Comparison of the Efficiency of Dynamic Phasor Models Derived from ABC and DQ0 Reference Frame in Power System Dynamic Simulations

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***Abstract*— Generally model equations of power system components such as transmission lines, transformers and loads etc. are given in the ABC reference frame. But for components with rotating masses such as synchronous machines, induction motors etc., the DQ0 reference frame is usually preferred. In this paper dynamic phasor models of synchronous machines and transmission lines will be derived in ABC and DQ0 reference frame. An efficiency and accuracy comparison in simulations will be made with SMIB test system between these differently derived dynamic phasor models.**

***Index Terms*— Dynamic Phasors, Time-domain Simulations, Transient simulations, EMT Simulations.**

I. Introduction

There are two major modeling techniques used in the simulation of power system dynamics. One is the ABC (Three-Phase) modeling technique, which is commonly used in "EMTP-like" detailed time domain simulations (e.g. PSCAD/EMTDC) [1]). Simulations with ABC reference frame models are accurate but inefficient due to the presence of AC quantities even at steady state conditions. Some production grade programs (e.g. SIMPOW [2]), prefer to use the DQ0 reference frame models for detailed time domain simulations. The advantage of using DQ0 based models for simulation is, that under balanced conditions (where we have only positive sequence quantities in the system) and with frequencies near to the system frequency the variations of the DQ0 transformed quantities are much slower than in the original ABC quantities, so that larger numerical integration step sizes can be used during numerical simulation. But if there are unbalanced conditions or other harmonics in the system, this advantage disappears, as the single reference DQ0 transformation is unable to simulate these harmonics efficiently.

In power systems the original phase quantities are periodical or nearly periodical. The idea behind dynamic phasors approach is now to approximate a system with nearly periodic quantities, with an appropriate set of time varying Fourier coefficients, which have slower variations but nevertheless reflect the system behavior very accurately. There has been many applications of this approach in recent years. In [3], the dynamic phasors approach has been applied to synchronous and induction machines. Reference [4] treats the application of the same approach to simulate asymmetrical faults in power systems. The application of dynamic phasors approach to power electronic based equipment has been treated in detail in [5], where a fundamental phasor TCSC model has been derived by selecting first, third and fifth Fourier coefficients as an appropriate approximation for the capacitor voltage.

In [6], a systematic comparison between mostly used modeling techniques (ABC and DQ0) and the phasor dynamics approach has been made, which is illustrated in Figure 1. In [6], the dynamic phasor models have been derived from the DQ0 based models, since the synchronous machine model equations are generally given in DQ0 reference frame [7]. The aim in the current work is to implement also the dynamic phasor models derived from the ABC reference frame and compare their efficiency and accuracy to those derived from DQ0 reference frame.

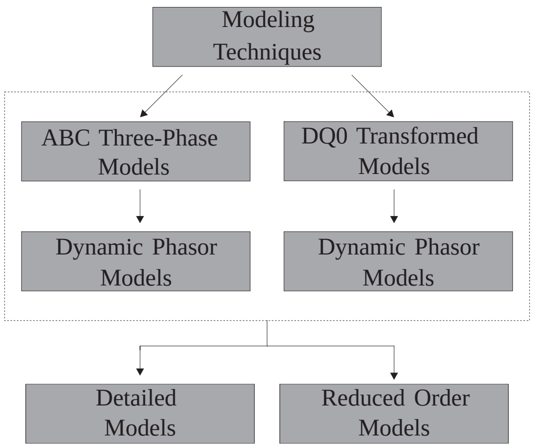

Fig. 1: Modeling Techniques

The paper is structured in the following way. In Section II, the concept of time varying Fourier coefficients is introduced. In Section III, the described analytical tool is used to derive models of some major power system components. Section IV shows some results with SMIB test case and makes a systematic efficiency and accuracy comparison between the two modeling techniques.

II. Time Varying Fourier Coefficients

A real valued periodic signal with period T [e.g. $x(\tau) = x(\tau - T)$] can be expressed with a Fourier series representation of the form given as

$$x(\tau) = Re\left\{\sum_{k=0}^{\infty} X_k \cdot e^{j\,k\,\omega_s\,\tau}\right\}$$

where $\omega_s = 2\pi/T$ and X_k is the k^{th} Fourier coefficient in complex form. It's important to note, since the signal $x(\tau)$ is periodic, the Fourier coefficients X_k are time invariant. In power system steady-state analysis, these Fourier coefficients X_k are also referred as *phasors*.

