VOLTAGE AND POWER INTERACTIONS IN MULTI-INFEED HVDC SYSTEMS

Draft

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**TABLE OF CONTENTS**

1. **INTRODUCTION**  
   1.1 Overview of Multi-Infeed HVDC Sensitivity Factors

2. **SYSTEM MODEL**  
   2.1 Mathematical Model

3. **DEFINITION OF SENSITIVITY FACTORS**  
   3.1 Primary and Secondary Interactions  
   3.2 Empirical Definition  
   3.3 Analytic Definitions

4. **ANALYSIS OF SENSITIVITY FACTORS**  
   4.1 Relationships Between Interaction Factors

5. **PRACTICAL APPLICATIONS OF SENSITIVITY FACTORS**  
   5.1 Derivation of JR Matrix From Interaction Factors  
   5.2 Derivation of Operation Charts From Interaction Factors

6. **PARAMETRIC INFLUENCE ON SENSITIVITY FACTORS**  
   6.1 Effective Short Circuit Ratio  
   6.1.1 Equivalent Effective Short Circuit Ratio  
   6.1.2 Equivalent Single-Infeed HVDC System  
   6.1.3 Per Unit Equivalent Single-Infeed HVDC Quantities  
   6.1.4 Critical Equivalent Effective Short Circuit Ratio  
   6.1.5 Relationship Between EESCR and MIESCR  
   6.1.6 Relationship Between EESCR and MESCIR  
   6.1.7 Application to Dual-Converter Multi-Infeed HVDC System

7. **REFERENCES**
1. INTRODUCTION

1.1 Overview of Multi-Infeed HVDC Sensitivity Factors

Historically, point-to-point hvdc links feeding into ac systems with low short-circuit level at the inverter ac bus typically experienced power and voltage instability problems. Such system configurations or commonly known as single-infeed hvdc systems had motivated a number of analysis methods to be proposed in the past, to study problems of this nature. Among the notable ones are the maximum available power and voltage stability factor method as introduced in [1] and [2], respectively. Essentially both approaches use the sensitivities of system states with respect to incremental changes in the controlling system quantities as an assessment of system stability.

As the use of hvdc transmission becomes increasingly common, there has also been a higher incidence of multiple hvdc converters terminating in a common ac area. Such system configurations are generally known as multi-infeed hvdc systems. It is natural that industry concern for multi-infeed hvdc systems, particularly those with constituent hvdc links having weak ac infeeds, also follow closely the historical context of the instability problems experienced by single-infeed hvdc systems. Thus, analysis methods similar to those for the single-infeed hvdc systems have been also extended to multi-infeed hvdc systems, such as the modal voltage sensitivity factor (MVSF) for voltage stability as in [3], [4], modal and nodal power sensitivity factor (MPSF and NPSF) for power stability as in [3], [5]. It should be noted that these analytic methods mentioned above for both single and multi-infeed hvdc systems are only applicable to line-commutated converters as distinguished from voltage-source converters. These latter converters, which are increasingly becoming common need special consideration which will be treated in another separate work.

Given the emergence of multi-infeed hvdc systems is of recent origin and that research on such system configurations has mostly been focused on instability problems similarly experienced by single-infeed hvdc systems, little attention has been given so far to other pertinent areas of concern. One of such is the voltage interaction among hvdc converters, which is currently being addressed by the Cigré Working Group B4-41 [7]. In this respect, the concept of a Multi-Infeed Interaction Factor (MIIF) is being proposed, which is closely allied to the concepts previously proposed for multi-infeed hvdc systems [3], [4], [6].

From these recent development as mentioned above, it is evident that there will be even more ideas and concepts that will be introduced as research on multi-infeed hvdc systems progresses. These new proposals will undoubtedly facilitate clearer understanding of multi-infeed hvdc systems, but this attainment will be made somewhat simpler if there is an

![Fig.1: Overview of sensitivity indices in multi-infeed hvdc systems](image)
overview of their roles, inter-relationships, and contextual applicability in voltage/power stability and interaction analysis. For this purpose, it is useful to map these multifarious indices into a hierarchical structure as shown in Fig.1, and to trace the inter-relationships among the analytic methods for the single and multi-infeed hvdc systems as shown in Fig.2. In Fig.1, it is seen that analysis of multi-infeed hvdc systems can be broadly classified into two domains. Where stability boundary and proximity to it are of concern, then stability indices such as $MVSF$, $MPSF$, and $NPSF$ as proposed in [3]-[5] may be useful. On the other hand, when the multi-infeed hvdc system is operating within the stable region, and operational and control issues such as voltage interactions are of concern, then interaction indices such as $MIIF$, $NVIF$, and $NPIF$ indices may be useful. Further, by inspection of such a hierarchical structure as in Fig.1, it is thus possible to make logical extension of the existing to derive new allied indices. In this respect, indices such as the Nodal Voltage Sensitivity Factor ($NVSF$) as allied to the $MVSF$ proposed in [3], [4], the Nodal Voltage and Power Interaction Factors ($NVIF$ and $NPIF$) as allied to the $MIIF$ proposed in [7], may be introduced. In Fig.2, the corresponding relationship between the voltage and power analytic methods, and the corresponding inter-relationship between these methods, for single and multi-infeed hvdc systems are illustrated as also had been shown in [3]-[5], [11]. The intra-relationship between the nodal and modal analytic methods for multi-infeed hvdc systems as also illustrated in Fig.2 will be shown in this work.

This work focuses on the development of these new indices and study their inter-relationship with those proposed recently, notably with the $MIIF$ which is currently still being studied. The parametric influence on these indices is also investigated. These indices may be useful for aiding control and operation of practical multi-infeed hvdc systems, and their use in developing operational charts and deriving conditions for operating regimes are also studied. Finally, time-domain simulations are carried out to verify that the theoretical foundations of these indices are consistent with the practical results obtained.

![Fig.2: Relationships between sensitivity indices of single and multi-infeed hvdc systems](image)

2. SYSTEM MODEL

2.1 Mathematical Model

The mathematical model for defining the analytic sensitivity indices is derived from the Jacobian for two power flow solution models, namely the unified ac/dc and eliminated dc-variable [8] power flow solution models.

Firstly, the Jacobian for the unified ac/dc power flow solution model given by the following may be used to derive the analytic sensitivity indices in this work;
Voltage and Power interactions in multi-infeed HVDC systems

\[
\begin{bmatrix}
\Delta P_{dc} \\
\Delta P_{ac} \\
\Delta Q_{ac}
\end{bmatrix} =
\begin{bmatrix}
J_{P_{dc}l} & J_{P_{dc}u} & J_{P_{dc}u} \\
J_{P_{ac}l} & J_{P_{ac}u} & J_{P_{ac}u} \\
J_{Q_{ac}l} & J_{Q_{ac}u} & J_{Q_{ac}u}
\end{bmatrix}
\begin{bmatrix}
\Delta I_{dc} \\
\Delta \delta \\
\Delta U / U
\end{bmatrix}
\]

where \( I_{dc} \), \( \delta \), and \( U \) are the defined state variables as:
- \( I_{dc} \) dc current
- \( U \) converter ac bus voltage
- \( P_{dc} \) dc power injection at converter ac bus
- \( P_{ac} \) active ac power injection at converter ac bus
- \( Q_{ac} \) reactive ac power injection at converter ac bus

Where there are no active and reactive ac power change at the converter ac buses, then \( \Delta P_{ac} \) and \( \Delta Q_{ac} \) may be assumed to be zero. Thus (1) becomes;

\[
\begin{bmatrix}
\Delta P_{dc} \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
J_{P_{dc}l} & J_{P_{dc}u} \\
J_{P_{ac}l} & J_{P_{ac}u} \\
J_{Q_{ac}l} & J_{Q_{ac}u}
\end{bmatrix}
\begin{bmatrix}
\Delta I_{dc} \\
\Delta \delta \\
\Delta U / U
\end{bmatrix}
\]

which reduces to;

\[
\begin{bmatrix}
\Delta \delta \\
\Delta U / U
\end{bmatrix} = -J_{dc} \Delta I_{dc}
\]

\[
\Delta P_{dc} = J_{dc} \Delta I_{dc}
\]

where;

\[
J_{dc} = J_{P_{dc}l} - \begin{bmatrix}
J_{P_{dc}l} & J_{P_{dc}u}
\end{bmatrix}^{-1}
\begin{bmatrix}
J_{P_{ac}l} & J_{P_{ac}u}
\end{bmatrix}
\]

If, however, there is only absence of active ac power change at the converter ac buses, then only \( \Delta P_{ac} \) may be assumed to be zero. Thus (1) becomes;

\[
\begin{bmatrix}
\Delta P_{dc} \\
\Delta Q_{ac}
\end{bmatrix} =
\begin{bmatrix}
J_{P_{dc}l} & J_{P_{dc}u} & J_{P_{dc}u} \\
J_{P_{ac}l} & J_{P_{ac}u} & J_{P_{ac}u} \\
J_{Q_{ac}l} & J_{Q_{ac}u} & J_{Q_{ac}u}
\end{bmatrix}
\begin{bmatrix}
\Delta I_{dc} \\
\Delta \delta \\
\Delta U / U
\end{bmatrix}
\]

which reduces to;

\[
\begin{bmatrix}
\Delta P_{dc} \\
\Delta Q_{ac}
\end{bmatrix} =
\begin{bmatrix}
J_{P_{dc}l} - J_{P_{dc}l} J_{P_{ac}u}^{-1} J_{P_{ac}l} & J_{P_{dc}u} - J_{P_{dc}u} J_{P_{ac}u}^{-1} J_{P_{ac}u} \\
J_{Q_{ac}l} - J_{Q_{ac}l} J_{P_{ac}u}^{-1} J_{P_{ac}l} & J_{Q_{ac}u} - J_{Q_{ac}u} J_{P_{ac}u}^{-1} J_{P_{ac}u}
\end{bmatrix}
\begin{bmatrix}
\Delta I_{dc} \\
\Delta U / U
\end{bmatrix}
\]

or more concisely as,

\[
\begin{bmatrix}
\Delta P_{dc} \\
\Delta Q_{ac}
\end{bmatrix} =
\begin{bmatrix}
J_{ac11} & J_{ac12} \\
J_{ac21} & J_{ac22}
\end{bmatrix}
\begin{bmatrix}
\Delta I_{dc} \\
\Delta U / U
\end{bmatrix}
\]

where;

\[
J_{ac11} = J_{P_{dc}l} - J_{P_{dc}l} J_{P_{ac}u}^{-1} J_{P_{ac}l} \\
J_{ac12} = J_{P_{dc}u} - J_{P_{dc}u} J_{P_{ac}u}^{-1} J_{P_{ac}u} \\
J_{ac21} = J_{Q_{ac}l} - J_{Q_{ac}l} J_{P_{ac}u}^{-1} J_{P_{ac}l} \\
J_{ac22} = J_{Q_{ac}u} - J_{Q_{ac}u} J_{P_{ac}u}^{-1} J_{P_{ac}u}
\]

The further reduction of (5) depends on the converter control modes. For constant dc power, and constant \( \gamma \) or constant dc voltage control, there is no dc power change at the converter ac buses. Thus \( \Delta P_{dc} \) may be assumed to be zero and (5) reduces to;
where

\[ \Delta Q_{ac} = J_{Rac} \Delta U / U \]

(6)

\[ J_{Rac} = J_{ac22} - J_{ac21} J_{ac11}^{-1} J_{ac12} \]

On the other hand, for constant dc current and constant \( \gamma \) control, there is no dc current change at the converter ac buses. Thus \( \Delta L_0 \) may be assumed to be zero and (5) reduces to;

\[ \Delta P_{dc} = J_{ac12} \Delta U / U \]

\[ \Delta Q_{dc} = J_{ac22} \Delta U / U \]

(7)

Secondly, the Jacobian for the eliminated dc-variable power flow solution model as given by the following may also be used to derive the analytic sensitivity indices in this work;

\[
\begin{bmatrix}
\Delta P_{ac} \\
\Delta Q_{ac}
\end{bmatrix} =
\begin{bmatrix}
J_{p ac} & J_{p ac} \delta \\
J_{q ac} & J_{q ac} \delta
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta U / U
\end{bmatrix}
\]

(8)

where \( \delta \) and \( U \) are the defined state variables. Note that in (8), the dc quantities are treated as dependent variables and their partial derivatives are expressed or so-called eliminated in terms of ac quantities. From (8), again where there is no active ac power change, then \( \Delta P_{ac} \) may be assumed to be zero. Thus;

\[
\begin{bmatrix}
0 \\
\Delta Q_{ac}
\end{bmatrix} =
\begin{bmatrix}
J_{p ac} & J_{p ac} \delta \\
J_{q ac} & J_{q ac} \delta
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta U / U
\end{bmatrix}
\]

which reduces to;

\[ \Delta Q_{ac} = J_R \Delta U / U \]

(9)

where \( J_R = J_{q ac} - J_{q ac} \delta J_{p ac} \delta J_{p ac}^{-1} J_{p ac} \).

