Network Flow Model for Multi-Energy Systems

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Abstract: - This paper describes a novel approach to model networks with multiple energy carrier. The proposed nodal matrix establishes a link between an optimization of enclosed areas and their interconnections via networks. In the envisioned network flow model typical energy carrier include electricity, natural gas, and district heating, which can be lossy as well as lossless. Furthermore, energy conversion inside the lines can be operated, altogether in a single model. The presented model is based on the energy hub concept, where hubs are representing the individual areas with local energy conversion and storage devices.


1 Introduction
The premise of any network flow theory is the graph theory. Basic goal of that theory is to answer questions such as maximum flow or minimal cost. In this concept neither of the extreme cases is suggested, but what is actually proposed is a model that enables an optimal solution considering minimum amount of investment into production of energy out of primary energy carriers. These two are generally in opposition, since fossil fuels are still the cheapest option in general. Along with these demanding requirements, also taken into the account is the local production of energy on sustainable bases, e.g. biomass, second-generation bio fuels, wind and solar production, small-scale hydropower or new chemical energy sources. More traditional fuels, such as natural gas, fuel oil and wood chips are presented as well. Co-generation based production of energy is also included in the objective design, via combined heat and power (CHP), and district heating elements. Since different energy carrier interact in a complex way, when they are stored and converted into each other, a general model, the so called “Energy Hub”, has been developed by the “Vision of Future Networks” (VoFEN) research group at ETH Zurich in the past six years.

2 Problem Formulation

2.1 Applying a Greenfield approach
Since the energy hub concept uses a Greenfield approach, which should enable unusual solutions, one can neglect certain legacy issues concerning the topology of the energy grid. Going into the hub layer, the energy consumption and production of an area is described. The energy exchange with other hubs is modeled with an overlying node layer, which was first discussed in [1].

One of basic advantages of this model compared to others is the fact it has the ability to combine different energy carriers and produce the best one available facing various constraints by synergies. Having a complete list of conversion factors between energy carriers can do that. This is depicted with the conversion matrix C in the model, leading to hub equation (1) from [2], where \( \mathbf{L} \) is the load, \( \mathbf{T} \) is the feed into the grid, \( \mathbf{P} \) is the consumption from the grid and \( \mathbf{R} \) is the local energy production.

\[
(\mathbf{L} + \mathbf{T}) = \mathbf{C} \cdot (\mathbf{P} + \mathbf{R})
\]  

(1)

The recent developments of the group’s model include network loss equations, built-in into the overall optimization process as additional optimization parameters. That feature takes into account e.g. district heating losses and provides perhaps a better solution in form of supplying natural gas directly to the consumers, in that way avoiding the costly heat distribution losses.
along the way. Those are typical optimization problems facing network design and daily operations alike.

In previous papers in the series there is a clear evolving path in the model by firstly adding inter-carrier energy optimization in the overall optimization process, and after, giving it a more realistic behavior in terms of providing a mathematical representation for energy storage. Given all that, the group’s proposal uses a mathematical model similar to the two-port network model in network analysis of electric circuits.

2.2 Network flow theory

In order to understand what is proposed the concept’s network flow and topology need to be brought in conjunction. An energy network is by design a mesh, or so called unsorted/unbalanced network. This means there is no general topology given for a standard design of an energy network. It evolves based on natural geographical configuration and demand on the terrain. Elements of such network are in this case hubs, nodes, and connecting lines. Each corresponds to approximately the same elements in the graph theory. When evaluating the needs of a network flow theory for the energy hub concept, one should compare the task with other applications.

Network flow theory is a well-known and documented area, especially in computer networks and other fields where complex routing problems are an issue [3]-[5]. Common uses of the theory can be found in telecommunications in form of “Routing Wavelength Assignment” (RWA) in optical networks. Such a system considers maximizing the efficiency of the communications channel, as the model does with energy lines, either in fixed wavelength mode using Dijkstra’s algorithm, or some sort of adaptive mode. Neither takes into account the state of the network, like it needs to be done in energy networks, and solely focuses on maximizing bandwidth.

Another example can be found in the way to manage data flow on basic TCP/IP levels. The proposal focuses on time-pricing schemes to level out the demand during peak hours in computer networks. Finally, traffic management can provide the best example of multi variable decision making in an environment, where each participant has an opposing agenda of travelling with minimum time and shortest route taken [6]. An applied case is described in [7], where an US-based package delivery company managed to deliver huge fuel savings in its delivery fleet, by not calculating the direct route from the warehouse to the customer, but by cutting down on engine idle time at left-hand traffic lights. This is an ideal case for presenting a multi variable optimization problem, in this instance, fuel, time and distance.