During transients, we don't have pure periodic but *nearly periodic signals*. The idea is now to extend this approach to nearly periodic signals [8] and to approximate $x(\tau)$ in the interval $\tau \in (t-T, t]$ with a Fourier series representation of the form given in (1).

$$x(\tau) = Re\left\{\sum_{k=0}^{\infty} X_k(t) \cdot e^{j\,k\,\omega_s\,\tau}\right\} \qquad (1)$$

In this representation, as the signal $x(\tau)$ is nearly periodic and since the interval under consideration slides as a function of time, the Fourier coefficients X_k are time varying. These time varying Fourier coefficients are also referred as *dynamic phasors*.

In this approach we are interested in cases where only a few coefficients provide a good approximation of the original waveform. $X_k(t)$ can be determined by the following averaging operations.

$$X_k(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau) \cdot e^{-j\,k\,\omega_s\,\tau} d\tau = \langle x \rangle_k (t) \qquad (2)$$

As notation, here lowercase letters $x(\tau)$ are used for instantaneous variables and uppercase letters $X_k(t)$ for dynamic phasors. Some important properties of dynamic phasors are:

- The relation between the derivatives of $x(\tau)$ and the derivatives of $X_k(t)$, which is given in (3), where the time argument t has been omitted to avoid the notational cluster. This can easily be verified by differentiating the formula given in (1)

$$\langle d_t x \rangle_k = d_t X_k + j \cdot k \cdot \omega_s \cdot X_k \qquad (3)$$

- The product of two time-domain variables equals a discrete time convolution of the two dynamic phasor sets of the variables, which is given in (4).

$$\langle x \cdot y \rangle_k = \sum_{l=-\infty}^{\infty} (X_{k-l} \cdot Y_l) \qquad (4)$$

- For real valued signal x, the relationship between X_k and X_{-k} is given as:

$$X_{-k} = \text{conjugate}(\, X_k \,) = X_k^* \qquad (5)$$

Here we have used the complex form representation of the equations. With $X_k(t) = X_{k,c}(t) - j \cdot X_{k,s}(t)$, these can easily be transformed into real form representation given in Appendix II, which are also used in MATLAB implementation.

The dynamic phasors approach offers a numbers of advantages over conventional methods.

+ The selection of k gives a wider bandwidth in the frequency domain than traditional slow quasi-stationary assumptions used in Transient Stability Programs.
+ As the variations of dynamic phasors X_k are much slower than the instantaneous quantities x, they can be used to compute the fast electromagnetic transients with larger numerical integration step sizes, so that it makes simulation potentially faster than conventional time domain EMTP like simulation.
+ The time domain simulations of such large systems with periodically switched power electronic based components have not only a computational burden, but also give little insight into the problem sensitivities to design controllers or protection schemes. Dynamic phasors approach also allows an analytical insight into such problems, as it approximates a periodically switched system with a continuous system.
– One disadvantage of the dynamic phasors approach is that the number of variables and equations in phasor dynamics approach is higher than in the original equations, which makes the simulation inefficient.

III. Power System Component Models

In the used simulation framework [6], the model equations of the power system components (in ABC and DQ0 reference frame) are normally given as

$$\begin{aligned} \dot{x} &= f(x, y) \\ 0 &= g(x, y) \end{aligned} \qquad (6)$$

in the **D**ifferential **S**witched-**A**lgebraic State-**R**eset (DSAR) structure [9]. Using the appropriate approximations for x, y in Equation (1) and the properties (2-5) in real form given in Appendix II, we can transform the set of f and g equations of the model into a new set of equations and get the definition of the dynamic phasor model in a new set of functions F and G as

$$\begin{aligned} \dot{X}_{k,c} &= F_{k,c}(X_{k,c}, X_{k,s}, Y_{k,c}, Y_{k,s}) - k \cdot \omega_s \cdot X_{k,s} \\ \dot{X}_{k,s} &= F_{k,s}(X_{k,c}, X_{k,s}, Y_{k,c}, Y_{k,s}) + k \cdot \omega_s \cdot X_{k,c} \\ 0 &= G_{k,c}(X_{k,c}, X_{k,s}, Y_{k,c}, Y_{k,s}) \\ 0 &= G_{k,s}(X_{k,c}, X_{k,s}, Y_{k,c}, Y_{k,s}) \end{aligned} \qquad (7)$$

where dynamic phasors X_k become the new continuous dynamic states and Y_k the new algebraic states.

In this section we will derive the dynamic phasor models of simple RL transmission line and synchronous machine from ABC and DQ0 reference frame. In [4] a similar derivation was made based on the space vector representation. Here we will use the DQ0 representation of the model equations.