(3)-(9) will be the main equations for defining the various analytic sensitivity indices in the next Section.

3  DEFINITION OF INTERACTION AND SENSITIVITY FACTORS

3.1  Primary and Secondary Interactions

In the previous Section, the small ac or dc change injection in the mathematical model represents a system disturbance in the corresponding system. For example, a small reactive ac power \( \Delta Q_{ac} \) or dc power \( \Delta P_{dc} \) change represents the switching in/out of a capacitor bank in the ac system or change in a dc link power order in the dc system, respectively. In the system where the power change or disturbance occurs there will be a direct interaction between the system variables, which will be known as a primary interaction. On the other hand, in the system where no power change or disturbance occurs there will also be an interaction, though indirect, between the system variables due to the electrical coupling between the ac and dc systems. This indirect interaction will be known as a secondary interaction. Table 1 summarizes the primary and associated secondary interactions due to the power change or system disturbance in the ac or dc system.

<table>
<thead>
<tr>
<th>Power Injection</th>
<th>Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta P_{dc} )</td>
<td>( \Delta P_{ac} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \neq 0 )</td>
<td>0</td>
</tr>
</tbody>
</table>
The analytic ac and dc interaction factors defined later in this Section will be associated with both primary and secondary interactions, accordingly to where the system disturbance occurs.

3.2 Empirical Definition

3.2.1 Multi Infeed Interaction Factor (MIIF)

According to [7], an index known as the Multi Infeed Interaction Factor (MIIF) has been proposed as the ratio between the voltage changes at inverter ac bus $j$ and $i$ due to a reactive ac power change at inverter ac bus $i$, and given by:

$$\text{MIIF}_{ji} = \frac{\Delta V_j}{\Delta V_i}$$

This is an empirical definition which can be used to provide a simple measure of the electrical coupling or interaction between the converter ac buses. To ensure that the voltage changes are small, it was suggested in [7], that the quantum of reactive power change at inverter ac bus $i$ should be such to induce only about 1% voltage change at the same bus.

3.3 Analytical Definitions

3.3.1 Nodal Voltage Interaction Factor and Nodal Voltage Sensitivity Factor

Corresponding to the empirical definition of (10), a similar definition could be derived analytically from the ac/dc power flow model given by (9). We will first consider the dual-converter system to gain an intuitive understanding of the interaction factors and then extend the results to the multiple converter case.

**Dual-converter case**

Using the eliminated-variable power flow model for a dual-converter multi-infeed hvdc system, eliminating all the non-converter ac buses and retaining only the converter ac buses, (9) is given by:

$$\begin{bmatrix} \Delta P_{ac(t)} \\ \Delta Q_{ac(t)} \end{bmatrix} = \begin{bmatrix} J_p & \delta_p \\ J_q & \delta_q \end{bmatrix} \begin{bmatrix} \Delta U_t \\ U_t \end{bmatrix}$$

where the subscript $t$ denotes a converter ac bus. For no active ac power change at all converter ac buses it is assumed that $\Delta P_{ac(t)}=0$. Thus (11) reduces to:

$$\begin{bmatrix} \Delta Q_{ac(t)} \end{bmatrix} = J_R \begin{bmatrix} \Delta U_t/U_t \end{bmatrix}$$

where $J_R = J_q - J_q \delta_p \delta_p^{-1} J_p$ is the QV Jacobian matrix.

For a dual-converter multi-infeed hvdc system, (12) is given by:

$$\begin{bmatrix} \Delta Q_{ac(1)} \\ \Delta Q_{ac(2)} \end{bmatrix} = \begin{bmatrix} J_{R11} & J_{R12} \\ J_{R21} & J_{R22} \end{bmatrix} \begin{bmatrix} \Delta U_{1t}/U_{1t} \\ \Delta U_{2t}/U_{2t} \end{bmatrix}$$

where the expressions for $J_{R11}, J_{R12}, J_{R21},$ and $J_{R22}$ can be found in [3].

If it is further assumed that there is only reactive ac power change at converter ac bus 1, then it can also be assumed that $\Delta Q_{ac(2)}=0$. Thus (13) gives:

$$\begin{align*}
\Delta Q_{ac(1)} &= J_{R11} \frac{\Delta U_{1t}}{U_{1t}} + J_{R12} \frac{\Delta U_{2t}}{U_{2t}} \\
0 &= J_{R21} \frac{\Delta U_{1t}}{U_{1t}} + J_{R22} \frac{\Delta U_{2t}}{U_{2t}}
\end{align*}$$
which reduces to;

$$\frac{\Delta U_{t2}}{U_{t2}} = -\frac{J_{R21}}{J_{R22}} \frac{\Delta U_{t1}}{U_{t1}}$$  \hspace{1cm} (14)$$

and substituting (14) into $\Delta Q_{ac(1)}$ gives;

$$\Delta Q_{ac(1)} = \frac{\text{det}(J_R)}{J_{R22}} \frac{\Delta U_{t1}}{U_{t1}}$$  \hspace{1cm} (15)$$

where $\text{det}(J_R) = J_{R11}J_{R22} - J_{R12}J_{R21}$ denotes the determinant of the $J_R$ matrix. From (15), a stability factor to be known as the Nodal Voltage Sensitivity Factor (NVSF) can be defined as;

$$\text{NVSF}_1 = \frac{\Delta U_{t1}}{\Delta Q_{ac(1)}} = \frac{J_{R22}}{\text{det}(J_R)}$$ \hspace{1cm} (16a)$$

Further from (14), corresponding to the empirical definition of MIIF of (10), an allied interaction factor to be known as Nodal Voltage Interaction Factor (NVIF) can be defined as;

$$\text{NVIF}_{21} = \text{NVIF}_{12} = \frac{\Delta Q_{ac(1)}}{\Delta U_{t2}/U_{t2}} = \frac{\Delta Q_{ac(1)}}{\Delta U_{t1}/U_{t1}} = \frac{J_{R21}}{J_{R22}}$$ \hspace{1cm} (17a)$$

and $\Delta Q_{ac(1)} = (\Delta U_{t1}/U_{t1})/\text{NVSF}_1$ is the necessary reactive ac power change at converter ac bus 1 to effect a per unit voltage change at the same bus. Similarly, where there is only reactive ac power change at converter ac bus 2, then it can be assumed that $\Delta Q_{ac(1)}=0$. Thus,

$$\text{NVSF}_2 = \frac{\Delta U_{t2}}{\Delta Q_{ac(2)}} = \frac{J_{R11}}{\text{det}(J_R)}$$ \hspace{1cm} (16b)$$

$$\text{NVIF}_{12} = \frac{\Delta Q_{ac(2)}}{\Delta U_{t1}/U_{t1}} = \frac{\Delta Q_{ac(2)}}{\Delta U_{t2}/U_{t2}} = \frac{J_{R12}}{J_{R11}}$$ \hspace{1cm} (17b)$$

Note that the NVSF and NVIF are sensitivity indices defined with respect to a single-degree freedom of reactive ac power variation, i.e. the reactive ac power variation is at a single-node while those at other nodes are constrained to zero. Thus they are nodal quantities which is in contrast with sensitivity indices defined with respect to multiple degrees of freedom of reactive ac power variation, i.e. the reactive ac power variation is in all the eigenmode directions, for example, the Modal Voltage Sensitivity Factor (MVSF) as proposed in [3], [4]. This comparison between the NVSF and MVSF is analogous to the comparison between the Nodal Power Sensitivity Factor (NPSF) and Modal Power Sensitivity Factor (MPSF) in [3], [5]. The NPSF is the dc power sensitivity with respect to a single-degree freedom of direct current variation in contrast with the multi-degree freedom of direct current variation for the MPSF.

Though the empirical definition of (10) and the analytical definition of (17) are very similar we shall distinguish between them by calling the latter as the Nodal Voltage Interaction Factor (NVIF).

**Multiple converter case**

The analytical definition of NVIF of (17) for the dual-converter multi-infeed hvdc system can be extended to the $N$-converter system.

For the $N$-converter system, we may also consider a reactive ac power change only at the $i$-th and none at other converter ac buses. Thus $\Delta Q_{ac(i)} \neq 0$ and $\Delta Q_{ac(j)}=0$: $i, j \in \{1, \ldots, N\}, j \neq i$. Then
similar to the case for the dual-converter multi-infeed HVDC system, (9) for the N-converter system becomes:

\[
\Delta Q_{ac(i)} = J_{R11} \frac{\Delta U_{i1}}{U_{i1}} + J_{R12} \frac{\Delta U_{i2}}{U_{i2}} + \ldots + J_{RiN} \frac{\Delta U_{iN}}{U_{iN}}
\]

which gives:

\[
\Delta Q_{ac(i)} = J_{R1i} \frac{\Delta U_{i1}}{U_{i1}} + J_{Rij} \frac{\Delta U_{ij}}{U_{ij}} + \ldots + J_{Rim} \frac{\Delta U_{im}}{U_{im}} = \sum_{m=1}^{N} J_{Rim} \frac{\Delta U_{im}}{U_{im}}
\]

Thus for an N-converter system, the primary Nodal Voltage Interaction Factor of the j-th converter ac bus with respect to a reactive ac power change at the i-th converter ac bus can be analytically defined as:

\[
NVIF_{ji} = \lim_{\Delta Q_{ac(i)} \to 0} \frac{\Delta Q_{ac(j)}}{\Delta Q_{ac(i)}} = \lim_{\Delta Q_{ac(i)} \to 0} \frac{\Delta U_{ij}}{U_{ij}} = -\sum_{k=1}^{N-1} \left[ \bar{J}^{-1}_{R} \right]_{jk} \left[ \bar{J}^{-1}_{Ri} \right]_{ki}
\]

where \( \bar{J}^{-1}_{R} \) is the \( R \) matrix having its i-th row and i-th column eliminated and \( \bar{J}_{Ri} = \left[ J_{R1i} \ldots J_{Rji} \ldots J_{Rmi} \right]_{j \neq i} \) is the i-th column vector of the \( R \) matrix having its i-th row element eliminated. Note that \( \bar{J}^{-1}_{R} \) and \( \bar{J}_{Ri} \) have dimensions \((N-1) \times (N-1)\) and \((N-1) \times 1\), respectively. Thus from (20):

\[
\left[ \frac{\Delta U_{i1}}{U_{i1}} \ldots \frac{\Delta U_{ij}}{U_{ij}} \ldots \frac{\Delta U_{iN}}{U_{iN}} \right]_{j \neq i} = \left[ \bar{J}^{-1}_{R} \right]_{jk} \left[ \bar{J}^{-1}_{Ri} \right]_{ki}
\]

and thus the multi-dimensional Nodal Voltage Sensitivity Factor (NVSF) can be defined as;
\[
\frac{\Delta U_{ii}}{U_{ii}} = NVSF_i = \frac{1}{J_{Rii} + \sum_{m=1, m \neq i}^{N} J_{Rim} \cdot NVIF_{mi} + \sum_{m=1}^{N} J_{Rim} \cdot NVIF_{mi}} \quad (24)
\]

