i^{t-1} & \text{otherwise}
\end{cases}
\]  

(32)

The uniform distribution of \(n_i\) ensures that the percentage of EWHs that change their block signal \(b_i^t\) at time step \(t\) is close to \(|\gamma|\). If the block signal for a particular heater is set to 1, it overrides a potential ON signal from the internal controller. If, on the other hand, the block signal is 0, then the ON/OFF state of the heating element is operated entirely according to the internal controller.

**B. Direct Temperature Feedback Control (C2)**

Here, the controller receives information about the ON/OFF state and the temperature of each EWH through individual communication links and it also knows \(T_{\text{min}}\) and \(T_{\text{max}}\). Unlike approach C1, here the controller can actively switch both ON and OFF individual devices. This strategy includes two parts: the selection of the subset of EWHs that is in principle available for external control and the ranking of this subset to specify the EWHs that will actually provide the reserve. The algorithm is outlined in the following.

1) **Selection**: In this step, the external controller divides the EWHs into subpopulations based on their temperature and ON/OFF state. In (33)-(36), \(N_{\text{OFF}}\) denotes the set of EWHs that are currently OFF, \(N_{\text{ON}}\) the set of EWHs that are currently ON, \(N_{\text{too cold}}\) and \(N_{\text{too hot}}\) are defined in Table IV for all investigated variations of strategy C2 and \(N_{\text{in deadband}} = \gamma(N_{\text{too cold}} \cup N_{\text{too hot}})\). Then, \(N_{\text{switch ON comp}}\) and \(N_{\text{switch OFF comp}}\) are the sets of the EWHs that will switch ON and OFF compulsorily due to very low and very high temperatures respectively.

\[
N_{\text{switch ON comp}} = N_{\text{OFF}} \cap N_{\text{too cold}}
\]  

(33)

\[
N_{\text{switch OFF comp}} = N_{\text{ON}} \cap N_{\text{too hot}}
\]  

(34)

\[
N_{\text{switch OFF candidates}} = N_{\text{ON}} \cap N_{\text{in deadband}}
\]  

(35)

\[
N_{\text{switch ON candidates}} = N_{\text{OFF}} \cap N_{\text{in deadband}}
\]  

(36)

From (33)-(36) and Table IV, it can be seen that, contrary to variations C2a, C2b and C2c, variations C2d, C2e and C2f do not respect the internal control signals since \(N_{\text{switch ON comp}}\) and \(N_{\text{switch OFF comp}}\) are not based on \(T_{\text{el}}\).

2) **Ranking**: The first step here is to calculate the difference between the setpoint and the actual aggregate power of the population:

\[
\Delta P = P_{\text{set}} - P_{\text{agg}}
\]  

(37)

Taking into account the compulsory switching actions of (33) and (34), the effective difference in power that needs to be achieved by additional switching actions is derived:

\[
\Delta P_{\text{ef}} = \Delta P + P_{\text{switch OFF comp}} - P_{\text{switch ON comp}}
\]  

(38)

If \(\Delta P_{\text{ef}} < 0\) additional switching OFF action is required, whereas if \(\Delta P_{\text{ef}} > 0\) switching ON action is desired. The final list of EWHs that will be toggled is constructed as follows:

- If \(\Delta P_{\text{ef}} < 0\), the EWHs \(N_{\text{switch OFF candidates}}\) are ranked according to parameter \(R\), which depends on the strategy variation and can be found in Table IV. The larger the \(R\) the hotter is the EWH and therefore the higher is its switching OFF priority. Then, the set of EWHs that will receive a switch OFF signal, \(N_{\text{active OFF}}\), is determined by finding the number of devices \(N_{\text{d}}\) that minimizes:

\[
\Delta P_{\text{ef}} + \sum_{k=0}^{N_{\text{d}}} P_{\text{el}}^k
\]  

(39)

where \(P_{\text{el}}^k\) is the power of the \(k\)th heater in the list.

- If \(\Delta P_{\text{ef}} > 0\), the EWHs \(N_{\text{switch ON candidates}}\) are ranked according to parameter \(R\). Now, the smaller the \(R\) the colder is the EWH and therefore the higher is its switching ON priority. Then, the switching ON list \(N_{\text{active ON}}\) is determined applying (39), but with a minus instead of a plus.