	ABC	$DQ0$
P	$\begin{bmatrix} \cos(\omega_s t + \theta_p) \\ \cos\left(\omega_s t + \theta_p - \frac{2\pi}{3}\right) \\ \cos\left(\omega_s t + \theta_p + \frac{2\pi}{3}\right) \end{bmatrix}$	$\begin{bmatrix} \cos(\theta_p) \\ \sin(\theta_p) \\ 0 \end{bmatrix}$
N	$\begin{bmatrix} \cos(\omega_s t + \theta_n) \\ \cos\left(\omega_s t + \theta_n + \frac{2\pi}{3}\right) \\ \cos\left(\omega_s t + \theta_n - \frac{2\pi}{3}\right) \end{bmatrix}$	$\begin{bmatrix} \cos(2\,\omega_s t + \theta_n) \\ -\sin(2\,\omega_s t + \theta_n) \\ 0 \end{bmatrix}$
Z	$\begin{bmatrix} \cos(\omega_s t + \theta_z) \\ \cos(\omega_s t + \theta_z) \\ \cos(\omega_s t + \theta_z) \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ \cos(\omega_s t + \theta_z) \end{bmatrix}$

TABLE I: DQ0 transformations of sequence components

If we are deriving models in DQ0 reference frame, we have to consider that under unbalanced conditions the system quantities will contain all three sequence components (Positive, Negative and Zero Sequence). As the Table I shows, the DQ0 transformation of the negative sequence component contains $\cos(2\omega_s t)$ and $\sin(2\omega_s t)$ terms and the DQ0 transformation of the zero sequence component contains $\cos(\omega_s t)$ and $\sin(\omega_s t)$ terms. During unbalanced conditions, we observe oscillations with double system frequency $2\,\omega_s$ in 'd' and 'q' coordinates, and oscillations with system frequency ω_s in '0' coordinate of DQ0 transformed quantities. Due to this fact, an appropriate selection for k in Equation (1) is

- $k = 0$ to include positive sequence quantities
- $k = 1$ to include zero sequence quantities
- $k = 2$ to include negative sequence quantities.

A. Transmission Line

1) ABC Reference Frame: The simple RL transmission line model equations in ABC reference frame is can be given as

$$L_{abc} \cdot d_t i_{abc} = e_{abc} - R_{abc} \cdot i_{abc} \tag{8}$$

with $e_{abc} = \begin{bmatrix} e_a & e_b & e_c \end{bmatrix}$, $i_{abc} = \begin{bmatrix} i_a & i_b & i_c \end{bmatrix}$, $R_{abc} = diag[R_a, R_b, R_c]$ and $L_{abc} = diag[L_a, L_b, L_c]$. Here we used different resistance and different inductance values for each phase to allow also asymmetrical component models. For ease of formulation the mutual inductances has been ignored, but these can easily be included in the model.

2) DQ0 Reference Frame: The simple RL transmission line model in DQ0 reference frame can easily be derived by applying the DQ0 transformation to the Equation (8). The DQ0 transformation matrix T_{dq0} is given as

$$T_{dq0} = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin(\theta) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

with $\theta = \omega_s t$. After the transformation the transmission line model equations in DQ0 reference become

$$L_{dq0} \cdot d_t i_{dq0} = e_{dq0} - R_{dq0} \cdot i_{dq0} + J_3 \cdot \omega_s \cdot L_{dq0} \cdot i_{dq0} \tag{9}$$

with $e_{dq0} = \begin{bmatrix} e_d & e_q & e_0 \end{bmatrix}$, $i_{dq0} = \begin{bmatrix} i_d & i_q & i_0 \end{bmatrix}$ and

$$J_3 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The expression for $L_{dq0}(\theta)$ is given in Appendix I-A. $R_{dqo,0}$ has also the same structure. It is important to note the time dependence ($\theta = \omega_s t$) and periodicity ($cos(\theta), cos(2\theta)$) of L_{dqo} due to the DQ0 transformation. But if the transmission line is symmetrical ($L_a = L_b = L_c$), the L_{dqo} equals to $diag[L_a, L_a, L_a]$, which simplifies the model equation (9) in DQ0 reference frame.

B. Synchronous Machine

The synchronous machine model equations in DQ0 reference frame are taken from [7] Chapter 3. The model equations in DQ0 and ABC reference frame are both based on these equations.

1) DQ0 Reference Frame: The per unit synchronous machine model equations in DQ0 reference frame in [7] can be written in the following form

$$\begin{aligned} d_t\Psi &= \omega_s \cdot (V + \omega_r \cdot J \cdot \Psi + R \cdot I) \\ d_t\omega_r &= \frac{1}{2H} \cdot (T_m - T_e - K_D \cdot \omega_r) \\ d_t\delta_r &= \omega_s \cdot (\omega_r - 1) \\ \Psi &= L \cdot I \\ T_e &= (\psi_d \cdot i_q - \psi_q \cdot i_d) \end{aligned} \tag{10}$$

with

$$\begin{aligned} \Psi &= \begin{bmatrix} \psi_d & \psi_q & \psi_0 & \psi_{fd} & \psi_{1d} & \psi_{1q} & \psi_{1q} \end{bmatrix}^T \\ I &= \begin{bmatrix} i_d & i_q & i_0 & i_{fd} & i_{1d} & i_{1q} & i_{1q} \end{bmatrix}^T \\ V &= \begin{bmatrix} e_d & e_q & e_0 & e_{fd} & 0 & 0 & 0 \end{bmatrix}^T \\ R &= \begin{bmatrix} R_a & R_a & R_a & R_{fd} & R_{1d} & R_{1q} & R_{1q} \end{bmatrix}^T \\ J &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

where the indices $d, q, 0$ refer to the stator quantities transformed to the rotor reference frame and