In order to derive a more convenient expression for \( NVIF_{ji} \) in (22), first (20) can be alternatively expressed as:

\[
- \tilde{J}_R \begin{bmatrix} \Delta U_{i1} \\
\Delta U_{ij} \\
\vdots \\
\Delta U_{iN}
\end{bmatrix} = J_{Ri} \Rightarrow - \tilde{J}_R \begin{bmatrix} NVIF_{i1} \\
NVIF_{ji} \\
\vdots \\
NVIF_{Ni}
\end{bmatrix} = J_{Ri} \quad (25)
\]

(25) is of the form \( AX = B \) where the solution for \( X \) is given by the Cramer’s rule [9] as:

\[
x_j = \frac{D_j}{D}, \quad \ldots, \quad x_j = \frac{D_j}{D}, \quad \ldots, \quad x_N = \frac{D_N}{D}, \quad (26)
\]

where \( D = \text{det}(A) \) and \( D_j \) is the determinant of \( A \) having its \( j \)-th column vector replaced by column vector \( B \). Thus (25) can be solved as follows:

\[
- NVIF_{ii} = \frac{\text{det}(J_{i1})}{\text{det}(\tilde{J}_R)}, \quad \ldots, \quad - NVIF_{ji} = \frac{\text{det}(J_{ji})}{\text{det}(\tilde{J}_R)}, \quad \ldots, \quad - NVIF_{Ni} = \frac{\text{det}(J_{Nj})}{\text{det}(\tilde{J}_R)} \quad (27)
\]

where \( \text{det}(\tilde{J}_R) \) is actually \( \text{det}(J_R) \) having its \( i \)-th row and \( i \)-th column eliminated, i.e. the minor \( M_{ii} \) of diagonal element \( J_{Rii} \) in \( \text{det}(J_R) \), as illustrated in the following:

\[
M_{ii} = \text{det}(\tilde{J}_R) = \begin{vmatrix} J_{R11} & \ldots & J_{R1N} \\
J_{R11} & \ldots & J_{R1N} \\
\vdots & \ddots & \vdots \\
J_{RN1} & \ldots & J_{RNN}
\end{vmatrix}
\]

and \( \text{det}(\tilde{J}_j) \) is \( \text{det}(\tilde{J}_R) \) having its \( j \)-th column vector replaced by column vector \( \tilde{J}_{Ri} \). Note that this is actually \( \text{det}(J_R) \) having its \( i \)-th row and \( j \)-th column eliminated, i.e. minor \( M_{ij} \) of element \( J_{Rij} \) in \( \text{det}(J_R) \), and then its \( i \)-th column vector consecutively interchanged with the immediate neighbouring column vector until it occupies the \( j \)-th column, as illustrated in the following:

\[
\text{det}(\tilde{J}_j) = \begin{vmatrix} J_{R11} & \ldots & J_{R1j} & \ldots & J_{R1N} \\
J_{R11} & \ldots & J_{R1j} & \ldots & J_{R1N} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
J_{RN1} & \ldots & J_{RNN} & \ldots & J_{RNN}
\end{vmatrix}
\]
becoming:

\[
\det(\tilde{J}_j) = \begin{vmatrix}
J_{R1} & J_{R1(j+1)} & \cdots & J_{R1(i-1)} & J_{R1(i+1)} & \cdots & J_{R1N} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
J_{RN1} & J_{RN(j+1)} & \cdots & J_{RN(i-1)} & J_{RN(i+2)} & \cdots & J_{RNN}
\end{vmatrix}
\]

Since an interchange of two columns in \(M_i\) results in just a sign reversal of the determinant value, then interchanging the \(i\)-th column \(i-(j+1)\) consecutive times with its immediate neighbouring column to migrate to the \(j\)-th column would result in \(i-(j+1)\) times sign reversal of the determinant value, i.e.

\[
\det(\tilde{J}_j) = (-1)^{i-(j+1)} \cdot M_{ij}
\]  

Thus from (27) and (28),

\[
NVIF_{li} = \frac{(-1)^{(i-1)} M_{il}}{M_{ii}}, \quad \cdots, \quad NVIF_{ji} = \frac{(-1)^{(i-j)} M_{ij}}{M_{ii}}, \quad \cdots, \quad NVIF_{Ni} = \frac{(-1)^{(i-N)} M_{IN}}{M_{ii}}
\]  

Since pre-multiplying (29) with \((-1)^{j}\) does not change their signs, then (29) becomes;

\[
NVIF_{li} = \frac{(-1)^{(i+1)} M_{il}}{M_{ii}}, \quad \cdots, \quad NVIF_{ji} = \frac{(-1)^{(i+j)} M_{ij}}{M_{ii}}, \quad \cdots, \quad NVIF_{Ni} = \frac{(-1)^{(i+N)} M_{IN}}{M_{ii}}
\]  

It is seen from (30) that the numerators are cofactors \(C_{ij}\) of \(J_R\) along the \(i\)-th row, i.e.

\[
NVIF_{li} = \frac{C_{ij}}{M_{ii}}, \quad \cdots, \quad NVIF_{ji} = \frac{C_{ij}}{M_{ii}}, \quad \cdots, \quad NVIF_{Ni} = \frac{C_{iN}}{M_{ii}}
\]  

where \(C_{ij} = (-1)^{i+j} M_{ij}\). Thus the NVSF and primary NVIF for an \(N\)-converter multi-infeed hvdc system can be computed from the \(J_R\) matrix, given by;

\[
NVSF_i = \frac{1}{\sum_{m=1}^{N} J_{Rim} \cdot \frac{C_{im}}{M_{ii}}} \quad \text{and} \quad NVIF_{ji} = \frac{C_{ij}}{M_{ii}}
\]  

\(NVSF_i\) in (32) is the interaction index due to the primary interactions occurring in the ac system caused by a reactive ac power change. As for the interaction index due to the secondary interactions occurring in the dc system caused by the ac/dc system coupling, (9) offers no convenient form to access the dc quantities since they have been ‘absorbed’ into the ac quantities. Thus, (6)-(7) derived from the unified ac/dc power flow solution model are made use for this purpose. For constant power control mode, from (6) and using the form of (32) for \(A\Delta U/U\) gives;

\[
A_{dc} = -J_{ac11}^{-1} \cdot J_{ac12} \begin{bmatrix} \Delta U \\ U \end{bmatrix} = -J_{Rac}^{-1} \begin{bmatrix} C_{Rac(i)} \frac{\Delta U}{U} \\ M_{Rac(i)} \frac{\Delta U}{U} \\ \vdots \\ C_{Rac(iN)} \frac{\Delta U}{U} \\ M_{Rac(iN)} \frac{\Delta U}{U} \end{bmatrix}
\]  

\[
\frac{\Delta U}{U} = \begin{bmatrix} C_{Rac(i)} \\ M_{Rac(i)} \\ \vdots \\ C_{Rac(iN)} \\ M_{Rac(iN)} \end{bmatrix} \end{bmatrix}
\]  

\[
A_{dc} = -J_{ac11}^{-1} \cdot J_{ac12} \begin{bmatrix} \Delta U \\ U \end{bmatrix} = -J_{Rac}^{-1} \begin{bmatrix} C_{Rac(i)} \frac{\Delta U}{U} \\ M_{Rac(i)} \frac{\Delta U}{U} \\ \vdots \\ C_{Rac(iN)} \frac{\Delta U}{U} \\ M_{Rac(iN)} \frac{\Delta U}{U} \end{bmatrix}
\]
where $C_{Rac(ij)}$ is the cofactor of the $i$-th row, $j$-th column elements and $M_{Rac(ij)}$ is the $i$-th row, $j$-th column minor, both of the $J_{Rac}$ matrix in (6). Thus, the secondary Nodal Power Interaction Factor $NPIF_{sec}$ with respect to dc current for constant power control mode is given by;

$$NPIF_{sec}^c(P_{dc(j)}) = \frac{\Delta I_{dc(j)}}{\Delta I_{dc(i)}} = \frac{\sum_{k=1}^{N} J'_{ac22}(ik) \cdot C_{ac22}(ik)}{\sum_{k=1}^{N} J'_{ac22}(ik) \cdot C_{ac22}(ik)}$$  (35)

where $J'_{ac22}(ik)$ is the cofactor of the $i$-th row, $k$-th column element of the $J_{ac}$ matrix.

For constant current control mode, from (7) and deriving in a similar approach as (33)-(34), the secondary Nodal Power Interaction Factor $NPIF_{sec}$ with respect to dc power for constant current control mode is given by;

$$NPIF_{sec}^c(P_{dc(j)}) = \frac{\Delta P_{dc(j)}}{\Delta P_{dc(i)}} = \frac{\sum_{k=1}^{N} J'_{ac22}(ik) \cdot C_{ac22}(ik)}{\sum_{k=1}^{N} J'_{ac22}(ik) \cdot C_{ac22}(ik)}$$  (35)

where $C_{ac22}(ik)$ is the cofactor of the $i$-th row, $k$-th column element of the $J_{ac22}$ matrix.

### 3.3.2 Nodal Power Interaction Factor (NPIF)

Based on the mathematical model for the multi-infeed hvdc sensitivity framework in Fig.1, it is thus intuitive to define an interaction factor with respect to dc power or current corresponding to the interaction factor with respect to voltage. This will be known as the Nodal Power Interaction Factor (NPIF) and defined as;

$$NPIF = \frac{\Delta P}{\Delta P_{dc(i)}} \quad NPIF = \frac{\Delta I}{\Delta I_{dc(i)}}$$  (36)

Here, the power interaction factors are defined with respect to dc power as well as dc current, since for certain hvdc control mode the interaction factors may be non-existent with respect to one dc quantity but not for the other, and vice versa. For example, the NPIF with respect to dc power is zero for hvdc links in constant power control mode since the dc power in hvdc link $j$ will remain constant for a small change in the dc power order in hvdc link $i$. However, the dc current in hvdc link $j$ may change to maintain constant dc power since its dc voltage may change due to its ac bus voltage affected by the power order change in hvdc link $i$. Table 2 shows the existence of the $NPIF_{Pdc}$ and $NPIF_{Idc}$ for different hvdc control modes.

<table>
<thead>
<tr>
<th>HVDC Control Mode</th>
<th>Nodal Power Interactor Factor based on:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dc Power: $\Delta P_{dc(j)} / \Delta P_{dc(i)}$</td>
</tr>
<tr>
<td>Constant dc power</td>
<td>0</td>
</tr>
<tr>
<td>Constant dc current</td>
<td>$\neq 0$</td>
</tr>
<tr>
<td>Constant dc voltage</td>
<td>$\neq 0$</td>
</tr>
</tbody>
</table>

To derive the analytic expressions for $NPIF$ in terms of the elements of the ac/dc Jacobian matrix, we rewrite (3) as;

$$\Delta P_{dc} = J_{dc} \Delta I_{dc}$$  (37)

where $J_{dc}$ is as given in (4).

