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Simulation of the space charge near coronating conductors of ac overhead transmission lines

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Abstract
In the case of ac high voltage overhead transmission lines, corona discharges may result in a broadband crackling and a humming noise at twice the mains frequency ($2f$). The computation of the emitted $2f$-levels presupposes the knowledge of the apparent power and force passed by the ions to the gas in the surroundings of the conductors. In this paper a simulation determining these magnitudes used in a previous publication is presented. For that purpose the ideas of earlier publications by other authors to compute corona losses are followed. Their central idea is to define an onset field strength on the surface of the conductor which is not exceeded by injecting the right amount of charges in the vicinity of the conductor. This injection together with the recombination determines the amount of drifting ions. The novelty of the method applied in this work is the consideration of different onset field strengths over the conductor surface in dependence of the polar angle. Further $2f$-force and $2f$-power acting on the gas in the vicinity of the conductor are determined out of the resulting ion drift. The corona currents thus computed coincide fairly well with measured results.

1. Introduction

The quantification of the tonal emission ($2f$) of ac corona presupposes the knowledge of $2f$-force and $2f$-power transferred from the ions to the gas surrounding the conductors [1]. These quantities which are the sources of the $2f$-emission are confined mainly to the drift zone in the vicinity of the conductors [1]. Hence that is the location to which the simulation presented in this publication is tailored. On the one hand, the distribution of the ions and electrical field for the force and on the other hand, the number of ions and drift velocity for the power has to be determined. Therefore, the drift and the rate of production and recombination of the ions have to be accounted for. As [1], the work presented here is based on the dissertation [2].

Very closely correlated with the goal and the problem at hand is the subject of the quantification of corona losses of ac-transmission lines. For the determination of the latter various publications can be found. In the following passages only some selected references investigating the mathematical description of the corona loss are examined. Those can be modified in such a way as to determine not only the corona loss but also the momentary power and the force heating and accelerating the gas around the conductor.

The approach of Cladé et al [3–5]—whose contents can be found in a similar form in the text book of Gary et al in sections [6, 7]—starts with a critical field strength $E_c$ in a given situation of conductor geometries and condition, above which corona sets in. The ions produced by the corona couple back on the corona activity by shielding the conductor from the electrical field. Now in the approach mentioned, the quantity of produced ions is defined by not permitting the electrical field on the surface of the conductor to exceed $E_c$. Hence, the number of ions in the drift zone is determined by their screening effect on the conductors and the magnitude of $E_c$.

In [3] and [6], respectively, a strongly simplified formula following the mentioned approach is deducted, by which the corona loss can be calculated. This formula is restricted to cylindrical symmetric geometries. Further, the drift of the ions is not accounted for by placing the ions produced by corona on a fixed cylinder surface. This cylinder is, according to the symmetry, arranged coaxially to the conductor, and its radius is chosen with an estimate for the average distance of the ions in the drift zone to the conductor.
Further, for the determination of the underside and small drops on the upside of the conductor. It is therefore assumed that the critical field strength is constant (i.e. two-dimensional). This is a good approach as well [8, 9], but in them the electric field due to the drifting charges is accounted for by means of the charge simulation method. This method makes it possible to leave the restriction to cylindrical symmetric geometries. However, in these works $E_c$ is still considered to be cylindrical symmetric. It is therefore assumed that the critical field strength is constant over the surface of the conductor, regardless of the distribution of drops on the conductor with, for example, large drops on the underside and small drops on the upside of the conductor.

However, the asymmetric magnitude of $E_c$ over the outline of the conductors must be considered to reach the goals here. In section 3 the deduction of $E_c$ in dependence on the polar angle is shown. Only that way asymmetrically mounted protrusions on the conductor can be considered—for a constant, cylindrical symmetric $E_c$ the total force is always zero otherwise (apart from a small asymmetry due to the influence from the neighbouring conductor bundles).