$fd, 1d, 1q, 2q$ to rotor quantities. The complete set of equations can be found in [7] on pages 86-87.

It is important to note, that the quantities $[e_d, e_q, e_0]^r$ and $[i_d, i_q, i_0]^r$ in these equations are in rotor DQ0 reference frame and have to be transformed into the stator DQ0 reference frame to be interfaced with the network quantities in stator DQ0 reference frame.

2) ABC Reference Frame: Again taking the equations in [7] as reference, the synchronous machine model equations in ABC reference frame can be expressed as

$$\begin{aligned} d_t\Psi &= \omega_s \cdot (V - R \cdot I) \\ d_t\omega_r &= \frac{1}{2H} \cdot (T_m - T_e - K_D \cdot \omega_r) \\ d_t\delta_r &= \omega_s \cdot (\omega_r - 1) \end{aligned}$$

$$\begin{aligned} \Psi &= L(\theta, \delta_r) \cdot I \\ T_e &= \frac{2\sqrt{3}}{9} \begin{pmatrix} i_a \cdot (\psi_c - \psi_b) + \\ i_b \cdot (\psi_a - \psi_c) + \\ i_c \cdot (\psi_b - \psi_a) \end{pmatrix} \end{aligned} \quad (11)$$

with

$$\begin{aligned} \Psi &= \begin{bmatrix} \psi_a & \psi_b & \psi_c & \psi_{fd} & \psi_{1d} & \psi_{1q} & \psi_{1q} \end{bmatrix}^T \\ I &= \begin{bmatrix} i_a & i_b & i_c & i_{fd} & i_{1d} & i_{1q} & i_{1q} \end{bmatrix}^T \\ V &= \begin{bmatrix} e_a & e_b & e_c & e_{fd} & 0 & 0 & 0 \end{bmatrix}^T \\ R &= \begin{bmatrix} R_a & R_a & R_a & R_{fd} & R_{1d} & R_{1q} & R_{1q} \end{bmatrix}^T \end{aligned}$$

where the indices a, b, c refer to the stator quantities and $fd, 1d, 1q, 2q$ to rotor quantities. The expression for $L(\theta, \delta_r)$ is given in Appendix I-B.

C. Dynamic Phasor Models

With the model equations (8,11) in ABC reference frame and (9,10) in DQ0 reference frame, we can derive now the dynamic phasor model equations of the power system components by using the Equations (1-5). A key point in the derivation of the dynamic phasor models is the appropriate selection of set k in Equation (1) for an adequate approximation of the model behavior. As stated before we have selected $k = 0, 1, 2$ for both modeling techniques in our derivations.

With $k = 0, 1, 2$ in Equations (1-5), all the model equations (8-11) can be transformed into the form given in (7).

IV. Simulation Results

As test case, we have used the "Single Machine Infinite Bus" (SMIB) system in [7] with minor modifications. Simulations have been made with both, *detailed* models and *reduced order* models. The detailed models capture both fast dynamics due to the electromagnetic phenomena, and slow dynamics due to the electromechanical phenomena. The reduced order models neglect the electromagnetic phenomena

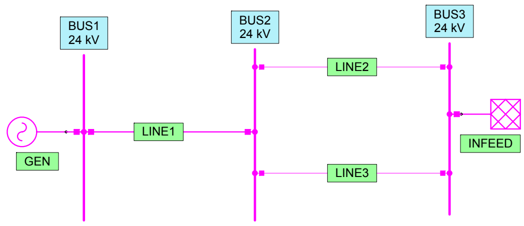

Fig. 2: Single-line diagram of SMIB system

and capture only the dynamics of the slow electromechanical phenomena as in the transient stability programs. The derivation of reduced order models from the detailed models is done by performing an eigenanalysis and analyzing the eigenvalues and their participation factors. Dynamics of continuous states with very low participation in the slow modes are neglected, which means for those continuous dynamic states the derivatives (d_tX) in model equations are set to zero and so they become algebraic states (Y). This leads to neglecting the fast transients due to the electromagnetic phenomena in the power system.