Last, the central controller sends switching signals to all EWHs according to the following rule \((t\) denotes again the time instance):

\[
u_i^t = \begin{cases} 
1 & \text{if } i \in N_{\text{active ON}} \lor i \in N_{\text{switch ON comp}} \\
0 & \text{if } i \in N_{\text{active OFF}} \lor i \in N_{\text{switch OFF comp}} \\
u_i^{t-1} & \text{otherwise}
\end{cases}
\]  

(40)

**C. Indirect Temperature Feedback Control (C3)**

In this scenario, the controller does not receive any direct temperature information. It is aware of the ON/OFF state of each EWH \(i\) and the temperature deadband crossing events, which can be communicated to the central controller by transmitting the following signal:

\[
s_i = \begin{cases} 
1 & \text{if } T_{\text{el}} \leq T_{\text{min}} \\
0 & \text{if } T_{\text{min}} < T_{\text{el}} < T_{\max} \\
-1 & \text{if } T_{\text{el}} \geq T_{\max}
\end{cases}
\]  

(41)

The signal \(s_i\) is sent only when a deadband crossing occurs, thus the communication burden is much lower compared to strategy C2. The general concept of the selection and ranking
steps, as explained in subsection V-B, can also be applied here. In this case, however, the subsets $N_{\text{too cold}}$, $N_{\text{too hot}}$ and $N_{\text{deadband}}$ are defined according to Table V. With these definitions, (37) and (38) can be used to calculate $\Delta P_{ef}$.

For implementation of this approach, the central controller should have a model as in (19)-(23) and historical data of hourly water consumption for all the EWHs in the aggregation. During real-time operation, the controller uses these data to simulate the population and estimate the temperatures of each EWH at each time step $t$. $T_{est}$. Additionally, every time a signal $s_i$ from an individual EWH is received, the vector $T_{est}^i$ is updated according to Table VI. The values used in this Table have been estimated from simulation results with a population of EWHs without any external control.

In this algorithm, the ranking parameter $R$ is the SOC as defined in (30), but calculated based on $T_{est}^i$. With this information, $N_{\text{active OFF}}$ and $N_{\text{active ON}}$ can be calculated applying (39) and the switching signals applying (40).

D. Aggregate Power Feedback Control (C4)

Bidirectional switching actions are possible in this strategy by broadcasting control signals to all EWHs. In each time step, the central controller receives only measurements of $P_{set}$ and $P_{agg}$ and calculates $\Delta P$ applying (37). Note that in this case the controller has no information about the ON/OFF state of individual EWHs. Therefore, the additional requirements discussed in subsection V-C also hold here to allow temperature estimation on-line.

The controller sets upper and lower limits on SOC, namely $SOC_{\text{max}}$ and $SOC_{\text{min}}$, to avoid using very hot or very cold devices for external control purposes. Based on these limits, $N_{\text{too cold}}$ and $N_{\text{too hot}}$ are defined as in strategies C2e and C2f (see Table IV), whereas $N_{\text{deadband}}$ is again defined as $\neg (N_{\text{too cold}} \cup N_{\text{too hot}})$. All EWHs in $N_{\text{too cold}}$ are expected to switch ON with a total power $P_{\text{too cold}}$, whereas all EWHs in $N_{\text{too hot}}$ to switch OFF with a total power $P_{\text{too hot}}$. Then, the controller calculates $\Delta P_{ef}$ according to:

$$\Delta P_{ef} = \Delta P + P_{\text{too hot}} - P_{\text{too cold}} .$$

If $\Delta P_{ef} < 0$, the EWHs in $N_{\text{deadband}}$ are ranked in descending SOC order, whereas if $\Delta P_{ef} > 0$ in ascending order. The controller makes the list of the EWHs that should toggle their state by finding the device number $N_d$ that minimizes (39). The SOC of the EWH that enters the list last is named threshold SOC and is denoted by $SOC_{th}$. Then, $SOC_{\text{min}}$, $SOC_{\text{max}}$ and $SOC_{th}$ are broadcasted to all devices along with an auxiliary binary signal defined as follows:

$$s = \begin{cases} 1 & \text{if } \Delta P_{ef} > 0 \\ 0 & \text{if } \Delta P_{ef} < 0 \end{cases} .$$

Each EWH responds to the broadcasted signal according to the following rule. In (44), $u_i^t$ denotes the actual ON/OFF state of EWH $i$ at time $t$.

$$u_i^t = \begin{cases} \text{ON} & \text{if } (s = 1 \land SOC \leq SOC_{th}) \lor SOC \leq SOC_{\text{min}} \\ \text{OFF} & \text{if } (s = 0 \land SOC \geq SOC_{\text{th}}) \lor SOC \geq SOC_{\text{max}} \\ u_i^{t-1} & \text{otherwise} \end{cases} .$$

VI. EVALUATION OF CONTROL STRATEGIES

In this Section, the proposed strategies are evaluated with respect to control quality, device operation and user comfort. The goal is to track a 24-hour target power profile with a time resolution of 10 seconds. Two different case studies are considered: a constructed target power profile and a target profile based on an actual LFC signal.