Another limitation of this whole concept of describing the corona activity ought to be noted here. In all the approaches mentioned above, the physical quantities along the conductor, such as $E_c$, the actual surface gradient and the corona, are supposed to be constant (i.e. two-dimensional). This is a good approximation only, if the distances between the protrusions along the conductor are small compared with the distance of the space charge shielding the conductor from the electrical field (thereby controlling the corona-activity). In the case of well aged, hydrophilic conductors under rain [11], potentially featuring only few water drops, or in the case of polluted conductors with only few protrusions, it is doubtful if this symmetry along the conductor actually holds.

The numerical implementation performed in the work presented here reproduces the ion drift for two-dimensional geometries by following the ideas of [3–5] and [8, 9]. In particular, the drift of the ions is simplified, as represented by line charges restricted to the so-called drift paths determined by the electric field unperturbed by corona, i.e. the case of space-charge-free situations (section 2). These line charges move according to the drift velocity defined by the known relationships on these drift paths, where recombination is also accounted for (section 4).

This numerical method is then applied to the two setups investigated in [1], namely an aluminium tube (20 mm diameter) with asymmetrically mounted conical protrusions and an Aldrey 600 (AAAC-conductor with 600 mm² cross section) double bundle under rain (with intensity of 3.8 mm h⁻¹). Both conductors are investigated in a coaxially arranged cage with a radius of 1.5 m, where they are energized at 120 kV (conical protrusions) and 167 kV (double bundle), respectively. The resulting corona currents are compared with the measured ones and summarized together with the other results of the simulation (section 5).

2. Discretization of space and time

The transmission lines are approximated by their two-dimensional cross section perpendicular to the line axis, which is continued by translation symmetry in the line axis. As this geometry is two dimensional, the charges on the conductors and the drifting ions are represented with line charges. In the following, the view will be limited to the two-dimensional cross section. Hence the locations of the charges are usually referred to as points rather than lines.

An overview to the discretization of this two-dimensional geometry is given by the schematic representation of figure 1.

The surface of the subconductors is discretized in $m$ points (with index $k$), to which the drift paths are connected. The latter represent the spatial location to which the line charges representing the drifting ions are restricted. The drift paths themselves are discretized in $r$ points (with index $s$). In the points $k$ of the surface of the conductor the electrical field is calculated. In the case of excess of the critical electrical field $E_{c6}$ this point becomes corona active, which means that charges of the corresponding polarity are injected at the first
The point of the drift path attached to \( k \) in such a way that the electric potentials are fulfilled and \( E_{pk} \) is not exceeded. The ions thus produced drift on the drift-path along the points \( s \).

The choice of this discretization of the geometry results in the following simplifications and assumptions:

- The drift paths are defined by the integrals of the electrical field unperturbed by the drifting ions. Thus the drift paths are only determined in the absence of drifting ions. The back coupling of the drifting ions on the distribution of the electrical field is neglected.
- When a charge reaches one of the ends of the drift paths, that charge disappears without substitution. At the drift path end near the conductor, a neutralization of the ions on the conductor ensues. At the end far from the conductor, the charge is simply neglected. This seems to be justified by the results of the simulation (figure 7). According to the considerations of [2] the ions far from the conductor do not relevantly contribute to the \( 2f \)-power and force at issue here.
- The charges due to corona are always introduced at the beginning of the drift path, even though onset streamers actually produce the ions further away from the conductor. This approximation in the simulation should represent the corona current fairly well, as in the case of the onset streamer in the positive half-wave the negative ions would drift to the conductor, thus producing a corona current fairly well, as in the case of the onset streamer the case of the positive ions at the beginning of the drift paths drifting away from the conductor. Regarding the force, an error occurs due to this approximation of the injection of the ions at the beginning of the drift paths.

The time \( t \) is discretized by the fixed interval \( \Delta t \) and denoted by \( t_l \) at the \( l \)th time step. The change in the quantity \( X(t) \) at such two points in time is expressed by

\[
\Delta X(t_l) = X(t_{l+1}) - X(t_l).
\]

In the following, the parameters of the discretization are chosen as eight drift paths per subconductor with 100 points per drift path and 10 periods with 200 time steps per period. The length of the drift path is 0.7 m, except for the drift path leading to a vanishing field, as drift path 6 in figure 5.