If all of the $N$ hvdc links operate in constant dc power-constant $\gamma$control mode, and only the $i$-th hvdc link is subjected to a small dc power order change, then it may be assumed that $\Delta P_{dc(j)}=0$ for all $j \neq i$. Thus (3) becomes;
\[
\begin{bmatrix}
0 \\
\vdots \\
\Delta P_{dci} \\
\vdots \\
0
\end{bmatrix} = \begin{bmatrix}
J_{dc} & \Delta M_{\text{dc}}
\end{bmatrix}
\]  
(38)

Separating out (38), we obtain;
\[
\Delta P_{dci} = J_{dc1} \Delta M_{dc1} + J_{dc2} \Delta M_{dc2} + \cdots + J_{dciN} \Delta M_{dcN}
\]  
(39)

\[
\begin{bmatrix}
0
\end{bmatrix} = \tilde{J}_{dc} \begin{bmatrix}
\Delta M_{dci}
\end{bmatrix}_{j \neq i} + \tilde{J}_{dci} \Delta M_{dci}
\]  
(40)

where \(\tilde{J}_{dc}\) is the \(J_{dc}\) matrix having its \(i\)-th row and \(i\)-th column eliminated and \(\tilde{J}_{dci} = [J_{dci}\ldots J_{dcj}\ldots J_{dci}]^T_{j
eq i}\) is the \(i\)-th column vector of the \(J_{dc}\) matrix having its \(i\)-th row element eliminated. Thus from (40), the primary Nodal Power Interaction Factor (NPIF) with respect to dc current for constant dc power control mode can be derived as;
\[
\text{NPIF}_{\text{dc}(ji)} = \frac{\Delta M_{dci}}{\Delta M_{dci}} = -\tilde{J}_{dc}^{-1} \tilde{J}_{dci}
\]

which can be expressed in a more convenient form by using the same approach as (25)-(32) for the NVIF, given by;
\[
\text{NPIF}_{\text{dc}(ji)} = \frac{C_{\text{dc}(ij)}}{M_{\text{dc}(ii)}}
\]  
(41)

where \(C_{\text{dc}(ij)}\) is the cofactor of the \(i\)-th row, \(j\)-th column element, and \(M_{\text{dc}(ii)}\) is the minor of the \(i\)-th row, \(i\)-th column element, both of the \(J_{dc}\) matrix.

If all of the \(N\) hvdc links operate in constant dc current-constant \(\gamma\) control mode, and only the \(i\)-th hvdc link is subjected to a small dc current order change, then it may be assumed that \(\Delta L_{dcj} = 0\) for all \(j \neq i\). Thus (3) becomes;
\[
\begin{bmatrix}
\Delta P_{dc} \\
\Delta M_{dci}
\end{bmatrix} = \begin{bmatrix}
0 \\
\vdots \\
\Delta P_{dci} \\
\vdots \\
0
\end{bmatrix} = \begin{bmatrix}
J_{dc1(i)} & \Delta J_{dci} \\
\vdots & \vdots \\
J_{dcN(i)} & \Delta J_{dci}
\end{bmatrix}
\]  
(42)

\[
\begin{bmatrix}
0 \\
\vdots \\
\Delta P_{dci} \\
\vdots \\
0
\end{bmatrix} = \begin{bmatrix}
J_{dc1(i)} \\
\vdots \\
J_{dcN(i)}
\end{bmatrix}
\]  
(43)

where \(J_{dc(ji)}\) is the \(j\)-th row, \(i\)-th column element of the \(J_{dc}\) matrix.

Thus from (43), the primary Nodal Power Interaction Factor (NPIF) with respect to dc power for constant dc current control mode can be derived as;
\[
\text{NPIF}_{\text{dc}(ji)} = \frac{\Delta P_{dci}}{\Delta P_{dci}} = \frac{J_{dc(ji)}}{J_{dc(ii)}}
\]  
(44)
(41) and (44) are the interaction indices due to the primary interactions occurring in the dc system caused by a small change in the controlled dc quantities. As for the interaction indices due to the secondary interactions occurring in the ac system caused by the ac/dc system coupling, (3) can be used to derive them. Rewriting (3) gives;
\[
\begin{bmatrix}
\Delta \delta \\
\Delta U/U
\end{bmatrix} = -J_{dc} \Delta I_{dc}
\]
and using (41) for constant dc power control mode, the secondary Nodal Voltage Interaction Factor \( NVIF_{sec} \) is thus defined as;
\[
NVIF_{ji}^{sec} = \frac{\Delta U_j/U_j}{\Delta U_i/U_i} = \frac{\sum_{k=1}^{N} J_{dc(jk)} \cdot \Delta I_{dc(k)}}{\sum_{k=1}^{N} J_{dc(ik)} \cdot \Delta I_{dc(k)}} = \frac{N}{C_{dc(ik)}} \sum_{k=1}^{N} J_{dc(jk)} \cdot C_{dc(ik)}
\]
Again using (3) and noting that \( \Delta I_{dc}=0 \) for \( j \neq i \) for constant dc current control mode, the secondary Nodal Voltage Interaction Factor \( NVIF_{sec} \) can thus be defined as;
\[
NVIF_{ji}^{sec} = \frac{J_{dc(ji)}}{J_{dc(ii)}}
\]
where \( J'_{dc(ji)} \) is the \( j \)-th row, \( i \)-th column element of the \( J'_{dc} \) matrix.

### 3.3.3 Summary and Numerical Computation of Interaction Factors

It is obvious that the numerous definitions of interaction indices in Section 3.3.1-3.3.2 may be somewhat confounding and it is instructive to summarize them in tabular form at this juncture, as shown in Table 3. Further, to be able to compute these interaction indices systematically and efficiently for analysis, a flow chart as shown in Fig.3 will also be useful.

<table>
<thead>
<tr>
<th>Sensitivity Index</th>
<th>Primary Governing equation</th>
<th>Secondary Governing equation</th>
<th>Small change in Power flow model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. NVIF</td>
<td>(32)</td>
<td>( NPIF ) (const. dc power)</td>
<td>reactive ac power</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( NPIF ) (const. dc current)</td>
<td>Eliminated variable or Unified ac/dc</td>
</tr>
<tr>
<td>2. NPIF (const. dc power)</td>
<td>(41)</td>
<td>( NVIF )</td>
<td>dc power order</td>
</tr>
<tr>
<td>3. NPIF (const. dc current)</td>
<td>(44)</td>
<td>( NVIF )</td>
<td>dc current order</td>
</tr>
</tbody>
</table>
4. ANALYSIS OF INTERACTION FACTORS

4.1 Relationships Between Sensitivity Factors

4.1.1 MVSF and NVIF-NVSF

Interaction factors only indicate the extent of electrical coupling between converter ac buses but they do not indicate the proximity to the stability boundary. It is thus of interest to find the relationship between the stability and interaction factors, viz. MVSF and NVIF.

For this purpose, (31) is multiplied on the left-hand and right-hand side by the corresponding \( i \)-th row element of \( J_R \), and given by;

\[
J_{R1} \cdot M_{ii} \cdot NVIF_{ii} = J_{R1} \cdot C_{i1}, \quad \ldots \quad J_{Rij} \cdot M_{ii} \cdot NVIF_{ji} = J_{Rij} \cdot C_{ij}, \quad \ldots \quad J_{RIN} \cdot M_{ii} \cdot NVIF_{Ni} = J_{RIN} \cdot C_{iN}
\]

Thus summing all the elements in (47) gives:

\[
J_{R1} \cdot M_{ii} \cdot NVIF_{i1} + \ldots + J_{Rij} \cdot M_{ii} \cdot NVIF_{ji} + \ldots + J_{RIN} \cdot M_{ii} \cdot NVIF_{Ni} = J_{R1} \cdot C_{i1} + \ldots + J_{Rij} \cdot C_{ij} + \ldots + J_{RIN} \cdot C_{iN}, \quad j \neq i
\]

Adding the left-hand and right-hand side of (48) with the cofactor of the \( J_{Rii} \) diagonal element;

\[
J_{R1} \cdot M_{ii} \cdot NVIF_{ii} + \ldots + J_{Rij} \cdot M_{ii} \cdot NVIF_{jj} + \ldots + J_{Rii} \cdot C_{ii} + \ldots + J_{RIN} \cdot M_{ii} \cdot NVIF_{Ni} = J_{R1} \cdot C_{i1} + \ldots + J_{Rij} \cdot C_{ij} + \ldots + J_{Rii} \cdot C_{ii} + \ldots + J_{RIN} \cdot C_{iN}
\]
Since $J_{Ri}C_{ii} = J_{Ri} (-1)^2 M_{ii}$ and the right-hand side expression of (49) is the sum of the adjoints of $J_R$ along its $i$-th row, (49) becomes:

$$J_{Ri1} \cdot M_{ii} \cdot NVIF_{i1} + \cdots + J_{Rij} \cdot M_{ii} \cdot NVIF_{ji} + \cdots + J_{Rin} \cdot M_{ii} \cdot NVIF_{ni} = \det(J_R)$$

giving:

$$J_{Ri1} + \sum_{j=1, j \neq i}^{N} J_{Rij} \cdot NVIF_{ji} = \det(J_R) \quad \Rightarrow \quad \sum_{j=1}^{N} J_{Rij} \cdot NVIF_{ji} = \frac{\det(J_R)}{M_{ii}}$$

Thus (50) relates the NVIF with the MVSF stability boundary since the latter is reached when $\det(J_R)$ diminishes to zero. However, the MVSF can also be related to the NVSF by using (32) in (50) to give:

$$\frac{\det(J_R)}{M_{ii}} = \frac{1}{NVSF_i}$$

From (51), it is seen that $\det(J_R)$ is zero if only NVSF becomes infinite. Since $\det(J_R)$ being zero corresponds to the minimum eigenvalue of $J_R$, i.e. MVSF, also being zero, (51) implies that the system becomes MVSF unstable if and only if NVSF goes infinite. This also means that all the information about the MVSF stability is contained in the NVSF. In other words, (51) relates a parameter-based quantity, i.e. $\det(J_R)$ or MVSF which are derived using the elements of the reduced Jacobian matrix $J_R$ computed from the network parameters and conditions, to a response-based quantity, i.e. NVSF which is derived from measurements of the converter ac bus voltage response to the switching of a network element such as a shunt capacitor. From a practical perspective, this offers considerable convenience to determine the MVSF stability of a multi-infeed hvdc system since it can be inferred from a response-based quantity, i.e. the NVSF, which can be practically measured from the network response.