Alternative models for future energy systems might also be a source of knowledge. One important advantage over the competing technologies, such as micro grids or smart grids, is the absence of the need to invest in supporting infrastructure, and complete reuse of existing infrastructure where possible. That leaves only investment in new features of the grid when needed, but that is true of every concept’s idea. Smart grids would require further investment in the IT layer of the grid. It is also true for the hub concept, but it drops the monolithic type of production of energy, substituting it with local production along the way, in a scalable way, following organic growth of that market.

The goal of network flow optimization for this issue can be summed up by stating the following: System should be able to provide current or superior level in quality of service with given restrictions in the energy connections and production with regards to the investment cost and ecological sustainability. Final goal is to produce a model that can provide us with the optimization solution, or even a Pareto front for every time step of the simulation. The operator in that case would undertake the ultimate decision.

2.3 Experiences from a case study

During the last several years the knowledge gained inside VoFEN was elaborated in a case study. Starting with at first three hubs, increasing number of hubs was described during over time. The interdependencies in between the hubs growing as the number of elements in the model went up. Aside from the size of model, the long time period of one year, enabling investigation about summer-winter behavior, increased the effort. For fast changes in the load a time step is currently defined as 15 min intervals between measurements. Such a system could prove to be extremely resilient to network congestion problems faced by today's networks when serving peak demand, in form of time-variant pricing. Due to the overall complexity, the necessity for a new way to describe the network arises.

A general method to determine the connectivity for each possible line between the hubs and the nodes could be useful. Each energy carrier line is considered, not simply the trunk line, and each is given a specific factor instead of main line efficiency. A node on its own does not serve a purpose in the Greenfield approach. However, it was needed in order to explain the concept on a viable test candidate in present time. It should be considered as a routing element in the overall network scheme. Primary difference between a node and a hub can be explained in a way that a node is a hub without active elements, therefore it doesn’t require the flow vector $\mathbf{F}$. If the network losses are not regarded in the calculation, it simply acts as a passive element of the network, neither contributing nor consuming energy from the network. If
losses are included, it should be described with an appropriate nodal matrix for losses, which describes each carrier line characteristic through the node. On the other hand, such topology allows for easy integration of future technologies into the network. Simply adding a hub with elements into existing nodes is a straightforward task in a determined network topology. In terms of what a node or a hub represent, boundaries need to be set only in the logical layout of the network. A hub may be physically represented with various elements over the entire geographical location, but it is only one logical element in the network. Given the current state of the affairs in real world, a hub can change ownership between various parties included in the process of generation, transmission and distribution of energy. Such flexibility can be provided by adding status flags to the vector in the simulation, providing an easy means of tracking each entities interests and assets throughout the entire period of planning and operation.

To sum up the discussion and formulate the task ahead, the nodal matrix is the most promising way to include all line losses in the network. That is true for every energy carrier included in the observed network. In such a way, the simulation is aware of all the variables involved and can produce a viable solution in terms of describing network flow, with minimum losses as possible, all the way contributing to reducing prices and the system’s carbon footprint.

3 Problem Solution

An ordinary network section can be described as in Fig. 1, with two nodes \( \mathcal{N}_i \) and \( \mathcal{N}_j \), the corresponding hubs \( \mathcal{H}_i \) and \( \mathcal{H}_j \), and all flows \( F \), like it was introduced in [1]. The connections beyond the network are indicated with the small letter \( x \) for external. The conversion of energy from one carrier \( \mathcal{E}_a \) into another carrier \( \mathcal{E}_b \) inside the network, except in the case of losses in chapter 3.1, is not postulated.