3. The critical cylindrical field strength \( E_c \)

As mentioned, the electric field strength of the corona onset on the investigated setups is needed. In the following, these critical field strengths will be given in terms of the critical cylindrical field strength \( E_c \), designating the field on a cylindrical conductor substitute unperturbed by conductor wires and protrusions.

In the simple case of cylindrical symmetric electrodes, the formula of Peek may be used, which links \( E_c \) with the relative air density \( \delta \) and the radius of the cylinder \( r \) (in cm) according to

\[
E_c = E_o \delta \left( 1 + \frac{K}{\sqrt{\delta r}} \right),
\]

with the constant \( K = 0.308 \) and \( E_o = 31 \text{kV cm}^{-1} \), the breakdown field strength of the homogeneous field and under normal conditions (values taken from [12]).

At the relevant points, namely at the location of the metallic protrusions or water drops, the cylindrical symmetry is cancelled. Therefore another onset criterion, as for example the streamer criterium, is needed. The choice of constants and ansatz of the ionization coefficients differ widely. Here the criterium

\[
\delta \int \left( \frac{E}{E_o} \right)^2 \left( \frac{1}{\delta} \right)^2 - 1 \, dr > K^2
\]

will be used, which is determined by the adaption of the streamer criterion to the formula of Peek [13, 14].

3.1. Conical protrusions

Figure 2 depicts the geometry of the conical protrusions. The electrical field (in dependence on voltage) in the vicinity of conical protrusions featuring the geometry shown in figure 2 is determined by means of an FEM software. The analysis of the electrical field in the axis of the protrusion with the streamer criterium (2) results in a critical cylindrical field strength \( E_c = 17.6 \text{kV cm}^{-1} \) for laboratory conditions with a density corrected by the barometric formula to an altitude of 480 m above mean sea level.

For the remaining part of the surface of the conductor, \( E_c \) is determined by the formula of Peek, resulting, again for laboratory conditions and an altitude of 480 m above mean sea level, in \( E_c = 38.5 \text{kV cm}^{-1} \).

3.2. Double bundle

Wetting can vary widely for different conductors. In particular, ageing of the conductor cables results in a considerable change in hydrophilicity of the cable surfaces [11]: on new conductors the upper part of the cable is covered by a multitude of small
drops, while on the sides of the cables bigger drops are situated, while on well aged conductors only large drops on the under side of the conductors are present.

The cables of the double bundle under consideration were relatively new, thus featuring a wetting behaviour under artificial rain (with an intensity of 3.8 mm h\(^{-1}\)) in the investigations as sketched in figure 3. The upper half of the cable was covered by small drops with diameters of up to 1 mm, while on a part of the lower side of the cable bigger drops with diameters of typically 3 mm were found [2]. This asymmetric distribution of the drops is of great relevance, as this leads to different values of \(E_c\) over the conductor surface. In the following the critical electrical field is investigated for the case of the drop distributions depicted in figure 3.

3.2.1. \(E_c\) for the fraction of bigger drops. The field strengths occurring in service typically are much higher than the onset fields for direct current [15]. The corona discharges are related in [15] to the so-called Taylor instabilities, which occur at the final stage of a gradually increased voltage. Due to the force exerted on the drop by the electric field, the drop is increasingly deformed with increasing electric field, until the Taylor cone is reached [16]. Beyond this point, the surface tension cannot balance this force any more, which results in a rupture of the tip of the cone and the formation of a jet of water in the direction of the electric field lines [16].

In the case of ac the processes are complicated much more by the nonlinearities within the drop movement. Further, the distribution of drop sizes over the conductor varies with rain intensity and field strengths [2]. A description of that seems to be too complicated to be performed purely theoretically. Therefore a value of 17 kV\(\text{peak} \text{ cm}^{-1}\), derived through various testing of corona onset fields [17], is used.

3.2.2. \(E_c\) for the fraction of smaller drops and dry cables. The small drops up to diameters of 1 mm show only small deformations due to the electrical field compared with the bigger drops. To determine the critical field strength for this fraction of drops the streamer criterium (2) is used again.