A. Simulations with Detailed Models

1) Unbalanced Fault: In this test case, a single-phase to ground fault is applied at BUS2 at 0.1 seconds and is removed after 0.07 seconds. Figures 3 and 4 show the simulation results of the test case using

- ABC reference frame models
- DQ0 reference frame models
- Dynamic Phasor models derived from ABC
- Dynamic Phasor models derived from DQ0

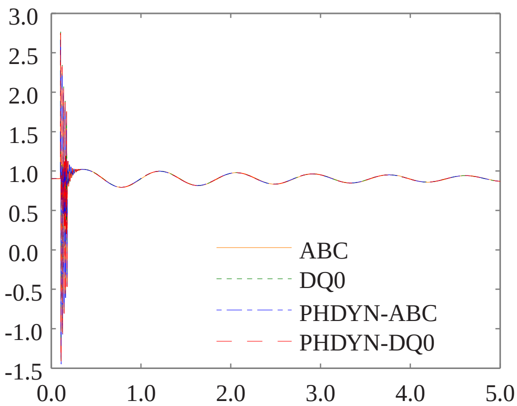

Fig. 3: Simulation of single phase to ground fault with detailed models (whole simulation)

ABC	DQ0	PHYDYN-ABC	PHYDYN-DQ0
238.1563	19.375	36.1719	24.6406

TABLE II: Simulation times in seconds - single phase to ground fault with detailed models

All plots in the Figures 3 and 4 show overall agreement. But if we compare the simulation times, which

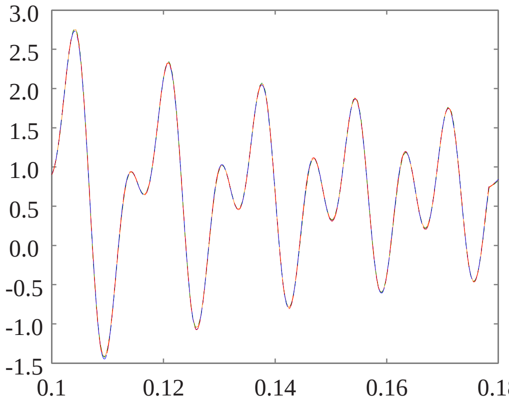

Fig. 4: Simulation of single phase to ground fault with detailed models (during fault)

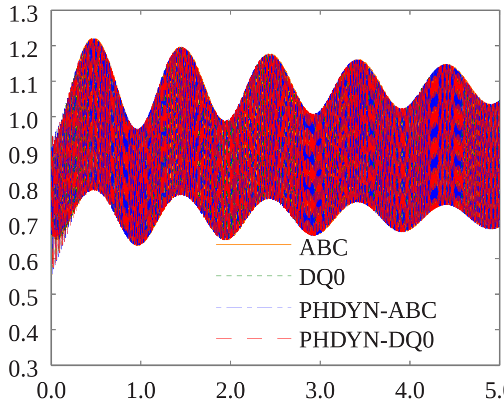

Fig. 5: Simulation of asymmetrical components with detailed models (whole simulation)

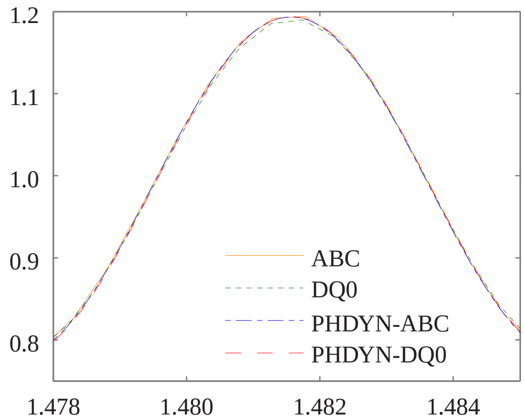

Fig. 6: Simulation of asymmetrical components with detailed models (zoomed section)

are given in Table II, we observe performance differences between different modeling techniques. In this example, the DQ0 is the most efficient modeling technique as the system is only 0.07 seconds in an unbalanced condition and in the remaining 4.93 seconds in a balanced condition. With symmetrical components and under balanced conditions DQ0 reference frame seems to be the best choice for efficient simulation, since there are only positive sequence quantities in the system and the variations of these DQ0 transformed quantities are much slower than the variations of the original ABC quantities. The dynamic phasor models derived from ABC and DQ0 are also much faster than the true physical ABC reference frame models. They are slower than the DQ0 reference frame models because of the increased number of states and equations. If we compare the efficiency of the two different dynamic phasor models (ABC - DQ0), we see that the DQ0 based models show a better performance, since the system is under balanced conditions most of the time.

2) Asymmetrical Components: This time, we change the inductance L in phase-c of LINE1 right at the beginning of the simulation ($t = 0.0001 sec$) and put the system under unbalanced conditions during the whole simulation.

All plots in the Figures 5 and 6 show overall agreement. The comparison of the simulation times (Table III) shows a change in the performance of some modeling techniques. The immense performance loss

ABC	DQ0	PHYDYN-ABC	PHYDYN-DQ0
214.6875	291.0781	22.2656	23.8438

TABLE III: Simulation times in seconds - asymmetrical components with detailed models

of DQ0 models attracts attention. In contrast to the previous case, we have unbalanced conditions during the whole simulation interval, which is not suitable for DQ0 models. The two dynamic phasor models (ABC-DQ0) show again a much better performance concerning the computational efficiency compared to the original ABC and DQ0 models. If we compare the performance of the dynamic phasor models derived from ABC and DQ0, we observe, that the ABC based models show a slightly better performance than the DQ0 based models. This results from the performance loss of the DQ0 based dynamic phasor models due to the permanent unbalanced conditions.

B. Simulations with Reduced Order Models

1) Unbalanced Fault: In this simulation we have used the same data and same fault as in Section IV-A.1, but this time we replaced the detailed models with the reduced order models. The reduced order models neglect the electromagnetic phenomena and capture only the electromechanical slow dynamics. This leads to 5-6 times faster simulation times than with detailed models. Figures 7 and 8 show the results with reduced order dynamic phasor models derived from ABC and DQ0 models. Due to the dominating balanced conditions in the system, the DQ0 based dynamic phasor models are faster than the ABC based models (see Table IV), which has been also observed in Section IV-A.1.

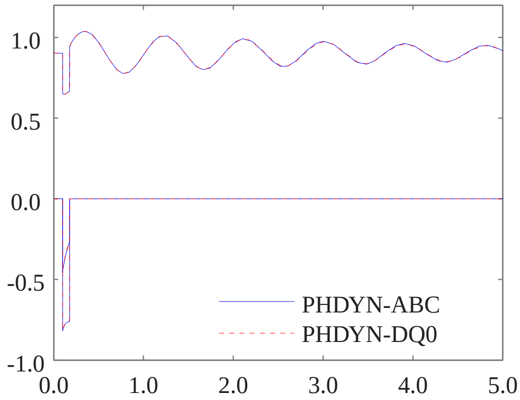

Fig. 7: Simulation of single phase to ground fault with reduced order models (whole simulation)

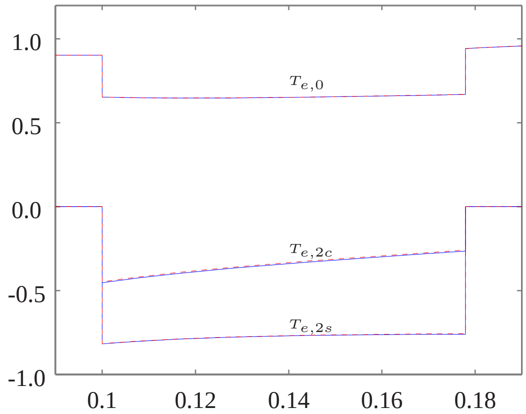

Fig. 8: Simulation of single phase to ground fault with reduced order models (during fault)

PHYDYN-ABC	PHYDYN-DQ0
6.4375	3.9531

TABLE IV: Simulation times in seconds - single phase to ground fault with reduced order models

2) Asymmetrical Components: In this simulation we have used reduced order models with the same data used as in Section IV-A.2. Figure 9 shows the results

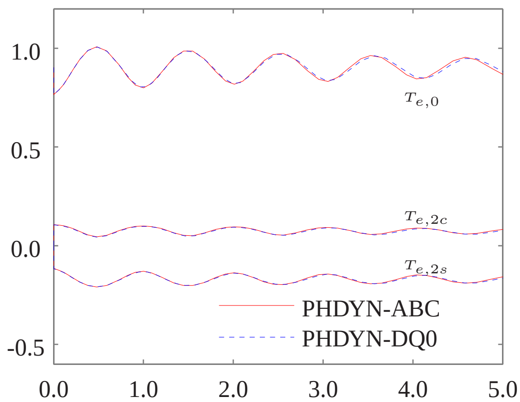

Fig. 9: Simulation of asymmetrical components with reduced order models (whole simulation)

PHYDYN-ABC	PHYDYN-DQ0
6.025	7.250

TABLE V: Simulation times in seconds - asymmetrical components with reduced order models

of the simulation and Table V simulation times. Both modeling techniques give the same result and in case of asymmetrical components the ABC based dynamic phasor models seem to be slightly faster then the DQ0 based model. Similar result has been observed also in Section IV-A.2.