A. Constructed Target Power Profile

In this case, the power profile includes ramps, step increases and decreases, as well as a sinusoidal section and it is shown in Fig. 4. This Figure presents also the tracking performance of the four more interesting strategies, whereas the evaluation results for all strategies are shown in Fig. 5.

---

**TABLE IV**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>C2a</th>
<th>C2b</th>
<th>C2c</th>
<th>C2d</th>
<th>C2e</th>
<th>C2f</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{too cold}}$</td>
<td>$T_d \leq T_{min}$</td>
<td>$T_d \geq T_{min}$</td>
<td>$T_d \leq T_{min}$</td>
<td>$T_{n+1} \geq T_{min}$</td>
<td>$SOC &lt; SOC_{\text{min}} = 0.3$</td>
<td>$SOC &lt; SOC_{\text{min}} = 0.3$</td>
</tr>
<tr>
<td>$N_{\text{too hot}}$</td>
<td>$T_d \geq T_{max}$</td>
<td>$T_d \leq T_{max}$</td>
<td>$T_d \leq T_{max}$</td>
<td>$SOC \geq SOC_{\text{max}} = 0.9$</td>
<td>$SOC &gt; SOC_{\text{max}} = 0.9$</td>
<td>$SOC &gt; SOC_{\text{max}} = 0.9$</td>
</tr>
<tr>
<td>$N_{\text{deadband}}$</td>
<td>$SOC &gt; SOC_{\text{min}} = 0.9$</td>
<td>$SOC &lt; SOC_{\text{max}} = 0.5$</td>
<td>$SOC &lt; SOC_{\text{max}} = 0.5$</td>
<td>$SOC &gt; SOC_{\text{min}} = 0.9$</td>
<td>$SOC &gt; SOC_{\text{min}} = 0.9$</td>
<td>$SOC &gt; SOC_{\text{min}} = 0.9$</td>
</tr>
</tbody>
</table>

**TABLE V**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{too cold}}$</td>
<td>Set of EWHs for which $s_i = 1$</td>
</tr>
<tr>
<td>$N_{\text{too hot}}$</td>
<td>Set of EWHs for which $s_i = -1$</td>
</tr>
<tr>
<td>$N_{\text{deadband}}$</td>
<td>Set of EWHs for which $s_i = 0$</td>
</tr>
<tr>
<td>$R$</td>
<td>SOC, eq. (30), both for $\Delta P_{ef} &lt; 0$ and $\Delta P_{ef} &gt; 0$</td>
</tr>
</tbody>
</table>

**TABLE VI**

<table>
<thead>
<tr>
<th>$s$</th>
<th>$T_{est}^i$ Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leftrightarrow 1$</td>
<td>$T_{est}^i = 0.5 \times (100.1 + 100.0)$</td>
</tr>
<tr>
<td>$0 \leftrightarrow 1$</td>
<td>$T_{est}^i = 0.5 \times (100.0 + 100.1)$</td>
</tr>
</tbody>
</table>
Apart from the sinusoidal part, the stochastic blocking approach follows the target profile resulting in a relatively small RMSE. However, it does not perform well with respect to user comfort since approximately 40% of the devices do not have warm water for at least half an hour during the day.

Among variations of strategy C2, the lowest tracking error is achieved by C2d, however, this comes at the cost of a high impact on customer comfort. Strategies C2a, C2b and C2c demonstrate the same performance both in terms of tracking error and user comfort. An interesting result is the superior performance of strategies C2e and C2f over strategies C2a, C2b and C2c. We observe that by using the SOC definition of 30, instead of 28, the tracking error is significantly reduced with very little compromise of user comfort. In addition, by comparing cases C2e and C2f, we conclude that parameters $SOC_{\text{min}}$ and $SOC_{\text{max}}$ can be tuned to achieve the desired tradeoff between tracking accuracy and customer comfort.

Strategy C3 respects the control signals from the EWH internal thermostatic controller, therefore the effects on user comfort are negligible. Due to the absence of continuous temperature feedback, the tracking quality deteriorates resulting in power spikes especially during the ramp phase of the profile.

It is interesting to note that strategy C4 has a much lower RMSE compared to C3, which is counterintuitive since the controller in C4 receives less information in real-time. Fig. 4 explains that the main reason for this result is the absence of spikes along the target profile in control approach C4.

According to Fig. 5, strategies C1, C2d and C4 result in the lowest average number of switching actions, whereas strategy C3 in the highest. Note that the direct temperature feedback control approaches based on (30) reduce the switching actions compared to the ones based on (28).

### B. Target Power Profile based on LFC Signal

To assess the algorithms’ performance in a more realistic case, a segment from an actual LFC signal from the control area of Switzerland has been used and the simulations have been repeated for strategies C1, C2e, C3 and C4. The LFC signal for 24 hours is shown in Fig. 7 as a percentage of the control band. To derive the target power profile for the EWH population the following procedure was adopted.