For this purpose drops in the form of a spherical calotte on cables are considered. The contact angles considered are 45\(^\circ\) and 90\(^\circ\), which cover the range of possible contact angles on the new cable. The actual form of the drops having a constant contact angle is complex due to the curvature of the conductor wires. In order to simplify this, the spherical calottes are placed on the wire in such a way that the above-mentioned contact angles and cross sections are reached along the wires, while perpendicularly to the cable they are not (figure 4).

The electrical field determined by the FEM software is again evaluated by the streamer criterium (2) for the laboratory conditions. The resulting values for \(E_c\) are arranged in table 1.

As the deformation of these drops is not accounted for, the more conservative value of \(E_c = 25 \text{ kV cm}^{-1}\) calculated for a contact angle of 90\(^\circ\) is chosen for this fraction of drops. For the dry sector of the cable (see figure 3), \(E_c\) is chosen to be 34 \text{kV cm}^{-1} according to the calculation (table 1).

The resulting critical field strength in dependence on the polar angle is summarized in figure 5, reflecting the wetting behaviour (see figure 3) of the cable.

The prospective electric field shown in figure 5 indicates that corona is to be expected only from the sector of larger drops. In the discrete points on the right cable of the bundle, the corona onset is first reached at point 3, followed closely by 5.

4. Implementation

4.1. Capacitance matrix

For determining the charges on the subconductors in the absence of space charges, they are approximated by one line charge each. Thus the subconductors may have a spatial extension, but the influenced asymmetry in the distribution of the charge is neglected. If there are \(n\) subconductors \(L_i\) with the potential differences \(\phi_i\) and charges \(q_i\), then the latter are
600 cable under rain (with an intensity of 3.8 mm h⁻¹). The critical field strength \(E_{\text{prosp}}\) is shown for the right subconductor of the bundle energized with 167 kV cm⁻¹ in the coaxially arranged cage.

linked to the charges by the potential matrix \(p_{ij}\) according to

\[
\phi_j = \sum_{i=1}^{n} p_{ij} q_i.
\]

In this investigation, the interesting geometries are conductor bundles coaxially enclosed by a grounded cage; a boundary condition which will be accounted for by the method of image charges. The image charge \(q'_i\) corresponding to the charge \(q_i\) to the centre of the cage with \(a_i\) and the radius of the cage with \(R\), the image charge \(q'_i\) has to be placed at the same polar angle as \(q_i\), with the distance \(a'_i = R^2/a_i\) to the centre of the cage. This result can be derived analogously to the case of a point charge near a grounded sphere, to be found in [18]. With these image charges, the boundary is not entirely fulfilled, as the potential on the cage is constant though, but not zero. But this can be accounted for in the potential coefficients; using (3) and (4), these then are

\[
p_{ij} = \frac{-1}{2\pi \varepsilon_0} \left( \log \left| \frac{a_i - a_j}{a_i - a'_j} \right| - \log \left( \frac{a'_i}{a'_j} \right) \right).
\]

With known potential coefficients the capacitance matrix is given by \(c = p^{-1}\), allowing the calculation of the charges in dependence of the potentials in the case of absent space charges.

4.2. Electrical field on the subconductor surface

According to Gauss’ law, the total electric flux originating from a subconductor is determined by the charge on the subconductor in question. This charge is in general not symmetrically distributed over its cylindric surface, as the presence of an exterior electric field \(E_{\text{exterior}}\), i.e. the sum of the electrical field originating from all exterior charges, induces an asymmetric redistribution of the charge on the subconductor.

Thus the electrical field \(E_i\) on the \(i\)th subconductor is determined by the symmetric field of the charge \(q_i\) of the subconductor and an asymmetric component \(E_{\text{induction}}\) by the exterior field

\[
E_i = \frac{1}{2\pi \varepsilon_0 r} q_i \mathbf{n} + E_{\text{induction}},
\]

where \(\mathbf{n}\) and \(r\) are the unit normal vector and the distance from the subconductor, respectively.

Over the surface of a subconductor \(L_i\) the exterior field \(E_{\text{exterior}}\) varies, as the charges from which the exterior field originates are generally situated several cross sections away from the conductor \(L_i\). For this reason the variation of the exterior electric field over the surface of the subconductors is neglected. Then the asymmetric component \(E_{\text{induction}}\) on the surface of the subconductor is

\[
E_{\text{induction}} = 2 \cdot \left( E_{\text{exterior}} \cdot \mathbf{n} \right) \cdot \frac{\mathbf{n}}{r_{\text{L}}},
\]

where \(r_{\text{L}}\) is the radius of the subconductor [19].

To determine the electric field on the surface of the subconductors, the following has to be accomplished:

- Determination of the charge on every subconductor. Here, this is accomplished by the capacitance matrix for all subconductors and the screening effect of the drifting ions by the reciprocity theorem (of Gauss) [20].
- Determination of the exterior field by summation of the contributions of all charges.

4.3. Charge on the conductor

To account for the shielding effect of the drifting ions on the charge on a subconductor, the reciprocity theorem is used, instead of performing an extension of the capacitance matrix to the discrete points on the drift paths. For a system of spatially fixed conductors with charges \(q_i\) and potentials \(\phi_i\), the reciprocity theorem is according to [20]

\[
\sum_i q_i \phi'_i = \sum_i q'_i \phi_i,
\]

where the quantities with and without slashes stand for two arbitrary different situations concerning charges and potentials as indicated in figure 6.

Let \(\phi^{(i)}(r)\) denote the solution of the boundary value problem

\[
\phi^{(i)}(r) = \delta_{ij} \quad \text{for} \quad r \in L_i, \ j = 1 \ldots n.
\]

By introducing this solution in the reciprocity theorem (7) for the slashed quantities, the screening effect on the subconductor \(L_i\) by a space charge \(q_d\) at location \(r_d\) results in an additional charge

\[
q^{\text{ind}}_i = -q_d \cdot \phi^{(i)}(r_d)
\]
situation 1 \[\begin{array}{c}
q_i \bullet \quad \phi_i \\
\end{array}\] situation 2 \[\begin{array}{c}
q_i' \bullet \quad \phi_i' \\
\end{array}\]

Figure 6. Schematic representation of two situations featuring the same geometry, but different charges and potentials to illustrate the reciprocity theorem.

on the subconductor, while the potential remains the same. As the subconductors are still approximately represented by one line charge for each subconductor, \(\phi^{(i)}(r) \phi^{(j)}(r)\) are determined in the considered coaxial arrangement by

\[
\phi^{(i)}(r) = -\frac{1}{2\pi\varepsilon_0} \left[ \sum_{j=1}^{n} C_{ij} \left( \log \frac{|r - a_j|}{|r - a_j'|} - \log \frac{a_j}{a_j'} \right) \right].
\] (9)

With the presence of the drifting ions not accounted for in the capacitance matrix, as mentioned above, the charges on the conductors are determined by the capacitance matrix and (8), thus the charge \(q_i\) on the \(i\)th subconductor is

\[
q_i + \sum_{k=1}^{m} g_{ik} \phi^{(i)}_k = \sum_{j=1}^{n} C_{ij} \phi - \sum_{s=1}^{r} q_{ds} \phi^{(i)}_d.
\] (10)

Here, \(q_{dl}\) and \(q_{sk}\) denote the space charges on the drift paths and the charges injected at the beginning of the drift path in the case of corona activity, respectively (see section 2), while \(\phi\) denotes the potential with which the conductor is energized.

4.4. The injection of ions

As for every subconductor one equation (10) is given, an equation exists for every unknown charge \(q_i\). What is missing are equations for the charges \(q_{sk}\) at the total \(m\) discharge points \(k\). These are formed by the following ansatz for the corona onset by the critical field \(E_c\).

If the absolute value of the field strength \(|E_i|\) at the discharge point \(k\) is smaller than the critical field \(E_c\), \(q_{sk}\) is zero. In the other case, \(|E_i|\) is kept equal to \(E_c\). To account for this, the vector

\[
b_k = \begin{cases} 
0 & \text{for } |E_i| < E_c \\
1 & \text{else}
\end{cases}
\] (11)

is defined, for which the field strengths \(E_k\) are given by (5) and (6). Then, with

\[
b_k |E_k| = b_k E_c,
\] (12)

\[
g_{sk}(1 - b_k) = 0,
\] (13)

\(m\) linearly independent equations, determining \(g_{sk}\), are present (in fact equation (12) is not unique as both algebraic signs are possible for \(E_c\); this can be determined by the algebraic sign of the charge on the conductor in question).

To solve the system of equations at a time \(t_1\), \(b_k(t_1)\) has to be determined first. For this purpose the electric field on the surface \(E_i\) is taken according to \(E_i = E_k(q_i(t_1), q_{dl}(t_{l1}))\) for comparison with \(E_c\) according to (11).

To determine the unknown \(\Delta q_i(t_{l+1})\) and \(q_{sk}(t_{l+1})\), equations (10), (12) and (13) are rewritten so that only the unknown quantities are on the left-hand side of the equations. Then the system of equations can be expressed in terms of matrices

\[
M \left[ \begin{array}{c}
\Delta q_i(t_{l+1}) \\
q_{sk}(t_{l+1})
\end{array} \right] = N \left[ \begin{array}{c}
\Delta \phi(t_{l+1}) \\
\arg(q_i(t_l)) \cdot E_c
\end{array} \right].
\]

The matrix \(M\) has a left inverse; hence the multiplication of the system of equations with this left inverse solves the system.

4.5. Drift and recombination of the ions

Between two time steps the drift and recombination of the ions are calculated. Obviously, the drift of positive and negative charges has to be treated separately, thus the space charge on the drift paths consists of negative \(q_{dl}\) and a positive \(q_{sk}\) species.

In the first step the drift of the ions in the electric field \(E\) over the time step \(\Delta t\) is determined with the widely used relation

\[
\Delta t \approx \pm E \mu \Delta t,
\] (14)

with a mobility \(\mu = 1.7 \times 10^{-4} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}\) according to [21]. The algebraic sign in (14) is determined by the direction of the electric field and the algebraic sign of the charge of the ions. Through the corresponding displacement by \(\Delta t\), the charge comes to lie between two neighbouring points on the drift path, between which it is divided according to the reciprocal distances of the charge to these points.

In the second step the recombination of positive and negative ions is accounted for. For this the relation

\[
\frac{dq_{sk}^+}{dt} = \frac{dq_{dl}^-}{dt} = r \frac{q_{dl}^+ q_{sk}^-}{V_{ds}}
\] (15)

is used, where the recombination coefficient is chosen as \(r \approx 1.5 \times 10^{-12} \text{ m}^3 \text{ s}^{-1}\) according to [22]. The volume \(V_{ds}\) is roughly approximated by the polar angle and the distances to the neighbouring points on the drift path. Therewith the recombination gets dependent on the number of drift paths, which is not physical. However, the results of the simulation show little dependence on the number of drift paths.

An impression of the injection of ions and their drift afterwards is given in figure 7, where the distribution of positive and negative ions along the drift path 3 of figure 5 over the first 6 periods is depicted. The comparison with the other drift paths confirms the assumption motivated by figure 5: the main corona activity of the right subconductor of the bundle is to be found on drift path 3. The thus injected ions shield the other discharge points sufficiently except for the one corresponding to drift path 5, where, in smaller degree, some injection of ions is calculated.

Clearly detectable in figure 7 is the starting phase of the corona simulation in the first periods, where the distributions of the ions do not yet possess the same amplitudes as in the subsequent periods. But the analysis of the subsequent periods reveals that after the first three periods the quantities derived out
of the simulation, such as the corona current or the force acting on the surrounding gas, are already approximately stationary.