5. PRACTICAL APPLICATIONS OF INTERACTION FACTORS

5.1 Derivation of $J_R$ matrix from interaction factors

![Fig. 4: A dual-converter multi-infeed hvdc system](image)

It is noted that the derivation of the indices NVSF and NVIF in (32) is parameter-based, i.e. they are derived using the elements of the reduced Jacobian matrix $J_R$. In analytical studies, such a mathematical approach is possible since the elements of $J_R$ can be computed from knowledge of the network parameters and conditions and the indices are then directly derived from these elements of $J_R$ to infer the stability of the multi-infeed hvdc system. On the other hand, in real time situations or in computer simulation studies only an empirical approach is possible since the elements of $J_R$ are non-existent or inaccessible but one can derive the indices directly from measurements of the network voltage response to switching actions. One is thus interested to compute the elements of $J_R$ from these indices measured in real-time or from computer simulation studies to infer the MVSF stability of the multi-infeed hvdc system.
In order to construct the elements of of \( J_R \) from these indices, it is essential to establish a mathematical relationship between them. Towards this end, (9) is re-written as:

\[
\frac{\Delta U}{U} = J_R^{-1} \Delta Q_{ac}
\]  

(55)

where \( J_R = J_{ac}^{-1} u - J_{ac}^{-1} J_p - J_{ac}^{-1} J_p \) is the reduced Jacobian matrix, and \( J_R^{-1} \) is its inverse which can be obtained as:

\[
J_R^{-1} = \frac{1}{\text{det}(J_R)} \begin{bmatrix}
C_{i1} & \cdots & C_{i1} & \cdots & C_{i1} & \cdots & C_{i1} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_{i1} & \cdots & C_{ij} & \cdots & C_{j1} & \cdots & C_{j1} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_{i1} & \cdots & C_{ij} & \cdots & C_{ij} & \cdots & C_{ij} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_{i1} & \cdots & C_{ij} & \cdots & C_{ij} & \cdots & C_{ij} \\
& \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
C_{i1} & \cdots & C_{ij} & \cdots & C_{ij} & \cdots & C_{ij} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
& \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
& & \ddots & \vdots & \ddots & \vdots & \vdots \\
& & & \ddots & \vdots & \ddots & \vdots \\
& & & & \ddots & \vdots & \ddots \\
& & & & & \ddots & \vdots \\
& & & & & & \ddots & \vdots \\
& & & & & & & \ddots
\end{bmatrix}
\]

(56)

where \( C_{ij} \) is the cofactor of the \( i \)-th row, \( j \)-th column element of the \( J_R \) matrix.

Now, consider a reactive power change only at the \( i \)-th converter ac bus, i.e. \( \Delta Q_{ac(i)} \) and \( \Delta Q_{ac(j)} = 0 \) for \( j \neq i \). Thus, using this in (55) and (56) we obtain:

\[
\frac{\Delta U_j}{U_j} = \frac{C_{ij}}{\text{det}(J_R)} \frac{\Delta Q_{ac(i)}}{\Delta U_j / U_j} \quad j = 1, \ldots, N, \ j \neq i \quad \text{and} \quad \frac{\Delta U_i}{U_i} = \frac{C_{ii}}{\text{det}(J_R)} \frac{\Delta Q_{ac(i)}}{\Delta U_i / U_i} \quad j = i
\]

(57)

Thus;

\[
\frac{C_{ij}}{\text{det}(J_R)} = NVIF_{ji} \cdot NVSF_i \quad j = 1, \ldots, N, \ j \neq i \quad \text{and} \quad \frac{C_{ii}}{\text{det}(J_R)} = NVSF_i \quad j = i
\]

(58)

Note that \( C_{ii} = M_i \) and (58) for \( j = i \) is identical to (51). Thus \( J_R^{-1} \) can be expressed as:

\[
J_R^{-1} = \begin{bmatrix}
NVSF_1 & \cdots & NVSF_1 \cdot NVIF_{i1} & \cdots & NVSF_1 \cdot NVIF_{i1} \cdot NVIF_{j1} & \cdots & NVSF_1 \cdot NVIF_{i1} \cdot NVIF_{j1} \cdot NVIF_{jN} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
NVSF_1 \cdot NVIF_{i1} & \cdots & NVSF_1 \cdot NVIF_{i1} & \cdots & NVSF_1 \cdot NVIF_{i1} \cdot NVIF_{j1} & \cdots & NVSF_1 \cdot NVIF_{i1} \cdot NVIF_{j1} \cdot NVIF_{jN} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
NVSF_1 \cdot NVIF_{j1} & \cdots & NVSF_1 \cdot NVIF_{j1} & \cdots & NVSF_1 \cdot NVIF_{j1} \cdot NVIF_{j1} & \cdots & NVSF_1 \cdot NVIF_{j1} \cdot NVIF_{j1} \cdot NVIF_{jN} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
NVSF_1 \cdot NVIF_{jN} & \cdots & NVSF_1 \cdot NVIF_{jN} & \cdots & NVSF_1 \cdot NVIF_{jN} \cdot NVIF_{jN} & \cdots & NVSF_1 \cdot NVIF_{jN} \cdot NVIF_{jN} \cdot NVIF_{jN}
\end{bmatrix}
\]

(59)

We first state two known operations on the determinant of a matrix, given by;
Voltage and Power interactions in multi-infeed HVDC systems

\[
\begin{align*}
\det\left(\begin{bmatrix}
a_{11} & \ldots & k \cdot a_{1j} & a_{1N} \\
\vdots & & \vdots & \vdots \\
a_{N1} & \ldots & k \cdot a_{Nj} & a_{NN}
\end{bmatrix}\right) = k \cdot \det\left(\begin{bmatrix}
a_{11} & \ldots & a_{1j} & a_{1N} \\
\vdots & & \vdots & \vdots \\
a_{N1} & \ldots & a_{Nj} & a_{NN}
\end{bmatrix}\right)
\end{align*}
\]

(60)

and:

\[
\begin{align*}
\det\left(\begin{bmatrix}
a_{11} & \ldots & a_{1j} + b_1 & a_{1N} \\
\vdots & & \vdots & \vdots \\
a_{N1} & \ldots & a_{Nj} + b_N & a_{NN}
\end{bmatrix}\right) = \det\left(\begin{bmatrix}
a_{11} & \ldots & a_{1j} & a_{1N} \\
\vdots & & \vdots & \vdots \\
a_{N1} & \ldots & a_{Nj} & a_{NN}
\end{bmatrix}\right) + \det\left(\begin{bmatrix}
a_{11} & \ldots & b_1 & a_{1N} \\
\vdots & & \vdots & \vdots \\
a_{N1} & \ldots & b_N & a_{NN}
\end{bmatrix}\right)
\end{align*}
\]

(61)

Applying (60) repetitively to the determinant of \( J_R^{-1} \) gives:

\[
\begin{align*}
\det(J_R^{-1}) = NVSF_1 \cdot NVSF_2 \cdots NVSF_N \cdot \det\left(\begin{bmatrix}
1 & \ldots & NVIF_{j_1} & \ldots & NVIF_{i_1} \\
\vdots & & \vdots & & \vdots \\
NVIF_{i_1} & \ldots & 1 & \ldots & NVIF_{i_1} \\
\vdots & & \vdots & & \vdots \\
NVIF_{j_1} & \ldots & NVIF_{j_1} & \ldots & 1 \\
\vdots & & \vdots & & \vdots \\
NVIF_{i_N} & \ldots & NVIF_{i_N} & \ldots & 1
\end{bmatrix}\right)
\end{align*}
\]

(62)

Applying (61) to \( \det(A) \) in (62) gives:

\[
\det(A) = \det\left(\begin{bmatrix}
0 + 1 & \ldots & NVIF_{j_1} & \ldots & NVIF_{i_1} \\
\vdots & & \vdots & & \vdots \\
NVIF_{i_1} + 0 & \ldots & 1 & \ldots & NVIF_{i_1} \\
\vdots & & \vdots & & \vdots \\
NVIF_{i_1} + 0 & \ldots & NVIF_{j_1} & \ldots & 1 \\
\vdots & & \vdots & & \vdots \\
NVIF_{i_N} + 0 & \ldots & NVIF_{i_N} & \ldots & 1
\end{bmatrix}\right)
\]

(62)

\[
\begin{align*}
\begin{bmatrix}
0 & \ldots & NVIF_{j_1} & \ldots & NVIF_{i_1} \\
\vdots & & \vdots & & \vdots \\
NVIF_{j_1} & \ldots & 1 & \ldots & NVIF_{i_1} \\
\vdots & & \vdots & & \vdots \\
NVIF_{i_1} & \ldots & NVIF_{j_1} & \ldots & 1 \\
\vdots & & \vdots & & \vdots \\
NVIF_{N_1} & \ldots & NVIF_{i_N} & \ldots & 1
\end{bmatrix}
\end{align*}
\]

Thus;
Voltage and Power interactions in multi-infeed HVDC systems

\[
det(A) = B_1 + \det(B_2) + \det(B_3) + \cdots + \det(B_{N-1}) \tag{63}
\]

Applying (61) repetitively to (63) gives;

\[
det(A) = B_1 + \det(B_2) + \det(B_3) + \cdots + \det(B_{N-1}) + \det(B_N) \tag{64}
\]

Thus, from (62) and (64);

\[
det(J_R^{-1}) = NVSF_1 \cdot NVSF_2 \cdots NVSF_N \cdot \sum_{j=1}^{N} B_j \tag{65}
\]

where;

\[
B_j = \det(\begin{bmatrix}
0 & NVIF_{j,j+1} & \cdots & NVIF_{j,N} \\
NVIF_{j+1,j} & 1 & \cdots & NVIF_{j+1,N} \\
\vdots & \vdots & \ddots & \vdots \\
NVIF_{N,j} & NVIF_{N,j+1} & \cdots & 1
\end{bmatrix}) \quad B_N = 1
\]

From (59) and (65), the reduced Jacobian matrix \( J_R \) and its determinant \( \det(J_R) \) can thus be derived as;
Voltage and Power interactions in multi-infeed HVDC systems

\[
J_R = (J_R^{-1})^{-1} = \begin{bmatrix}
NVSF_1 & \cdots & NVSF_j \cdot NVIF_i & \cdots & NVSF_j \cdot NVIF_{iN} & \cdots & NVSF_N \cdot NVIF_{iN} \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
NVSF_1 \cdot NVIF_1 & \cdots & NVSF_i & \cdots & NVSF_i \cdot NVIF_{iN} & \cdots & NVSF_N \cdot NVIF_{iN} \\
NVSF_j \cdot NVIF_j & \cdots & NVSF_j \cdot NVIF_{j1} & \cdots & NVSF_j \cdot NVIF_j & \cdots & NVSF_N \cdot NVIF_{jN} \\
NVSF_N \cdot NVIF_N & \cdots & NVSF_N \cdot NVIF_{N1} & \cdots & NVSF_N \cdot NVIF_{Nj} & \cdots & NVSF_N \\
\end{bmatrix}^{-1}
\]

(66)

and;

\[
\det(J_R^{-1}) = \frac{1}{\det(J_R)} = \frac{1}{NVSF_1 \cdot NVSF_2 \cdots NVSF_N \cdot (1 + \sum_{j=1}^{N-1} B_j)}
\]

(67)

The individual elements of \(J_R\) can be obtained from (66) after numerically inverting \(J_R^{-1}\). However, the off-diagonal element \(J_{Rij}\) and diagonal element \(J_{Rii}\) can also be directly obtained from (59) as;

\[
J_{Rij} = \frac{\tilde{C}_{ji}}{\det(J_R^{-1})} \quad \text{and} \quad J_{Rii} = \frac{\tilde{C}_{ii}}{\det(J_R^{-1})}
\]

where \(\tilde{C}_{ji}\) is the cofactor of the \(j\)-th row, \(i\)-th column element of \(J_R^{-1}\), given by;

\[
\tilde{C}_{ji} = (-1)^{i+j} \cdot \frac{\det(NVSF_1 \cdot NVSF_2 \cdots NVSF_k \cdots NVSF_N)_{k \neq i}}{\det(NVSF)}
\]

Thus;

\[
J_{Rij} = \frac{(-1)^{i+j} \cdot B_{ji}}{NVSF_j \cdot (1 + \sum_{k=1}^{N-1} B_k)} \quad \text{and} \quad J_{Rii} = \frac{1}{NVSF_1} \frac{B_{ii}}{(1 + \sum_{k=1}^{N-1} B_k)}
\]

(68)

where;

\[
B_{ji} = \det(1 \cdots NVIF_{i,j-1} NVIF_{i,j+1} \cdots NVIF_{iN} \\
\vdots \ddots \vdots \ddots \vdots \ddots \\
NVIF_{j-1,1} \cdots 1 \cdots NVIF_{j-1,N} \\
NVIF_{j+1,1} \cdots 1 \cdots NVIF_{j+1,N} \\
\vdots \ddots \vdots \ddots \vdots \ddots \\
NVIF_{N,j-1} \cdots NVIF_{N,j+1} \cdots 1)
\]