The daily energy consumption without external control, $E_{\text{day}}$, was calculated and a base power profile $P_{\text{base}}$ was constructed using $E_{\text{day}}$ and the water draw probability profile of Fig. 2. It was assumed that the population can participate in secondary frequency control with hourly bids and a control band defined as $P_{\text{cb}} = \pm 20\% P_{\text{base}}$. Then, the target power profile was obtained by superimposing the LFC signal on the base power profile:

$$P_{\text{tp}} = P_{\text{base}} + LFC P_{\text{cb}} \quad .$$

Fig. 6 presents the tracking performance, Fig. 7 the relative instantaneous tracking error and Fig. 8 the strategy evaluation results. It can be seen that in the case of control approaches without temperature feedback (C1 and C4), there are intervals with very high tracking errors that can exceed 50%. As expected, strategy C2e achieves the lowest tracking error with RMSE = 1.18%. Strategy C3 performs significantly better with the actual LFC signal than the constructed target profile. In this case, the maximum relative error is approximately 30%, whereas the RMSE is 8.7%. Note also that the impact of both C2e and C3 on customer comfort is minimal.

Based on these results, the direct temperature feedback control approach is in principle suitable for LFC programs with aggregations of EWHs. Indirect temperature feedback control demonstrates a limited potential for LFC applications. From Fig. 6 and Fig. 7, it is observed that the larger tracking errors occur during the two peak power intervals, i.e. between hours 8-10 a.m. and 6-7 p.m. This indicates that performance of strategy C3 decreases with larger control bands. The stochastic blocking and aggregate power feedback approaches are clearly not suitable for LFC. Nevertheless, strategies C3 and C4 might be of interest in applications with lower accuracy requirements, such as load shifting. For a detailed analysis of such applications the reader is referred to [3].

Another interesting conclusion is drawn by comparing strategies C1 and C4. Although the RMSE of C4 is approximately 45% greater than that of C1, the number of users who are affected for more than half an hour is reduced by a factor of seven. Thus, the additional complexity introduced in strategy C4 due to water consumption prediction and simulation of EWH population by the central controller, results in a more acceptable overall performance.

### VII. Conclusion

In this paper, we presented a dynamic model of an EWH which captures the thermal stratification inside the water tank,
as well as a statistical model for populations of EWHs. The validation results show that the developed models describe the power consumption of EWH populations with sufficient accuracy. Four rule-based control approaches for aggregate power setpoint tracking with different complexities and communication requirements have been introduced. The control strategies have been evaluated with respect to tracking quality, customer comfort and device operation. It has been shown that the introduction of the SOC concept improves control performance, since the whole temperature distribution inside the tank is considered when taking control actions.

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Evangelos Vrettos (S’09) was born in Athens, Greece. He received the Dipl.-Ing degree in electrical and computer engineering from the National Technical University of Athens (NTUA), Greece, in 2010. After his graduation he worked as a research assistant at the same University. He is currently working towards the Ph.D. degree at the Power Systems Laboratory, Swiss Federal Institute of Technology (ETH) Zurich, Switzerland, where he joined in September 2011.

His research interests are demand response, energy storage, integration of renewable energy sources, modeling and optimization. He is a member of the Technical Chamber of Greece.

Stephan Koch (S’08) was born in Bielefeld, Germany. He received the Dipl.-Ing. degree in engineering cybernetics from the University of Bremen, Bremen, Germany, in 2007. He is currently working towards the Ph.D. degree at the Power Systems Laboratory, Swiss Federal Institute of Technology (ETH) Zurich, Switzerland, where he joined in October 2007.

His work for the project Local Load Management is focused on control and operation strategies for flexible household loads and their technical and economic integration into power systems and electricity markets. His further research interests are automatic control in power systems and system integration of distributed and renewable energy resources.

Göran Andersson (M’86, SM’91, F’97) was born in Malmö, Sweden. He obtained his M.S. and Ph.D. degree from the University of Lund in 1975 and 1980, respectively. In 1980 he joined the HVDC division of ASEA, now ABB, in Ludvika, Sweden, and in 1986 he was appointed full professor in electric power systems at the Royal Institute of Technology (KTH), Stockholm, Sweden. Since 2000 he has been a full professor in electric power systems at the Swiss Federal Institute of Technology (ETH), Zurich.

His research interests are power system analysis, simulation and control. Another research interest is future energy and power systems. He is a member of the Royal Swedish Academy of Engineering Sciences and Royal Swedish Academy of Sciences, and he is active in IEEE PES. He was the recipient of the IEEE PES Outstanding Power Educator Award 2007.