In figure 7 the drift can also be seen to a certain degree: the injections set in soon after a polarity reversal of the voltage. Thereafter the ions of this polarity move along drift path 3 further away from the cable. With the reversal of the electrical field they change direction and again drift towards the conductor. Due to the influence of the ions on the field, the charges near the conductor start to change direction towards the conductor before polarity reversal in contrast to the ions far from the conductor. Within the first two periods, a further sign of the field perturbed by the drifting ions is evident in the increasingly numerous injected amounts of ions in each half-wave. The reason is that the net charge is dominated by the remaining ions of the preceding half-wave, which has the opposite polarity and which thereby increases the electric field on the conductor, whereby \( E_{ck} \) is reached earlier within the half-wave.

Figure 7 also shows the effect of the recombination of the ions: before the injection of the ions of one polarity starts, the amount of the ions left over from the preceding period is small. An idea of time and place of the most efficient recombination can be deduced in the product of the two ion distributions depicted in figure 7. This picture would even be accentuated, if the product not of the distributions but of the ion densities were depicted, as with the distance from the conductor the geometric attenuation would lead to a further compaction of the equipotential surfaces in figure 7 in the vicinity of the cable.

4.6. Processing of the simulated quantities to the corona current and the \( 2f \)-force source

The charge in the system, i.e. all charges on the bundle and in the drift region, is summed together:

\[
q_{\text{tot}}(t) = \sum_{i=1}^{n} q_i(t) + \sum_{k=1}^{m} q_{\text{wet}}(t) + \sum_{s=1}^{r} q_{\text{dry}}(t).
\]  

The difference of this summed charge \( q_{\text{tot}}(t) \) to the purely capacitive charge, i.e. where corona is absent, results in a relatively good approximation of the charge due to corona. The derivative of this charge with respect to time results in the corresponding currents. The simulated currents for the 6th period after corona inception on the double bundle are shown in figure 8.

For the determination of the force acting on the gas due to the drifting ions, the contributions of every single point on the drift paths formed by the charge and electrical field at the corresponding point are summed together:

\[
F(t) = \sum_{s=1}^{r} q_{\text{dry}}(t) E_{ds}(t).
\]

The determination of the second harmonic is again performed by a Fourier series.

In the case of the double bundle, the symmetry in horizontal and asymmetry in the vertical direction result in a vertical force. The latter is depicted, again for the double bundle, in figure 9, together with its \( 2f \)-component (the second harmonic of the mains frequency) which is further discussed in section 5.1.
It is striking that the force features a constant component. This may arise because of the recombination of the ions: recombining ions contribute to the total force on the entire way of drifting from the conductor, while the same contribution on the way back to the conductor is smaller, as the drift is interrupted in the moment of recombination.

5. Results

As deducted in [1], the relevant components of the corona current are the first and third harmonics, while the relevant component of the force is the second harmonic of the mains frequency.

5.1. Double bundle conductor

The comparison of calculation and measurement of the corona currents of the double bundle is given in figure 10. The simulated corona currents shown in this figure 10 are the same as in figure 8 multiplied by the estimated corona active length of 6 m by artificial rain. In the case of the measurement, the corona current is approximated by the difference of the currents measured in rainy (intensity of 3.8 mm h⁻¹) and dry conditions shown in [1].

The thus resulting simulated corona currents in figure 10 match the measured ones fairly well according to amplitude. Interestingly, the 2f-relevant components of the corona current depicted in figure 10 are largely identical with the whole corona currents. However, the big phase between the measured and simulated corona currents (as well as between their 2f-components) is striking. At first sight, an explanation for this difference between measurement and simulation could be expected to lie in the inertia of the oscillating drops. But this is contradicted by the similar phase which is also noted in the case of the conductor with the mechanically fixed, conical protrusions (see section 5.2).

The interesting parameters of figures 10 and 9 concerning 2f-emission are summarized in table 2. These are the amplitude of the 2f-power and the corona loss (both simulated and measured), as well as the amplitude of the 2f-component of the force and the phase between the 2f-force and 2f-power. Because of the phase between measured and simulated 2f-corona currents, the question arises whether the simulation also results in a wrong phase between 2f-force and 2f-power, for which no answer can yet be given.

In the considered situations the 2f-relevant quantities show little dependence on the critical field strengths $E_c$. Wetting of new conductors is not fundamentally changed from figure 3 if the rain intensity is increased [17]. Therefore, no massive reduction in $E_c$ can be expected for stronger rains and the increase in the 2f-relevant quantities and therewith of the 2f-emission should be small. This is in agreement with the experimental findings where for rain rates of several mm h⁻¹ (such as 3.8 mm h⁻¹) changes in the rate result generally in relatively small variations of the levels of the 2f-emission (as well as of the levels of the overall broadband noise component) [2].

5.2. Conical protrusions

For the implemented coaxial arrangement of the laboratory experiments with the conductor with conical protrusions, the simulated and measured 2f-component of the corona current is depicted in figure 11.

As the simulation only accounts for two dimensional situations, the fraction of the length of the conductor where this two-dimensional picture of the calculation is accurate is roughly estimated. Obviously the drift region of the ions and therewith the effects thus produced onto the surrounding gas exceed the length of the part of the conductor where the protrusions are mounted (76 cm, see figure 2). In the following this corona-active length of the conductor is simply rounded to 1 m as a rough estimate.

Also in the case of the conical protrusions the 2f-corona current determined by means of the simulation shows a phase ahead compared with the measured current (see figure 11). The calculated and measured powers coincide fairly well, as

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measuring</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{P}_{2f}$ (VA m⁻¹)</td>
<td>74</td>
<td>53</td>
</tr>
<tr>
<td>Loss (W m⁻¹)</td>
<td>54</td>
<td>42</td>
</tr>
<tr>
<td>$\hat{F}_{2f}$ (N m⁻¹)</td>
<td>—</td>
<td>0.17</td>
</tr>
<tr>
<td>$\delta$ (°)</td>
<td>—</td>
<td>50</td>
</tr>
</tbody>
</table>
a dependence of the critical field strength. The rain rate could be accounted for by corona-active length of 1 m. Values are converted to values per unit length by assuming a force for the conductor with conical protrusions. The measured force and the phase (δ) between the 2f-force and 2f-power are shown.

The simulation does not yet account for different rain ageing of the conductors and its influence on corona activity. Further, ageing of the conductors and its influence on corona activity and tonal noise is not yet investigated in this numerical method.

Table 3. Measured and simulated 2f-powers and corona losses, as well as 2f-force (F_f) and the phase (δ) between the 2f-force and 2f-power for the conductor with conical protrusions. The measured values are converted to values per unit length by assuming a corona-active length of 1 m.

<table>
<thead>
<tr>
<th></th>
<th>Measuring</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ_{2f} (VA m^-1)</td>
<td>30</td>
<td>24</td>
</tr>
<tr>
<td>loss (W m^-1)</td>
<td>24</td>
<td>19</td>
</tr>
<tr>
<td>F_{2f} (N m^-1)</td>
<td>—</td>
<td>0.15</td>
</tr>
<tr>
<td>δ (°)</td>
<td>—</td>
<td>48</td>
</tr>
</tbody>
</table>

can be seen in Table 3. In the same table also the 2f-force and its phase to the 2f-power are shown.

6. Conclusions

As far as a comparison with laboratory measurement is possible, the simulation reproduces the measured currents fairly well, except for the phase of the corona currents with respect to the voltage: the corona currents resulting from the simulation precede the voltage with a significantly larger phase than in the measurement.

As [1] shows, the contribution of the 2f-force to the tonal emission is relevant. Thus the determination of E_c in dependence on the polar angle is essential for the calculation of 2f-levels, as only that way the 2f-force can be assessed.

The simulation does not yet account for different rain intensities. The rain rate could be accounted for by a dependence of the critical field strength E_c. Further, ageing of the conductors and its influence on corona activity and tonal noise is not yet investigated in this numerical method.

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References


Figure 11. The 2f-component of the corona current from the conductor with the conical protrusions energized at 120 kV determined by measurement and the simulation. The simulated corona current is calculated for a corona active length of 1 m.