V. Conclusions

In this paper we have derived dynamic phasor models of major power system components such as synchronous machines, transmission lines etc. from ABC reference frame and DQ0 reference frame models and compared their performance in simulations. In systems, where the balanced conditions are dominating, the DQ0 based dynamic phasor models are more efficient. But in systems with unbalanced conditions (e.g. with asymmetrical components) the ABC based dynamic phasor models show slightly better performance. Besides its efficiency and accuracy, the dynamic phasor modeling technique is also a systematic and analytical tool in modeling of power system components.

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Appendix I
Details of Model Equations

A. L_{dq0} in Transmission Line Equations in DQ0 reference frame

$$
\begin{aligned}
L_0 &= \frac{1}{3}\left(L_a + L_b + L_c\right) \\
L_\alpha &= \frac{1}{3}\left(2L_a - L_b - L_c\right) \\
L_\beta &= \frac{1}{\sqrt{3}}\left(L_b - L_c\right) \\
L_{dqo} &= L_{dqo,0} + L_{dqo,1c}\cos(\theta) + L_{dqo,1s}\sin(\theta) + L_{dqo,2c}\cos(2\theta) + L_{dqo,2s}\sin(2\theta) \\
L_{dqo,0} &= \begin{bmatrix} L_0 & 0 & 0 \\ 0 & L_0 & 0 \\ 0 & 0 & L_0 \end{bmatrix} \\
L_{dqo,1c} &= \begin{bmatrix} 0 & 0 & L_\alpha \\ 0 & 0 & L_\beta \\ \frac{1}{2}L_\alpha & \frac{1}{2}L_\beta & 0 \end{bmatrix}, L_{dqo,1s} = \begin{bmatrix} 0 & 0 & L_\beta \\ 0 & 0 & -L_\alpha \\ \frac{1}{2}L_\beta & -\frac{1}{2}L_\alpha & 0 \end{bmatrix} \\
L_{dqo,2c} &= \begin{bmatrix} \frac{1}{2}L_\alpha & -\frac{1}{2}L_\beta & 0 \\ -\frac{1}{2}L_\beta & -\frac{1}{2}L_\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}, L_{dqo,2s} = \begin{bmatrix} -\frac{1}{2}L_\beta & -\frac{1}{2}L_\alpha & 0 \\ -\frac{1}{2}L_\alpha & \frac{1}{2}L_\beta & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

B. L in Synchronous Machine Equations in ABC reference frame

$$
\begin{aligned}
L &= \begin{bmatrix} L_{ss} & L_{sr} \\ L_{rs} & L_{rr} \end{bmatrix} \\
L_{ss} &= L_{ss,0} + L_{ss,2c}\cos(2\theta) + L_{ss,2s}\sin(2\theta) \\
L_{sr} &= L_{sr,1c}\cos(\theta) + L_{sr,1s}\sin(\theta) \\
L_{rs} &= L_{rs,1c}\cos(\theta) + L_{rs,1s}\sin(\theta) \\
L_{rr} &= L_{rr,0}
\end{aligned}
$$

$$
\begin{aligned}
L_{ss,0} &= \begin{bmatrix} -\frac{1}{3}\cdot\left(L_{ad} + L_0 + L_{aq} + 2L_l\right) & \frac{1}{6}\left(L_{aq} + L_{ad} - 2L_0 + 2L_l\right) & \frac{1}{6}\left(L_{aq} + L_{ad} - 2L_0 + 2L_l\right) \\ \frac{1}{6}\left(L_{aq} + L_{ad} - 2L_0 + 2L_l\right) & -\frac{1}{3}\cdot\left(L_{ad} + L_0 + L_{aq} + 2L_l\right) & \frac{1}{6}\left(L_{aq} + L_{ad} - 2L_0 + 2L_l\right) \\ \frac{1}{6}\left(L_{aq} + L_{ad} - 2L_0 + 2L_l\right) & \frac{1}{6}\left(L_{aq} + L_{ad} - 2L_0 + 2L_l\right) & -\frac{1}{3}\cdot\left(L_{ad} + L_0 + L_{aq} + 2L_l\right) \end{bmatrix} \\
L_{rr,0} &= \begin{bmatrix} L_{ad} + L_{fd} & L_{ad} & 0 & 0 \\ L_{ad} & L_{ad} + L_{1d} & 0 & 0 \\ 0 & 0 & L_{aq} + L_{1q} & L_{aq} \\ 0 & 0 & L_{aq} & L_{aq} + L_{2q} \end{bmatrix} \\
L_{sr,1c} &= \begin{bmatrix} L_{ad}\cos(\delta_r) & L_{ad}\cos(\delta_r) & -L_{aq}\sin(\delta_r) & -L_{aq}\sin(\delta_r) \\ L_{ad}\cos(\delta_r - \frac{2\pi}{3}) & L_{ad}\cos(\delta_r - \frac{2\pi}{3}) & -L_{aq}\sin(\delta_r - \frac{2\pi}{3}) & -L_{aq}\sin(\delta_r - \frac{2\pi}{3}) \\ L_{ad}\cos(\delta_r + \frac{2\pi}{3}) & L_{ad}\cos(\delta_r + \frac{2\pi}{3}) & -L_{aq}\sin(\delta_r + \frac{2\pi}{3}) & -L_{aq}\sin(\delta_r + \frac{2\pi}{3}) \end{bmatrix} \\
L_{sr,1s} &= \begin{bmatrix} -L_{ad}\sin(\delta_r) & -L_{ad}\sin(\delta_r) & -L_{aq}\cos(\delta_r) & -L_{aq}\cos(\delta_r) \\ -L_{ad}\sin(\delta_r - \frac{2\pi}{3}) & -L_{ad}\sin(\delta_r - \frac{2\pi}{3}) & -L_{aq}\cos(\delta_r - \frac{2\pi}{3}) & -L_{aq}\cos(\delta_r - \frac{2\pi}{3}) \\ -L_{ad}\sin(\delta_r + \frac{2\pi}{3}) & -L_{ad}\sin(\delta_r + \frac{2\pi}{3}) & -L_{aq}\cos(\delta_r + \frac{2\pi}{3}) & -L_{aq}\cos(\delta_r + \frac{2\pi}{3}) \end{bmatrix} \\
L_{rs,1c} &= -\frac{2}{3}L_{sr,1c}^T \\
L_{rs,1s} &= -\frac{2}{3}L_{sr,1s}^T \\
L_{ss,2c} &= -\frac{1}{3}\left(L_{ad} - L_{aq}\right)\cdot\begin{bmatrix} \cos(2\delta_r) & \cos\left(2\delta_r - \frac{2\pi}{3}\right) & \cos\left(2\delta_r + \frac{2\pi}{3}\right) \\ \cos\left(2\delta_r - \frac{2\pi}{3}\right) & \cos\left(2\delta_r + \frac{2\pi}{3}\right) & \cos(2\delta_r) \\ \cos\left(2\delta_r + \frac{2\pi}{3}\right) & \cos(2\delta_r) & \cos\left(2\delta_r - \frac{2\pi}{3}\right) \end{bmatrix} \\
L_{ss,2s} &= \frac{1}{3}\left(L_{ad} - L_{aq}\right)\cdot\begin{bmatrix} \sin(2\delta_r) & \sin\left(2\delta_r - \frac{2\pi}{3}\right) & \sin\left(2\delta_r + \frac{2\pi}{3}\right) \\ \sin\left(2\delta_r - \frac{2\pi}{3}\right) & \sin\left(2\delta_r + \frac{2\pi}{3}\right) & \sin(2\delta_r) \\ \sin\left(2\delta_r + \frac{2\pi}{3}\right) & \sin(2\delta_r) & \sin\left(2\delta_r - \frac{2\pi}{3}\right) \end{bmatrix}
\end{aligned}
$$

Appendix II
Fourier coefficients in real form

With $X_k(t) = X_{k,c}(t) - j\,X_{k,s}(t)$ the equation (1) becomes

$$x(\tau) = \sum_{k=0}^{\infty} \left(X_{k,c}(t) \cdot \cos(k\,\omega_s\,\tau) + X_{k,s}(t) \cdot \sin(k\,\omega_s\,\tau) \right) \tag{12a}$$

the equation (2) becomes

$$X_{k,c}(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau) \cdot \cos(k\,\omega_s\,\tau) \cdot d\tau = \langle x \rangle_{k,c}(t) \tag{12b}$$

$$X_{k,s}(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau) \cdot \sin(k\,\omega_s\,\tau) \cdot d\tau = \langle x \rangle_{k,s}(t) \tag{12c}$$

the equation (3) becomes

$$\langle d_t x \rangle_{k,c} = d_t X_{k,c} + k \cdot \omega_s \cdot X_{k,s} \tag{13a}$$

$$\langle d_t x \rangle_{k,s} = d_t X_{k,s} - k \cdot \omega_s \cdot X_{k,c} \tag{13b}$$

the equation (4) becomes

$$\langle x \cdot y \rangle_{k,c} = \frac{1}{2} \cdot \sum_{l=0}^{\infty} \left(X_{k-l,c} \cdot Y_{l,c} + X_{k-l,s} \cdot Y_{l,s} \right) \tag{14a}$$

$$\langle x \cdot y \rangle_{k,s} = \frac{1}{2} \cdot \sum_{l=0}^{\infty} \left(X_{k-l,c} \cdot Y_{l,s} + X_{k-l,s} \cdot Y_{l,c} \right) \tag{14b}$$

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