![Fig. 1: Simple network with two nodes](image)

The nodal equation for \( \mathcal{N}_i \) in (2) is the sum of all flows from the node, \( F_i \), towards the hub \( \mathcal{H}_i \), \( F_x \) to the external node \( \mathcal{N}_i \), and to all other nodes \( \mathcal{N}_j \).

\[
0 = F_i + \sum_j F_{ij} + F_x \quad \forall i, j \in \mathcal{H} \forall i, j \in \mathcal{N} \forall \mathcal{E} \quad (2)
\]

Equation (2) is valid for all energy carriers \( \mathcal{E} \), all hubs and all nodes. In the case of a strategically sited node without corresponding hub, \( F_i \) can be omitted. For a lossless network, the network flow in an opposite direction can be formulated as in (3).

\[
F_{ij} = -F_{ji} \quad (3)
\]

3.1 Network with losses

When introducing losses into the consideration, (3) can be rewritten as in (4). The ratio between flow \( F_i \) and reversed flow \( -F_{ji} \) is stated as \( n_{ij} \).

\[
F_i = -F_{ji} + F_{ij}^{Losses} \quad (4)
\]

\[
n_{ij} = \frac{-F_{ji}}{F_{ij}} \quad (5)
\]

Depending on the direction of the positive flow, (5) decomposes to (6), where the efficiency \( \eta_i \) appears.

\[
\begin{align*}
&\text{if } F_{ji} > 0 \Rightarrow n_{ij} = \eta_{ij} \\
&\text{if } F_{ji} < 0 \Rightarrow n_{ij} = \frac{1}{\eta_{ij}}
\end{align*} \quad (6)
\]

For the case of no physical connection between two nodes \( \mathcal{N}_i \) and \( \mathcal{N}_j \) or efficiency \( \eta_{ij} = 0 \), the value \( n_{ij} \) is left empty (7).

\[
\forall F_i = 0 \Rightarrow n_{ij} = 0 \quad (7)
\]

3.2 Nodal matrix

Assuming that a nodal matrix for the whole network can be established as aspired in (8), it is necessary to determine the coefficients \( n_{ij} \) like in (5). For each energy carrier one nodal matrix can be designed, if no energy conversion within \( F_i \) is allowed.

\[
F_i = \mathbf{N}_{ij} \cdot F_{ij} \quad (8)
\]

\[
\mathbf{N}_{ij} = \begin{bmatrix}
n_{11} & n_{12} & \cdots & n_{ij} \\
n_{21} & n_{22} & \cdots & n_{2j} \\
\vdots & \vdots & \ddots & \vdots \\
n_{i1} & n_{i2} & \cdots & n_{ij}
\end{bmatrix} \quad \text{and } \quad F_{ij} = \begin{bmatrix} F_{1j} \\ F_{2j} \\ \vdots \\ F_{ij} \end{bmatrix} \quad (9)
\]

The nodal matrix and the connection vector in general, as in (9), contain three cases for the relation between \( \mathcal{N}_i \) and \( \mathcal{N}_j \). It is quite evident that a flow from one node into the same node in (10) is not defined, similar to (7).

\[
i = j \Rightarrow n_{ij} = 0 \quad (10)
\]

The representation of flows in [1] is written from smaller to larger indices. In addition, an overall positive flow from the node to somewhere else requires a negative flow to the hub. Both result to (11).

\[
i < j \Rightarrow n_{ij} = -1 \quad (11)
\]

Building up the nodal matrix leads to (12), where the case of even \( i \) and \( j \) (10), \( i \) smaller than \( j \) (11) and \( j \) smaller than \( i \) (6) are integrated.
\[
N_y = \begin{bmatrix}
0 & -1 & \cdots & -1 \\
1 & 0 & -1 & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
n_{i1} & n_{i2} & \cdots & 0
\end{bmatrix}
\] (12)

\[
n_y = \begin{cases} 
\eta_y & \text{if } F_{ji} > 0 \\
1/\eta_y & \text{if } F_{ji} < 0 \\
0 & \text{if } F_{ji} = 0
\end{cases} \quad \forall i > j
\] (13)

A similar relation in (13) can be developed for the connections of nodes and external, with a vector for \(N_{ix}\) and the case differentiation for the size of \(F_{ix}\).

\[
N_{ix} = \begin{bmatrix} n_{ix} \end{bmatrix}
\] with \(n_{ix} = \begin{cases} 
\eta_{ix} & \text{if } F_{ix} > 0 \\
1/\eta_{ix} & \text{if } F_{ix} < 0 \\
0 & \text{if } F_{ix} = 0
\end{cases} \quad \forall i > j
\]

Finally the nodal equation for a network of nodes and hubs can be accumulated to (14).

\[
F = N_y F_y - N_{ix} F_{ix}
\] (14)

The efficiencies for \(\eta_{ij}\) and \(\eta_{ix}\) can be designated separately, e.g. as a function of the distance and the specific losses, linear or nonlinear. For different energy carrier each network has an own nodal equation.