As an example, consider the dual-converter multi-infeed Hvdc system in Fig.4. In this case \(N=2\) which gives;
Voltage and Power interactions in multi-infeed HVDC systems

\[ B_{11} = 1, \quad B_{22} = 1, \quad B_{12} = NVIF_{21}, \quad B_{21} = NVIF_{12}, \quad B_{1} = -NVIF_{12} \cdot NVIF_{21}, \]

and substituting into (68) gives;

\[
J_{R11} = \frac{1}{NVSF_1} \cdot \frac{1}{1 - NVIF_{12} \cdot NVIF_{21}}, \quad J_{R12} = \frac{-1}{NVSF_1} \cdot \frac{NVIF_{12}}{1 - NVIF_{12} \cdot NVIF_{21}}, \\
J_{R21} = \frac{-1}{NVSF_2} \cdot \frac{NVIF_{21}}{1 - NVIF_{12} \cdot NVIF_{21}}, \quad J_{R22} = \frac{1}{NVSF_2} \cdot \frac{1}{1 - NVIF_{12} \cdot NVIF_{21}}.
\]

From (67), det(\(J_R\)) is obtained as;

\[
\det(J_R) = \frac{1}{NVSF_1} \cdot \frac{1}{NVSF_2} \cdot \frac{1}{1 - NVIF_{12} \cdot NVIF_{21}}.
\]

It is seen that \(\det(J_R)\) depends on NVSF and NVIF of the converter ac buses. However for a regular system, i.e. \(0 < NVIF < 1\) meaning that a change of a bus voltage induces a change in the same direction and of smaller magnitude of the other bus voltages, \(\det(J_R)\) can only be zero if and only if the NVSF of either converter ac bus becomes infinite. Thus for a regular dual-converter multi-infeed hvdc system, its MVSF stability is determined only by the NVSF of either converter ac bus although the NVIF can influence the rate at which it becomes unstable, i.e. \(\det(J_R)\) becoming zero. If NVSF \(<\) 1, i.e. there is only a small interaction between the converter ac buses due to being electrically far apart, then the MVSF stability of the multi-infeed system can be determined from only the VSF of each constituent ac/dc system. Also for a regular system, the voltage interactions are positive since increasing NVIF will increase the value of \(\det(J_R)\). This is intuitive since in a regular system, the closer the constituent ac/dc systems are to each other the stronger voltage support will they mutually lend to the total system due to their short-circuit power contribution, thus implying positive voltage interactions between the converters. For a non-regular system, i.e. NVIF \(<\) 0 meaning that a change of a bus voltage induces a change in the opposite direction of the some of the other bus voltages, then the voltage interactions are negative since increasing the magnitude of NVIF will decrease the value of \(\det(J_R)\).

As another example, consider a triple-converter multi-infeed hvdc system. In this case \(N=3\) which gives;

\[
B_{11} = 1 - NVIF_{23} \cdot NVIF_{32}, \quad B_{22} = 1 - NVIF_{13} \cdot NVIF_{31}, \quad B_{33} = 1 - NVIF_{12} \cdot NVIF_{21}, \\
B_{12} = NVIF_{21} - NVIF_{23} \cdot NVIF_{31}, \quad B_{13} = -(NVIF_{31} - NVIF_{12} \cdot NVIF_{21}), \\
B_{21} = NVIF_{12} - NVIF_{32} \cdot NVIF_{13}, \quad B_{23} = NVIF_{32} - NVIF_{12} \cdot NVIF_{21}, \\
B_{31} = -(NVIF_{13} - NVIF_{12} \cdot NVIF_{23}), \quad B_{32} = NVIF_{23} - NVIF_{12} \cdot NVIF_{31}, \\
B = -NVIF_{12} - NVIF_{13} \cdot NVIF_{23}, \quad B = -NVIF_{12} \cdot NVIF_{13}.
\]

and substituting into (68) gives;

\[
J_{R11} = \frac{1}{NVSF_1} \cdot \frac{1 - NVIF_{23} \cdot NVIF_{32}}{1 - S}, \quad J_{R12} = \frac{-1}{NVSF_1} \cdot \frac{(NVIF_{12} \cdot NVIF_{32} \cdot NVIF_{13})}{1 - S}, \quad J_{R13} = \frac{-1}{NVSF_1} \cdot \frac{(NVIF_{13} \cdot NVIF_{12} \cdot NVIF_{23})}{1 - S}, \\
J_{R21} = \frac{-1}{NVSF_2} \cdot \frac{(NVIF_{21} - NVIF_{23} \cdot NVIF_{31})}{1 - S}, \quad J_{R22} = \frac{1}{NVSF_2} \cdot \frac{1 - NVIF_{13} \cdot NVIF_{31}}{1 - S}, \quad J_{R23} = \frac{-1}{NVSF_2} \cdot \frac{(NVIF_{23} - NVIF_{12} \cdot NVIF_{13})}{1 - S}, \\
J_{R31} = \frac{-1}{NVSF_3} \cdot \frac{(NVIF_{31} - NVIF_{32} \cdot NVIF_{23})}{1 - S}, \quad J_{R32} = \frac{-1}{NVSF_3} \cdot \frac{1 - NVIF_{12} \cdot NVIF_{32}}{1 - S}, \quad J_{R33} = \frac{1}{NVSF_3} \cdot \frac{1 - NVIF_{12} \cdot NVIF_{21}}{1 - S},
\]

where;
As seen in this Section, the elements of $J_R$ and $\det(J_R)$ of a general order multi-infeed HVDC system can be computed from the values of NVSF and NVIF of its converter ac buses. In practical situations such as computer simulations using a transient stability program, these indices may be derived in a systematic manner as outlined in the following steps;

**STEP 1**: Solve a load flow case for a particular system configuration and conditions.

**STEP 2**: Initialise all the electrical machines and controllers with the solved load flow case.

**STEP 3**: Run the dynamic simulation up to $t=0^-$ sec. Record all the converter ac bus voltages $U_j$.

**STEP 4**: Switch in a shunt capacitor at the $i$-th converter ac bus at $t=0$ sec. Record the MVar change at this bus $\Delta Q_i$.

**STEP 5**: Run the dynamic simulation up to $t=0^+$ sec. Record all the new converter ac bus voltages $U_j+\Delta U_j$.

**STEP 6**: Compute the voltage change in all the converter ac buses $\Delta U_j$. Then compute NVIF$_{ji}$ and NVSF$_i$ from their definitions.

**STEP 7**: Compute the $i$-th row elements of $J_R$ with (68).

**STEP 8**: Repeat **STEP 2** to **STEP 7** until all rows of $J_R$ are computed.

**STEP 9**: Compute $\det(J_R)$ with (67).

### 5.2 Derivation of operations charts from interaction factors
Voltage and Power interactions in multi-infeed HVDC systems

Fig. 5a: Surface plot of $\log(1+\det(J_R))$ versus NVSF and NVIF

Fig. 5b: Contour plot of $\log(1+\det(J_R))$ versus NVSF and NVIF
As seen previously, the NVIF and NVSF can be used to compute the stability of the multi-infeed HVDC system. Furthermore, in practical situations it is expected to be relatively straightforward to derive these indices in real time based on field measurements of converter ac bus voltages in response to switching of shunt equipment. As such it is useful to derive operational charts based on these indices to indicate secure and insecure system operating regions, so that system operators may be guided by such charts to operate and control the system based on periodic actual field measurements of these indices.

An example of an operational chart for a dual-converter multi-infeed HVDC system is shown in Fig.5b which is the contour plot of det($J_R$) derived from the surface plot of det($J_R$) in Fig.5a. In Fig.5b, the contour lines of det($J_R$) computed with (67) are plotted versus NVSF$_1$, NVSF$_2$, NVIF$_{12}$, and NVIF$_{21}$. From this chart, an area of high voltage sensitivity and low positive interaction indicating insecure operating region and an area of low voltage sensitivity and high positive interaction indicating secure operating region can be identified. Thus, when the indices are obtained from actual field measurements, the operator can check against this chart to determine if the system is operating in a secure or insecure regime, and thus can decide whether to take the appropriate remedial action.

6. PARAMETRIC INFLUENCE ON INTERACTION FACTORS

6.1 Effective Short Circuit Ratio

6.1.1 Equivalent Effective Short Circuit Ratio

Historically, an index known as the Effective Short-Circuit Ratio (ESCR) was defined and intended to indicate the ac system strength at the ac/dc interconnection point with respect to the power rating of the single-infeed HVDC system, as:

$$ ESCR = \frac{S_{MVA} - Q_c}{P_d} $$

where:
- $S_{MVA}$: ac short-circuit level at the converter ac bus.
- $Q_c$: reactive shunt compensation at the converter ac bus.
- $P_d$: dc power rating of the single-infeed HVDC system.

In [7], a similar index known as the Multi-Infeed Effective Short-Circuit Ratio (MIESCR) was defined for a multi-infeed HVDC system, as:

$$ MIESCR = \frac{S_{MVA_i} - Q_{ci}}{P_{di}} + \sum_{j=1, j \neq i}^{N} MIIF_{ji} \cdot P_{dj} $$

where:
- $S_{MVA_i}$: ac short-circuit level at the $i$-th converter ac bus [MVA].
- $Q_{ci}$: reactive compensation at the $i$-th converter ac bus [MVar].
- $P_{di}$: dc power rating of the HVDC link terminating at the $i$-th converter ac bus [MW].
- $P_{dj}$: dc power rating of the HVDC link terminating at the $j$-th converter ac bus [MW].
- $MIIF_{ji}$: voltage interaction factor between the $j$-th and $i$-th converter ac bus due to a small reactive power change at the $i$-th converter ac bus.

Here, the influence of the dc power ratings of the other HVDC links on the ac system strength at the converter ac bus of the reference HVDC link is also taken into account by incorporating their scaled power ratings into the index as given in (55). The scaling factors used is the $MIIF$ given by (10), as it was deemed in [7] that these represent the extent of the influence of the neighbouring HVDC links on the referenced one.

In [12], another index $MESCRI$ was also defined for the multi-infeed HVDC system as;

$$ MESCRI_i = \frac{1}{\sum_{j=1}^{N} Z_{ij} \cdot P_{dj}} $$
where:

\( Z_{ij} \): \( i \)-th row, \( j \)-th column element of the \( Z_{BUS} \) impedance matrix of the multi-infeed hvdc configuration, including all the reactive shunt compensation at the inverter ac buses of the ac system, [per unit of system MVA base].

\( P_{dj} \): dc power rating of \( j \)-th constituent hvdc link, respectively, of the multi-infeed hvdc system [per unit of system MVA base].

In (56), \( Z_{ij}, j \neq i \) is the transfer impedance between the \( j \)-th and \( i \)-th converter ac bus, and \( Z_{ii} \) is the driving-point impedance of the \( i \)-th converter ac bus. It is deemed that the transfer impedances take into account of the influence of the remote hvdc links on the ac system strength at the converter ac bus of the reference hvdc link.

In this work, another index known as the \textit{Equivalent Single-Infeed Effective Short-Circuit Ratio}, \( EESCR \) is defined for the ac system strength at the converter ac bus of the \( i \)-th constituent hvdc link of an \( N \)-converter multi-infeed hvdc system, as:

\[
EESCR_i = \frac{V_R^2 / Z_{ii} - Q_{ci}}{P_{di} + \sum_{j=1,j\neq i}^{N} \frac{Z_{ij}}{Z_{ii}} P_{dj}} \quad \text{assuming} \quad |V_i| = |V_j| = V_R, \quad \delta_i = \delta_j = \delta \quad (57-1)
\]

or in a more general case:

\[
EESCR_i = \frac{V_R^2 / Z_{ii} - Q_{ci}}{P_{di} + \sum_{j=1,j\neq i}^{N} \frac{Z_{ij}}{Z_{ii}} (P_{dj} \cdot \cos \delta_{ij} + Q_{dj} \cdot \sin \delta_{ij})} \quad \text{assuming} \quad |V_i| = |V_j| = V_R \quad (57-2)
\]

where:

\( Z_{ij} \): \( i \)-th row, \( j \)-th column element of the \( Z_{BUS} \) impedance matrix of the ac system of the multi-infeed hvdc configuration, i.e. the transfer impedance [ohm].

\( Z_{ii} \): \( i \)-th row, \( i \)-th column element of the \( Z_{BUS} \) impedance matrix of the ac system of the multi-infeed hvdc configuration, i.e. the self or driving-point impedance [ohm].

\( Q_{ci} \): reactive compensation at the \( i \)-th converter ac bus [MVar].

\( P_{di}, P_{dj} \): dc power rating of the \( i \)-th and \( j \)-th constituent hvdc link, respectively, of the multi-infeed hvdc system [MW].

\( V_R \): rated system voltage [kV].

\( V_i, \delta_i \): \( i \)-th bus voltage magnitude and angle.

\( \delta_{ij} \): voltage angle between \( i \)-th and \( j \)-th bus.

The motivation for defining the ac system strength at the \( i \)-th converter ac bus in a multi-infeed hvdc system as in (57) will become apparent later in this Section.

6.1.2 Equivalent Single-Infeed HVDC System

In view of the various indices as proposed in (55)-(57), our objectives in this Section are thus to establish their inter-relationships and to find out whether they can adequately represent the ac system strength at the converter ac bus in the context of a multi-infeed hvdc configuration. First, consider an \( N \)-converter multi-infeed hvdc system as shown in Fig.6.

The converter dc currents \( I_{dc} \) can be considered as ac currents \( I_{ac} \) injected into the ac system since they are related by \( I_{ac} = \sqrt{\frac{\delta}{\pi}} I_{dc} \). Thus the nodal voltage equations for the \( N \)-converter multi-infeed hvdc system can be written as:

\[
V_{ac} = Z_{BUS} \cdot I_{ac} \quad (58)
\]

where \( V_{ac}, I_{ac} \) are column vectors of bus voltages and injected ac currents, respectively. \( Z_{BUS} \) is the \( N \times N \) bus impedance matrix of the ac system, excluding all the reactive compensation at the converter ac buses. Consider a current \( I_i \) injected into the \( j \)-th bus only and none
elsewhere, i.e. $I_k=0$, $k \neq j$. Then the bus voltage $\Delta V_{ij}$ at the $i$-th node due to this current is given by (58) as:

$$\Delta V_{ij} = 0 + \cdots + Z_{ij} \cdot I_j + \cdots 0 + \cdots 0$$

(59)

From (59) and (60),

$$\tilde{I}_{ij} = \frac{Z_{ij}}{Z_{ii}} \cdot I_j$$

(61)

This means that the effect at the $i$-th bus due to the injected current $I_j$ into the $j$-th bus can be represented by an equivalent current $\tilde{I}_{ij}$ injected into the $i$-th bus and of magnitude given by (61), as shown diagrammatically in Fig.7. Thus as seen from Fig.7, the converter at $j$-th bus is now’ absorbed’ into the $i$-th bus, i.e. the converters at the $i$-th and $j$-th bus are now feeding into the ac system at the same bus which is equivalent to a single-infeed hvdc configuration at
the \( i \)-th bus. Likewise for all the \( N-1 \) remote converters in the multi-infeed hvdc system, their effects at the reference \( i \)-th converter ac bus due to their currents injected into their respective ac buses can be similarly represented by their equivalent currents injected into the \( i \)-th bus. Thus, the \( N \)-converter multi-infeed hvdc can be represented by an Equivalent Single-Infeed HVDC System as shown in Fig.8.

Further from (61), the equivalent power \( \tilde{s}_{ij} \) injected into the \( i \)-th converter ac bus due to the \( j \)-th dc link can be obtained as;

\[
\tilde{s}_{ij} = V_i \cdot I_{ij}^* = V_i \cdot \frac{Z_{ij}}{Z_{ii}} \cdot I_j^* = \frac{Z_{ij}}{Z_{ii}} \cdot (V_j I_j^*)
\]  

(62)

where the asterisk denotes the complex conjugate, and it is assumed that all impedance angles are 90\(^\circ\), i.e. \( Z_{ij}/Z_{ii} = |Z_{ij}|/|Z_{ii}| = |Z_{ij}|/|Z_{ii}| \). Thus from (62),

\[
\tilde{s}_{ij} = \frac{Z_{ij}}{Z_{ii}} \cdot \frac{V_i}{V_j} \cdot \frac{V_i}{V_j} \cdot \frac{I_j}{S_j}
\]  

(63)

where \( s_j \) is power injected into the \( j \)-th converter ac bus. \( \tilde{s}_{ij} \) and \( s_j \) are also given by;

\[
\tilde{s}_{ij} = \bar{P}_{dij} - j \cdot \bar{Q}_{dij}, \quad S_j = S_j | \angle \phi_j = P_{dij} - j \cdot Q_{dij}
\]  

(64)

where;

- \( \bar{P}_{dij} - \bar{Q}_{dij} \) : equivalent active power, reactive power injected into the \( i \)-th converter ac bus due to the \( j \)-th dc link, respectively.
- \( P_{dij}, Q_{dij} \) : active power, reactive power injected into the \( j \)-th converter ac bus, respectively.
- \( \phi_j \) : power factor of the power injected into the \( j \)-th converter ac bus.

Note that a positive sign is defined for flow of reactive power out of the converter ac bus, i.e. into the converter, thus the negative sign in (64) denotes an injection into the converter ac bus.

Similar to the case of the single-infeed hvdc system, if rated voltage magnitude is assumed for all the converter ac buses, then \( |V_i| = |V_j| = V_R \) where \( V_i = |V_i| \angle \delta_i \), \( V_j = |V_j| \angle \delta_j \). Thus (63) becomes;

\[
\tilde{s}_{ij} = \bar{P}_{dij} - j \cdot \bar{Q}_{dij} = \frac{Z_{ij}}{Z_{ii}} \cdot S_j \angle (\phi_j + \delta_{ij}) \quad \text{where} \quad \delta_{ij} = \delta_i - \delta_j
\]  

(65)

Therefore;
Voltage and Power interactions in multi-infeed HVDC systems

Thus comparing LHS and RHS of (65) gives;

\[
\begin{align*}
\tilde{P}_{ij} &= \frac{Z_{ij}}{Z_{ii}} \left( P_{ij} \cos \delta_{ij} + Q_{ij} \sin \delta_{ij} \right), \\
\tilde{Q}_{ij} &= \frac{Z_{ij}}{Z_{ii}} \left( Q_{ij} \cos \delta_{ij} - P_{ij} \sin \delta_{ij} \right)
\end{align*}
\]  

(66)

If rated voltage magnitude and angle are assumed for all the converter ac buses, then \( \mid V_1 \mid = \mid V_j \mid = V_R \). Thus (67) becomes;

\[
\begin{align*}
\tilde{P}_{ij} &= \frac{Z_{ij}}{Z_{ii}} \cdot P_{ij}, \\
\tilde{Q}_{ij} &= \frac{Z_{ij}}{Z_{ii}} \cdot Q_{ij}
\end{align*}
\]  

(68)

Note that the assumption of rated voltage magnitude and angle for all converter ac buses imply zero power transfer in the \( i-j \) branch between the \( i \)-th and \( j \)-th converter ac bus, if it exists. Thus this assumption only applies to a particular system condition of interest. A more general system condition is the assumption of rated voltage magnitude only for all converter ac buses, and this results in the equivalent power injections as given in (67). From (67), the total equivalent power injection into the \( i \)-th converter ac bus due to the power injections of the other remote N-1 converters into their respective converter ac bus, is thus given by;

\[
\begin{align*}
\tilde{P}_{di} &= \sum_{j=1, j \neq i}^{N} \frac{Z_{ij}}{Z_{ii}} \left( P_{ij} \cos \delta_{ij} + Q_{ij} \sin \delta_{ij} \right), \\
\tilde{Q}_{di} &= \sum_{j=1, j \neq i}^{N} \frac{Z_{ij}}{Z_{ii}} \left( Q_{ij} \cos \delta_{ij} - P_{ij} \sin \delta_{ij} \right)
\end{align*}
\]

and the power injection of the equivalent single-infeed hvdc system \( P_{di_eq}, Q_{di_eq} \) into the reference \( i \)-th converter ac bus, are thus given by;

\[
\begin{align*}
P_{di_eq} &= P_{di} + \tilde{P}_{di} = \sum_{j=1}^{N} \frac{Z_{ij}}{Z_{ii}} \left( P_{ij} \cos \delta_{ij} + Q_{ij} \sin \delta_{ij} \right), \\
Q_{di_eq} &= Q_{di} + \tilde{Q}_{di} = \sum_{j=1}^{N} \frac{Z_{ij}}{Z_{ii}} \left( Q_{ij} \cos \delta_{ij} - P_{ij} \sin \delta_{ij} \right)
\end{align*}
\]

(69)

As seen from (58)-(69), the N-converter multi-infeed hvdc system is now reduced to an equivalent single-infeed configuration with lumped power injections, i.e. \( P_{di_eq}, Q_{di_eq} \), which is similar to the single-infeed hvdc configuration. Therefore, the effective short-circuit ratio for this equivalent system \( EESCR \) at the \( i \)-th bus can be similarly defined like the single-infeed hvdc configuration as;

\[
EESCR_i = \frac{S_{MVA_i} - Q_{ci}}{P_{deq}} = \frac{S_{MVA_i} - Q_{ci}}{P_{di} + \sum_{j=1, j \neq i}^{N} \frac{Z_{ij}}{Z_{ii}} \left( P_{ij} \cos \delta_{ij} + Q_{ij} \sin \delta_{ij} \right)}
\]

(70)

where the notations in (70) are as defined in (57) and \( S_{MVA_i} \) is the ac short-circuit level at the \( i \)-th converter ac bus which can be determined from the \( Z_{BUS} \) impedance matrix as \( S_{MVA_i} = U_R^2/Z_n \). \( Z_n \) is the \( i \)-th bus self-impedance of the \( Z_{BUS} \) impedance matrix, which is also the Thevenin impedance of the ac system at the \( i \)-th converter ac bus.
Since (70) is derived from aggregating the effect of the remote converters at the reference \(i\)-th converter ac bus, it thus truly represents the equivalent ac system strength of the multi-infeed hvdc system at the \(i\)-th converter ac bus which has motivated its definition as the \textit{Equivalent Effective Short-Circuit Ratio EESCR} as in (57).

### 6.1.3 Per Unit Equivalent Single-Infeed HVDC Quantities

Since there is only an equivalent single-infeed configuration at the \(i\)-th converter ac bus now, it is convenient to choose the system base \(MVA_{\text{base}}\) as the sum of the rated dc power of all the \(N\) dc links. Thus;

\[
MVA_{\text{base}} = \sum_{j=1}^{N} P_{dj} = P_{di} \sum_{j=1}^{N} \frac{P_{dj}}{P_{di}} = P_{di} \cdot \sum_{j=1}^{N} PBR_{ij}
\]

where \(PBR_{ij}\) is the ratio between the rated dc power of the \(j\)-th and the reference \(i\)-th dc link, or known as the \textit{Power Base Ratio} as defined in [3], [4].

Thus from (69), the per unit power injection of the equivalent single-infeed hvdc system are;

\[
\begin{align*}
\bar{P}_{di\_eq} &= \frac{N}{MVA_{\text{base}}} \sum_{j=1}^{N} \frac{Z_{ij}}{Z_{ii}} (P_{dj} \cos \delta_{ij} + Q_{dj} \sin \delta_{ij}) \\
\bar{Q}_{di\_eq} &= \frac{N}{MVA_{\text{base}}} \sum_{j=1}^{N} \frac{Z_{ij}}{Z_{ii}} (Q_{dj} \cos \delta_{ij} - P_{dj} \sin \delta_{ij})
\end{align*}
\]

where \(QBR_{ij} = \frac{Q_{dj}}{P_{di}}\) is the ratio between the \(j\)-th dc link reactive power and the reference \(i\)-th dc link rated power, or called the \textit{ Reactive Power Base Ratio QBR}. By differentiating (69) with respect to the reference \(i\)-th converter ac bus voltage magnitude \(V_i\), the power sensitivities of the equivalent single-infeed hvdc system are obtained as;

\[
\frac{\partial \bar{P}_{di\_eq}}{\partial V_i} = \sum_{j=1}^{N} \frac{Z_{ij}}{Z_{ii}} (\frac{\partial P_{dj}}{\partial V_i} \cos \delta_{ij} + \frac{\partial Q_{dj}}{\partial V_i} \sin \delta_{ij}) = \frac{\partial P_{di}}{\partial V_i} \quad \text{since} \quad \frac{\partial P_{dj}}{\partial V_i} = 0 \quad \text{for} \quad j \neq i
\]

\[
\frac{\partial \bar{Q}_{di\_eq}}{\partial V_i} = \sum_{j=1}^{N} \frac{Z_{ij}}{Z_{ii}} (\frac{\partial Q_{dj}}{\partial V_i} \cos \delta_{ij} - \frac{\partial P_{dj}}{\partial V_i} \sin \delta_{ij}) = \frac{\partial Q_{di}}{\partial V_i} \quad \text{since} \quad \frac{\partial Q_{dj}}{\partial V_i} = 0 \quad \text{for} \quad j \neq i
\]

The expressions of \(\frac{\partial P_{di}}{\partial V_i}\) and \(\frac{\partial Q_{di}}{\partial V_i}\) for the various hvdc control modes are given in [3].

### 6.1.4 Critical Equivalent Effective Short-Circuit Ratio

Similar to the necessary condition for power instability in the single-infeed hvdc system as shown in [3], a similar necessary condition can be derived for the equivalent system as;

\[
(V_i^2 \cdot EESCR_i + Q_{di\_eq})^2 + (V_i \frac{\partial Q_{di\_eq}}{\partial V_i} - 2Q_{di\_eq}) \cdot (V_i^2 \cdot EESCR_i + Q_{di\_eq}) - P_{di\_eq} \cdot (P_{di\_eq} - V_i \frac{\partial P_{di\_eq}}{\partial V_i}) = 0
\]

The LHS of (74) is the \textit{equivalent maximum available power expression MAP_{eq}} for the single-infeed equivalent of the multi-infeed hvdc system, and the \textit{EESCR} is said to be critical \textit{CEESCR} when the condition in (74) is satisfied. Thus solving (74) gives;
Voltage and Power interactions in multi-infeed HVDC systems

\[ CEESCR_i = \frac{1}{2V_i^2} \left[ -\frac{\partial Q_{di(eq)}}{\partial V_i} \pm \sqrt{\left(\frac{\partial Q_{di(eq)}}{\partial V_i} - 2\frac{\partial Q_{di(eq)}}{\partial P_{di(eq)}}\right)^2 + 4\frac{\partial P_{di(eq)}}{\partial V_i}} - \frac{\partial P_{di(eq)}}{\partial V_i} \right] \quad (75) \]

### 6.1.5 Relationship between EESCR and MIESCR

For small changes in bus voltages and injected currents, the nodal voltage equations for the N-converter multi-infeed hvdc system in (57) becomes;

\[ \Delta V_{ac} = Z_{BUS} \cdot \Delta I_{ac} \quad (76) \]

Consider a small change in the current injected at the \( i \)-th converter ac bus, \( \Delta I_i \) and none elsewhere, i.e. \( I_k = 0, \ k \neq i \). Thus from (76), the voltage changes at the \( i \)-th and \( j \)-th bus due to this current injection are given by:

\[
\begin{align*}
\Delta V_i &= 0 + \cdots + Z_{ii} \cdot \Delta I_i + \cdots + 0 \\
\Delta V_j &= 0 + \cdots + Z_{jj} \cdot \Delta I_j + \cdots + 0
\end{align*}
\]

Dividing the two equations gives;

\[
\frac{\Delta V_j}{\Delta V_i} = \frac{Z_{jj}}{Z_{ii}}
\]

Thus, the EESCR and MIESCR are mathematically equivalent though the former and latter are respectively parameter-based and response-based definitions. Therefore, the EESCR is suitable for analytical studies where one is interested in the system strength in relation to parametric values while the MIESCR is more applicable in computer studies where one is interested in finding the system strength corresponding to a particular system response obtained from the simulations.

### 6.1.6 Relationship between EESCR and MESCR

If a system MVA \( S_B \) is chosen and the numerator and denominator of (57-1) are divided by it, it becomes;

\[
\begin{align*}
\frac{S_{MVAl} - Q_{ci}}{P_{di} + \sum_{j=1, j\neq i}^{N} \frac{Z_{ij}}{Z_{ii}} \cdot P_{dj}} &= \frac{S_{MVAl} - Q_{ci}}{P_{di} + \sum_{j=1, j\neq i}^{N} \Delta V_j / \Delta V_i \cdot \Delta V_i / \Delta V_i} = MIESCR
\end{align*}
\]

Thus, the EESCR and MIESCR are mathematically equivalent though the former and latter are respectively parameter-based and response-based definitions. Therefore, the EESCR is suitable for analytical studies where one is interested in the system strength in relation to parametric values while the MIESCR is more applicable in computer studies where one is interested in finding the system strength corresponding to a particular system response obtained from the simulations.
Thus the \( EESCR \) and \( MESC \) are also mathematically equivalent and both are parameter-based quantities. Note that although they are equivalent, there is a subtle difference in the interpretation of their definitions.

### 6.1.7 Application to dual-converter multi-infeed hvdc system

As an example in applying (54)-(56), consider the dual-converter multi-infeed hvdc system in Fig.4. By using a step-by-step procedure to construct the \( Z_{bus} \) as shown in [13], the elements of the \( Z_{bus} \) of the multi-infeed hvdc system can be obtained as:

\[
Z_{11} = \frac{z_1 z_2^2 + z_2 z_3 (z_1 + z_2) + z_3 z_3 z_2 (z_1 + z_3) + z_3 z_3 z_2 (z_2 + z_3)}{(z_1 + z_2) z_p + z_3 (z_2 + z_3)}
\]

\[
Z_{12} = z_2 \frac{(z_3 + z_3 (z_1 + z_2) + z_3 z_3 z_2)}{(z_1 + z_2) z_p + z_3 (z_2 + z_3)}
\]

\[
Z_{21} = Z_{12}, \quad Z_{22} = z_2 \frac{(z_3 + z_3 (z_1 + z_2) + z_3 z_3 z_2)}{(z_1 + z_2) z_p + z_3 (z_2 + z_3)}
\]  
(80)

### Converters electrically far apart

Then from (80) \( Z_{ij}/Z_{ii} \) can be expressed as;

\[
Z_{23} = \frac{z_3 (1 + \frac{z_1 + z_2}{z_3})}{(1 + \frac{z_3}{z_3}) (z_1 + z_2) + z_3}
\]

\[
Z_{32} = \frac{z_2 (1 + \frac{z_1 + z_2}{z_3})}{(1 + \frac{z_2}{z_3}) (z_1 + z_2) + z_2}
\]

Now consider the two converters being electrically very far apart so that \( z_1 \) tends to infinity in the limit and further, if the converters are completely decoupled then \( z_1 \) or \( z_2 \) must also tend to infinity in the limit. Thus;

\[
\lim_{z_1 \to \infty} Z_{23} = 0 \quad \text{and similarly} \quad \lim_{z_1 \to \infty} Z_{32} = 0
\]

and from (81);

\[
\lim_{z_2 \to \infty} z_1 = \frac{1}{2}, \quad \lim_{z_2 \to \infty} z_2 = \infty, \quad \lim_{z_2 \to \infty} z_3 = \frac{1}{2}
\]  
(82)

and from (69),

\[
p_{di \_eq} = p_{di}, \quad Q_{di \_eq} = Q_{di}, \quad \frac{\partial p_{di \_eq}}{\partial v_i} = \frac{\partial p_{di}}{\partial v_i}, \quad \frac{\partial Q_{di \_eq}}{\partial v_i} = \frac{\partial Q_{di}}{\partial v_i}
\]  
(83)

As seen from (82) and (83), the equivalent ac and dc quantities reduce to those corresponding to the \( i \)-th constituent ac/dc system. Thus, the equivalent single-infeed hvdc system reduces to the decoupled constituent ac/dc system of the dual-converter multi-infeed hvdc system as shown in Fig.9a.

### Converters electrically near together

From (80), \( Z_{ij}/Z_{ii} \) can also be alternatively expressed as;

\[
Z_{23} = \frac{z_3 (z_1 + z_2 + z_3)}{(z_3 + z_3 (z_1 + z_2) + z_3 z_3 z_2)}, \quad Z_{32} = \frac{z_2 (z_1 + z_2 + z_3)}{(z_2 + z_3 (z_1 + z_2) + z_2 z_3 z_2)}
\]
Now consider the converters being electrically very near together so that \( z_3 \) tends to zero in the limit and \( \delta_{i}=0, \ V_i=V_j \). Thus,

\[
\lim_{z_3 \to 0} \frac{Z_{23}}{Z_{22}} = \lim_{z_3 \to 0} \frac{Z_{32}}{Z_{33}} = 1
\]

and from (81);

\[
\lim_{z_3 \to 0} \tau_1 = \frac{z_1}{z_1+ z_2}, \quad \lim_{z_3 \to 0} \tau_2 = 0, \quad \lim_{z_3 \to 0} \tau_3 = 0 \tag{84}
\]

and so from (69),

\[
P_{di\_eq} = P_{di} + P_{dj}, \quad Q_{di\_eq} = Q_{di} + Q_{dj}, \quad \frac{\partial P_{di\_eq}}{\partial V_i} = \frac{\partial P_{di}}{\partial V_i} + \frac{\partial P_{dj}}{\partial V_j}, \quad \frac{\partial Q_{di\_eq}}{\partial V_i} = \frac{\partial Q_{di}}{\partial V_i} + \frac{\partial Q_{dj}}{\partial V_j} \tag{85}
\]

As seen from (84) and (85), the equivalent quantities reduce to those corresponding to the parallel combination of the \( i \)-th and \( j \)-th constituent ac/dc system. Thus, the equivalent single-infeed hvdc system reduces to the amalgamated constituent ac/dc systems of the dual-converter multi-infeed hvdc system as shown in Fig.9b.

![Diagram](image)

*Fig.9: Reduction of dual-converter multi-infeed hvdc system into equivalent single-infeed configuration*

Although the results of (82) and (85) are derived from a voltage-current model of the dual-converter multi-infeed hvdc system, they are identical to those derived from a power-voltage model in Section 4.3.5-4.3.6 of [3]. Such consistency lends credence to the theoretical foundations of representing the multi-infeed hvdc system by a equivalent single-infeed hvdc configuration, as presented in this Section.
7 REFERENCES