### 3.3 Energy conversion inside the network

Synergy effects between different energy carriers were expected to be found when starting the VoFEN project. Several studies carried out had the interconnections as the object of investigation, as in [8] and [9]. Physical parameters of gaseous or electrical flows were in the spotlight. Thus, an integrated view over all carriers like it has been realized in the coupling matrix \(C\) should be obtained. First of all the energy flow vector to the hubs \(i\) has to be expanded to all energies \(\mathcal{E}\) in (15), which also leads to a bigger nodal matrix \(N_{ij}\). Note that \(F_y\) is expanded in the same manner as \(F_x\).

\[
F^e_{ix} = \begin{bmatrix} F^e_1 \\
\vdots \\
F^e_{ix} \\
\vdots \\
F^e_n
\end{bmatrix} = \begin{bmatrix}
n_{i1}^{aa} & \cdots & n_{i1}^{ao} & \cdots & n_{ij}^{ao} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
n_{i1}^{oa} & \cdots & n_{i1}^{oo} & \cdots & n_{ij}^{oo} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
n_{i1}^{oa} & \cdots & n_{i1}^{oo} & \cdots & n_{ij}^{oo}
\end{bmatrix} \cdot F^e_y
\] (15)

For the former diagonal elements of the nodal matrix in (12) the conditions do not change (16). The upper right triangle of the matrix split as the energy is split during the conversion, from one carrier into another as in (17).

\[
i = j \Rightarrow 0 \Rightarrow \begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0
\end{bmatrix}
\] (16)

Similarly, the lower left triangle in (18) will be transformed, whereas \(\eta_{ij}^{ee}\) are the efficiencies of the energy conversion and \(\eta_{ij}^{eo}\) are the transmission efficiencies, as in (4).

\[
i > j \Rightarrow n_{ij} = \begin{cases} 
\eta_{ij}^{aa} & \cdots & \eta_{ij}^{ao} \\
\vdots & \ddots & \vdots \\
\eta_{ij}^{oa} & \cdots & \eta_{ij}^{oo}
\end{cases} \quad \forall i > j
\] (18)

Consequently, the interconnections in (19) split into a sum of all converted flows \(\mathcal{E}(i) \neq \mathcal{E}(j)\) and the single not converted flow \(\mathcal{E}(i) = \mathcal{E}(j)\).

\[
F^e_y = \sum \mathcal{E}(i) \mathcal{E}(i) = \mathcal{E}(j)
\] (19)

### 3.4 Example network configuration

The concept of energy hubs in general and of the propounded nodal matrix is not limited to any number of nodes or hubs respectively. The ongoing case study utilizes a maximum of eleven nodes, in actual publications are typically three hubs present, as in [9] and [10]. Hence, and for the sake of clarity an example with three hubs/nodes is used as well. The procedural method in order to establish ties in between the reality and the modal is shown in Fig. 2.
Out of the geographic information system (GIS) three areas are chosen. The specific characteristics and behavior of the areas leads to the coupling matrix $C$, the local renewable energy production $R$ and the load $L$ for each of the three hubs $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$. Since the Greenfield approach allows a general interconnection of hubs, this task is implemented by using the nodes, like in [1] and [10]. The illustrated hubs are situated in a peripheral region, therefore only one connection to the grid, from node $\mathcal{N}_1$ to external $\mathcal{X}$ is assumed. Data for this generic example is provided in Table 1. Due to the limited space only the nodal matrix as in chapter 3.2 is taken, with global efficiencies for each energy carrier and no energy conversion inside the connections.

Table 1: Example flows for three energy carrier and global efficiency

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Electricity</th>
<th>Natural gas</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node to node</td>
<td>$F_{12}$</td>
<td>-1.74</td>
<td>-1.40</td>
<td>-4.65</td>
</tr>
<tr>
<td>Node to node</td>
<td>$F_{32}$</td>
<td>0.04</td>
<td>No line</td>
<td>2.87</td>
</tr>
<tr>
<td>Node to node</td>
<td>$F_{33}$</td>
<td>2.33</td>
<td>1.01</td>
<td>0.93</td>
</tr>
<tr>
<td>Node to external</td>
<td>$F_{3x}$</td>
<td>-5.20</td>
<td>-2.41</td>
<td>No line</td>
</tr>
<tr>
<td>Efficiency</td>
<td>$\eta$</td>
<td>0.98</td>
<td>1.00</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Assembling the data of Table 1 directly into the nodal equation of (14) yields to the equation system in (20). Again each energy carrier has its own nodal matrix, which represents the network in the nodal layer of Fig. 1.

\[
\begin{align*}
F_{1e}^{el} &= \begin{bmatrix} 0 & -1 & -1 \\ 1.02 & 0 & -1 \end{bmatrix} \cdot F_{ij}^{el} - \begin{bmatrix} 0 \\ 1.02 \end{bmatrix} \cdot F_{ix}^{el} = \begin{bmatrix} 1.70 \\ 1.20 \end{bmatrix} \\
F_{1e}^{ng} &= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \cdot F_{ij}^{ng} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot F_{ix}^{ng} = \begin{bmatrix} 1.40 \\ 1.01 \end{bmatrix} \\
F_{1e}^{he} &= \begin{bmatrix} 0 & -1 & -1 \\ 1.14 & 0 & -1 \end{bmatrix} \cdot F_{ij}^{he} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot F_{ix}^{he} = \begin{bmatrix} 1.78 \\ -6.23 \end{bmatrix} \\
F_{1e}^{he} &= \begin{bmatrix} 0 & 0 & 0 \\ 0.88 & 0.88 & 0 \end{bmatrix} \cdot F_{ij}^{he} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot F_{ix}^{he} = \begin{bmatrix} 3.34 \\ 0 \end{bmatrix}
\end{align*}
\]

For the electricity flow in (20) it is apparent that all hubs are consumers and the total demand has to be routed loss via $F_{3x}$ and node $\mathcal{N}_2$. In contrast, the natural gas network is assumed lossless, because of the unidirectional flow from high to low pressure. Also line $F_{3}$ is not present, as in (7), and hub $\mathcal{H}_2$ does not use gas, which is no problem for the model. Finally, the district-heating network builds up a closed ring structure, without any connection to the outside $\mathcal{X}$. Hub $\mathcal{H}_2$ produces heat for both other hubs, $\mathcal{H}_1$ and $\mathcal{H}_3$.

Even when not considering energy conversion inside the interconnections, the example demonstrates the benefits of an enclosed modeling of the network topology by using the suggested nodal matrix.

3.4 Optimization of the network

An optimization of the nodal flows intends usually to minimize the costs of the grid by reducing the losses. Due to the fact that line losses reduce the hub’s efficiencies, some new goals for the objective function area required in (21). The formulation of [10] can be adopted to the current case.

Minimize: $\overline{\mathcal{S}}(F, TSC)$

Subject to: $\begin{align*}
L_i + T_i &= C_i \cdot (P_i + T_i) \quad \forall i \in \mathcal{H} \\
F_i &= P_i - T_i \quad \forall i \in \mathcal{H} \\
F_i &= N_{ij} \cdot F_{ij} - N_{ix} \cdot F_{ix} \quad \forall i, j \in \mathcal{H} \\
TSC &= \sum_t \sum_i THC_i(t) \quad \forall i \in \mathcal{H} \\
THC_i &= \Psi_i \cdot \Phi_i - \Phi_i \cdot T_i \quad \forall i \in \mathcal{H}
\end{align*}$

The minimization is dedicated directly to the flows into the hubs, according to the total system costs $TSC$, which derive from the individual hub costs $THC$ over the time. $\Psi$ and $\Phi$ are the marginal costs and benefits respectively. Only the main equalities in (21) are specified, of course there are additional inequalities and boundaries.

4 Conclusion

This paper gives a general overview for a novel approach in distributed energy generation and optimization. It is the author’s opinion such a system could provide significant savings in both generation and day-to-day operations on such a system. Proposed method is further extension of the group’s mathematical model by means of a nodal matrix describing network losses, with the added ability for executing the simulation with or without losses. The existing model was adapted to accommodate such possibility and proved highly scalable in terms of code reuse. It is hoped that the work up to date would create added value and knowledge to further an ever-increasing body of knowledge and expertise in this vital area of research. In order to achieve the goal in it entirety, future work should presume more simulation time with an increased set of data covering the ongoing case study. The mentioned data set analysis should also include cost-benefit analysis compared to the existing situation and with the evolution versions of this model in particular.

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References: