An Optimal Power Flow Formulation Including Risk of Cascading Events

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SUMMARY

Higher penetration of fluctuating renewable energy sources (RES) and market liberalization increase both the demand for transmission capacity and the focus on cost efficient operation of the grid. To find a better trade-off between system security and the cost of operation, we propose to use a risk-based approach to system security. Risk-based security criteria might help to maintain system security while decreasing operational cost by utilizing additional information about the security level in the system.

This paper proposes a new, risk-based security measure, which intends to reflect the risk of cascading events in a risk-based optimal power flow (OPF) formulation. The proposed risk measure limits the probability that a cascade is initiated, focusing on cascading through a sequence of line outages. It accounts both for the probability of the initial N-1 contingency (primary event) and probability of further, dependent outages (secondary trip). The probability of a secondary trip is assumed to be a function of both the pre- and the post-contingency line flows.

The risk measure is implemented in a DC OPF formulation, forming a risk-based security constrained OPF (RB-SCOPF) where so-called “risk-based N-1 constraints” are used to limit the risk of cascade initiation. To understand how the proposed formulation influences not only the risk of initiating a cascade, but also the impact of the full cascade in terms of expected load shed, the risk-based criterion is implemented in a Monte Carlo framework. Within this framework, we simulate the full cascade and compute the expected load shed based on the probability of the initial outage and the probability of the secondary trip.

The proposed method is demonstrated through a case study for the IEEE 118 bus system. We show how the RB-SCOPF can be used to control the risk of cascade initiation, and compare the RB-SCOPF solution with the solution of a standard security constrained OPF (SCOPF). Further, we investigate how the choice of the risk limit influences both the generation cost and the expected load shed. A lower risk limit increases the cost of generation, and decrease the expected load shed in most cases. However, in some cases a lower risk limit might also increase the expected load shed, since the RB-SCOPF does not consider how the cascade evolves after it has been initiated.

KEYWORDS
Risk-based OPF, cascading events, security constrained OPF, Monte Carlo based security assessment
1. Introduction

Market liberalization and increasing penetration of fluctuating renewable energy sources (RES) are leading to a situation where power is not necessarily produced close to where it is consumed, but rather where production is cheap, where the wind is blowing or the sun is shining. This trend leads to a higher demand for transmission capacity and an increased pressure on efficient use of existing grid infrastructure. In order to reduce cost, the transmission system operators (TSOs) are often forced to operate the system closer to the limits, which increases operational risk. To cope with this new situation, methods to find a better trade-off between system security and cost of operation are needed. In [1], it is argued that additional information is needed to operate the system securely while maintaining a reasonably low cost. As [1] suggests, this additional information could be provided in terms of a risk-based measure of system security. In this paper, we propose a risk measure for use in operational planning, by introducing a method to include the risk of cascading events in an optimal power flow (OPF) formulation.

There exist two main approaches to model risk in power system operation. On the one hand, risk, can be modelled through overall reliability parameters such as Expected Energy Not Served (EENS). This type of risk measure reflects the impact on the consumers in the system, and accounts for interruptions of supply that occur due to cascading events, e.g., through cascading trips of transmission lines. However, the evaluation of those risk measures requires extensive calculations (e.g., Monte Carlo simulations), and they are thus better suited for analysis of the risk in a given operating condition [1-2], as opposed to inclusion in an optimization problem.

On the other hand, risk can be modelled in terms of violation of technical limits, e.g., as a function of the line flow or the voltage magnitude after an N-1 outage. This type of risk measure does not consider how a potential cascade would develop further and thus does not directly reflect the impact of a cascade on the consumers (in contrast to e.g. Monte Carlo-based risk measures). However, when risk is modelled as a function of line flows or voltages, there is a direct relation to operational parameters, and the risk is more easily understood and influenced by the system operators. This second type of risk measure is also relatively easy to evaluate and have been proposed for inclusion in OPF formulations by, e.g., [3-5].

Although several OPF formulations assume that a larger violation of an N-1 limit is more dangerous than a small violation [3-6], there are, to the best of the authors’ knowledge, currently no OPF formulations that explicitly account for the risk of cascading events within the optimization algorithm. This paper aims at introducing a risk measure that better reflects the risk of cascading events in the OPF formulation while maintaining computational tractability. To assess the effectiveness of the proposed risk measure, the actual risk level measured in terms of expected load shed is evaluated through a Monte Carlo simulation, and compared with the risk level achieved with a standard security constrained OPF (SCOPF).

The contribution of this paper is threefold. First, we introduce a risk-based security criterion reflecting the risk that one initial N-1 contingency (primary disturbance) can lead to further, dependent outages (secondary trips) and thus initiate a cascading event. We model the probability of secondary trips as a function of both the post-contingency line flow and the change in line flow from the pre- to the post-contingency state, based on the formulation in [3]. Second, we formulate a risk-based security constrained OPF (RB-SCOPF) where so-called “risk-based N-1 constraints” are used to limit the risk of cascade initiation. Finally, we implement the probabilistic model for secondary trips in a Monte Carlo simulation. The Monte Carlo framework allows us to assess the impact of the full cascade, and to assess whether the RB-SCOPF can help to control the risk faced by consumers.

In a case study for the IEEE 118 bus system, the RB-SCOPF is compared with a standard SCOPF. We compare the two OPF formulations in terms of the risk of cascade initiation (evaluated directly from the OPF solution), the expected load shed (obtained from the Monte Carlo simulation), and the cost of the generation dispatch.

The remainder of this paper is organized as follows: Section 2 introduces the risk model, Section 3 formulates the OPF problem and Section 4 describes the setup of the Monte Carlo simulation. The case study is presented in Section 5, and Section 6 summarises and concludes the paper.
2. Risk modelling

The aim of this paper is to propose a risk measure that better reflects the risk of cascading events in an OPF formulation. As mentioned above, it is hard to model the full length of a cascading event within the OPF due to the high computational requirements. Instead, we consider the risk of cascade initiation, i.e., the risk that an initial N-1 outage might lead to further outages, and compute the risk based only on the operating condition after the N-1 outage. Formulating the risk in this way complies with the N-1 security criterion, which requires TSOs to operate the grid such that no single contingency leads to a major disturbance [7]. However, the risk measure represents two main improvements compared with the deterministic N-1 criterion:

First, the risk measure accounts explicitly for the probability of the initial outage, while the N-1 criterion treats all outages as if they have the same probability. By accounting for the outage probability, the risk-based formulation can accept lower security margins for outages that are unlikely to happen, and increase the margins for contingencies that have a high probability of occurrence.

Second, the risk measure assumes that the probability of a secondary trip is a piecewise continuous function of the pre- and post-contingency line flow, whereas the deterministic N-1 criterion assumes that the probability of a secondary trip rises from 0 to 1 as the line flow reaches the line capacity limit. By including additional information about the magnitude of the power flows and the extent of the N-1 violation, the risk measure allows us to model effects that can otherwise not be considered.

2.1. Definition of the risk measure

The risk measure reflects both the probability of the initial outage and the severity of the resulting operating condition. Here, \( Pr_k \) is the probability of contingency \( k \), and \( Sev_{l|k} \) is the severity of the operating condition on line \( l \) after outage \( k \). Since we consider the risk of cascade initiation, severity is defined as the probability that outage \( k \) will lead to a trip of line \( l \), denoted as the probability of secondary trip \( Pr_{l|k} \). Thus, the risk related to line \( l \) after contingency \( k \) is expressed as

\[
Risk_{l|k} = Pr_k \cdot Sev_{l|k} = Pr_k \cdot Pr_{l|k}.
\]  

(1)

We assume that the probability of each initial contingency can be estimated based on historical data (e.g., from [8]), and be updated according to seasonal variations and weather effects. In particular, we assume that it is independent of operational variables such as the generation dispatch or the line flows.

2.2. Probability of Secondary Trip

It is well known that errors in the protection system can lead to secondary trips. Statistics for outages in Germany [8] show that about 0.5% of all outages in the transmission grid (220 and 380 kV) lead to tripping of equipment which should have remained in service. It is relatively common to assume that higher line flows lead to higher probability of failure, due to increased conductor sag and due to increased probability of hidden failures in the protection system (e.g., [6, 9]). However, investigations of the power flow on transmission lines during three major events involving cascading outages in the US, presented in [10], show that the tripped lines experienced not only high post-contingency flows, but also a large change in power flow from the pre- to the post-contingency state. Based on this data, the authors of [10] suggest that the probability of failure should be expressed as a function of both the post-contingency power flow and the power flow change and fit a logistic model to the obtained data. The argumentation in [3] follows along the same lines, assuming that 1) a higher post-contingency line flow increases the probability of a trip due to increased conductor sag and 2) a larger change in the line flow from the pre- to the post-contingency state reflects higher transients and thus higher probability of inadvertent tripping. The model for secondary trips used here is based on the model proposed in [3], though extended to a more general form for inclusion in the OPF.

| Nl lines | Nk contingencies on the N-1 contingency list. \( P_k^L \in \mathbb{R}^{N_L} \) denotes the line flows in the \( k^{th} \) operating condition, with \( k = 0 \) being normal operation and \( k = 1, \ldots, N_k \) being the operating condition after the \( k^{th} \) contingency. The line flow of line \( l \) is given by the \( l^{th} \) entry, denoted by \( P_{L(l)}^k \). Based on the formulation proposed in [3], we model the probability of |
secondary trip \( Pr_{l|k} \) as a function of the power flow \( P_{L(l)}^0 \) on line \( l \) in normal operation, the post-contingency power flow \( P_{L(l)}^k \) on line \( l \) and the threshold \( P_{L(l)}^{trip} \), which is the power flow leading to a certain trip of line \( l \). Assuming that the line flow jumps to a higher value, \( Pr_{l|k} \) is expressed as [3] \[
Pr_{l|k} = \frac{P_{L(l)}^k - P_{L(l)}^0}{P_{L(l)}^{trip} - P_{L(l)}^0}.
\] (2)

By construction, (2) fulfills the assumptions on the probability of secondary trip, as \( Pr_{l|k} \) increases when either the post-contingency flow \( P_{L(l)}^k \) or the power flow change \( P_{L(l)}^k - P_{L(l)}^0 \) increase. Eq. (2) can be interpreted as the ratio between the change in power flow, \( \Delta P_{L(l)} = P_{L(l)}^k - P_{L(l)}^0 \), and the initial distance to the threshold of certain trip, \( \Delta P_{L(l)} = |P_{L(l)}^{trip} - P_{L(l)}^0| \), for a positive change in line flow. However, the line flow can also “jump” in negative direction. In this case, the initial distance to the threshold is given by \( \Delta P_{L(l)} = |P_{L(l)}^{trip} - P_{L(l)}^0| \) as illustrated in Fig. 1. To account for both cases, we formulate \( Pr_{l|k} \) as

\[
Pr_{l|k} = \max \left( \frac{P_{L(l)}^k - P_{L(l)}^0}{P_{L(l)}^{trip} - P_{L(l)}^0}, \frac{P_{L(l)}^k - P_{L(l)}^0}{-P_{L(l)}^{trip} - P_{L(l)}^0} \right) .
\] (3)

Finally, we assume that \( Pr_{l|k} = 0 \) if \( P_{L(l)}^k \) is below 90% of the capacity limit \( P_{L(l)}^{max} \), and set \( Pr_{l|k} = 1 \) if \( P_{L(l)}^k \) exceeds \( P_{L(l)}^{trip} \). The final expression for \( Pr_{l|k} \) is thus given by

\[
Pr_{l|k} = \begin{cases} 
\max \left( \frac{P_{L(l)}^k - P_{L(l)}^0}{P_{L(l)}^{trip} - P_{L(l)}^0}, \frac{P_{L(l)}^k - P_{L(l)}^0}{-P_{L(l)}^{trip} - P_{L(l)}^0} \right), & \text{if } |P_{L(l)}^k| \leq 0.9 \times P_{L(l)}^{max} \\
0, & \text{if } 0.9 \times P_{L(l)}^{max} < |P_{L(l)}^k| < P_{L(l)}^{trip} \\
1, & \text{if } P_{L(l)}^{trip} \leq |P_{L(l)}^k| 
\end{cases}
\] (4)

Fig. 2 illustrates \( Pr_{l|k} \) for positive post-contingency line flows and different values of \( P_{L(l)}^{0} \).
3. Formulation of the risk-based optimal power flow

In this section, the risk measure introduced above is incorporated in an OPF formulation to form a risk-based security constrained OPF (RB-SCOPF). Starting from a standard SCOPF, we replace the standard security constraints limiting the post-contingency line flow with risk-based constraints that account for the risk of cascade initiation. The reformulation of the RB-SCOPF to a tractable optimization problem is explained, and a method to define reasonable risk limits is proposed.

3.1. Standard SCOPF

The OPF is based on a DC power flow formulation. We consider a system with \( N_B \) buses, \( N_L \) lines and \( N_G \) generators. The vector \( P^k_G \in \mathbb{R}^{N_G} \) describes the active power injections of the generators and the loads at each bus in the operating condition \( k \), and the vectors \( P_D \in \mathbb{R}^{N_B} \) and \( \theta \in \mathbb{R}^{N_B} \) describes the load and voltage angles at each bus. The post-contingency generator in-feeds and the post-contingency line flows are modelled using linear sensitivity factors. The generation at generator \( g \) after outage of generator \( k \) is given by

\[
P^k_{G(g)} = P^0_{G(g)} + d_{(g,k)}P^0_{G(k)},
\]

where the matrix \( d \in \mathbb{R}^{N_G \times N_G} \) models how the lost generation is balanced by the remaining generators. Assuming that all generators contribute based on their nominal maximum output, we set \( d_{(g,k)} = P^\text{max}_{G(g)}/\sum_{g \neq k} P^\text{max}_{G(g)} \) for \( g \neq k \) and \( d_{(k,k)} = -1 \). Further, we choose \( P^\text{trip}_L = 1.25 P^\text{max}_L \). The post-contingency line flow on line \( l \) after the outage \( k \) of a generator or a line are given by (6) and (7).

\[
P^k_{L(l)} = P^0_{L(l)} + GGDF_{(l,k)}P^0_{G(k)} \tag{6}
\]

\[
P^k_{L(l)} = P^0_{L(l)} + LODF_{(l,k)}P^0_{L(k)} \tag{7}
\]

The Generalized Generation Distribution Factors \( GGDF \in \mathbb{R}^{N_L \times N_G} \) describe how a change in generation at generator \( k \) changes the line flow on line \( l \) [11], given that the power mismatch is balanced according to \( d \). The Line Outage Distribution Factors \( LODF \in \mathbb{R}^{N_L \times N_L} \) describe how the outage of line \( k \) influences the flow on line \( l \) [12]. The resulting SCOPF problem considering \( N_K = N_L + N_G \) outages is given by

\[
\min_{P_G^0} c^T P^0_G \tag{8}
\]

subject to

\[
C_G P^0_G - B \theta = P_D, \tag{9}
\]

\[
\theta_{\text{ref}} = 0, \tag{10}
\]

\[
-P^\text{max}_L \leq P^0_L \leq P^\text{max}_L, \tag{11}
\]

\[
P^\text{min}_G \leq P^0_G \leq P^\text{max}_G, \tag{12}
\]

\[
-P^\text{max}_L + LODF_{(l,k)}P^0_{L(k)} \leq P^\text{max}_L, \quad \text{for } k = 1, \ldots, N_L, \tag{13}
\]

\[
-P^\text{max}_L + GGDF_{(l,k)}P^0_{G(k)} \leq P^\text{max}_L, \quad \text{for } k = N_G, \ldots, N_K, \tag{14}
\]

\[
P^\text{min}_G \leq P^0_G + d_{(l,k)}P^0_{G(k)} \leq P^\text{max}_G, \quad \text{for } k = N_G, \ldots, N_K. \tag{15}
\]

The cost function is given by (8), with \( c \in \mathbb{R}^{N_G} \) being the cost coefficients of the generators. Eq. (9) is the nodal power balance constraint, with \( B \in \mathbb{R}^{N_B \times N_B} \) being the nodal admittance matrix and \( C_G \in \mathbb{R}^{N_B \times N_G} \) a matrix relating the generators to the buses. Eq. (10) sets the angle of the slack bus to zero. The remaining constraints refer to the limits on line flows and generators in normal operation (Eq. (11) and (12)) and in post-contingency conditions. Eq. (13) and (14) limits the post-contingency line flow after line and generator outages, and (15) limits the post-contingency generator output.

3.2. Risk-based SCOPF

For the RB-SCOPF, we consider risk-based constraints for the post-contingency line flows following either a generator outage or a line outage. The generator constraints (15) are not changed, since we do not consider generator trips as part of the cascading mechanism, but rather as a hard limit. To obtain the RB-SCOPF, we replace (13) and (14) by constraints that limit the risk as formulated in (1).
\[ Risk_{l|k} = Pr_k \cdot Pr_{l|k} \leq R_{limit}, \]  
where \( R_{limit} \) is the largest acceptable risk. Assuming that the outage probability \( Pr_k \) of the initial outage is known a-priori, (16) can be reformulated as a constraint on the probability of secondary trip \[ Pr_{l|k} \leq C_k, \]  
where \( C_k = \frac{R_{limit}}{Pr_k} \) is a contingency specific limit on \( Pr_{l|k} \). To include (17) in the OPF problem, we need to reformulate (17) as a limit on the post-contingency flow \( P_{L(l)}^k \). Since \( Pr_{l|k} \) is a piecewise affine function of \( P_{L(l)}^k \) (as given by (4) and shown in Fig. 2) and \( P_{L(l)}^k \) is not known a-priori, the reformulation is a quite difficult task. We therefore use an iterative approach which is explained in Section 3.3. However, before we explain the iterative process, we will analyse how constraint (17) is expressed for the three different cases in (4), i.e., for post-contingency flows in the range \( |P_{L(l)}^k| < 0.9 P_{L(l)}^{max}, 0.9 P_{L(l)}^{max} \leq |P_{L(l)}^k| < p_{L(l)}^{trip} \) or \( p_{L(l)}^{trip} \leq |P_{L(l)}^k| \).

We first formulate constraint (17) for post-contingency line flows \( 0.9 P_{L(l)}^{max} < |P_{L(l)}^k| < p_{L(l)}^{trip} \). For this case, \( Pr_{l|k} \) is given as the point-wise maximum of two functions in (4). Since (17) represents an upper bound on \( Pr_{l|k} \), we can avoid the max function by including both constraints in the optimization problem. For a line outage \( k \), we express the power flow change on line \( l \) as \( P_{L(l)}^k - P_{L(l)}^0 = LODF_{(L(k))} P_{G(k)}^0 \), and replace (17) by (18) and (19).

\[ \frac{LODF_{(L(k))} P_{G(k)}^0}{P_{L(l)}^k - P_{L(l)}^0} < C_k, \] \[ \frac{LODF_{(L(k))} P_{G(k)}^0}{P_{L(l)}^k - P_{L(l)}^0} < C_k. \]

With some manipulations, (18) and (19) can be expressed as the risk-based N-1 constraint

\[ -P_{L(l)}^{trip} < P_{L(l)}^0 + \frac{1}{C_k} LODF_{(L(k))} P_{G(k)}^0 < P_{L(l)}^{trip}. \]

For a generator outage \( k \), we express the line flow change on line \( l \) as \( P_{L(l)}^k - P_{L(l)}^0 = GGDF_{(L(k))} P_{G(k)}^0 \), and get a similar risk-based constraint

\[ -P_{L(l)}^{trip} < P_{L(l)}^0 + \frac{1}{C_k} GGDF_{(L(k))} P_{G(k)}^0 < P_{L(l)}^{trip}. \]

Comparing (20) to (13) and (21) to (14) reveals a very similar structure. However, when the probability of secondary trip is \( Pr_{l|k} \leq C_k < 1 \), the power flow change has a higher weight compared with the SCOPF formulation. On the other hand, since we assume that \( P_{L(l)}^{trip} > P_{L(l)}^{max} \) in most cases, the risk-based constraints (20), (21) will often be less tight than (13), (14).

For the case with high or low post-contingency line flows (i.e., \( |P_{L(l)}^k| \geq P_{L(l)}^{trip} \) or \( |P_{L(l)}^k| \leq 0.9 P_{L(l)}^{max} \)), the probability of secondary trip \( Pr_{l|k} \) is constant, and do not depend on the line flow. For the case of a high post-contingency line flow \( |P_{L(l)}^k| \geq P_{L(l)}^{trip} \), \( Pr_{l|k} = 1 \). This means that the outage \( k \) will lead to a trip of line \( l \) independent of how much \( P_{L(l)}^k \) exceeds \( P_{L(l)}^{trip} \). If we choose \( C_k \geq 1 \), we allow \( Pr_{l|k} = 1 \) and thus allow \( P_{L(l)}^k \) to become arbitrarily large, i.e., \( |P_{L(l)}^k| \leq \infty \). Thus, we can just delete constraint (17) for contingencies \( k \) with \( C_k \geq 1 \). For the case with low post-contingency line flow \( |P_{L(l)}^k| < 0.9 P_{L(l)}^{max} \), \( Pr_{l|k} = 0 \) independent of the value of \( P_{L(l)}^k \). This means that we do not need to constrain the post-contingency flow to a value lower than \( 0.9 P_{L(l)}^{max} \), and thus the tightest possible constraint on the line flow of line \( l \) after outage \( k \) is given by

\[ -0.9 P_{L(l)}^{max} \leq P_{L(l)}^0 + LODF_{(L(k))} P_{G(k)}^0 \leq 0.9 P_{L(l)}^{max}, \] \[ -0.9 P_{L(l)}^{max} \leq P_{L(l)}^0 + GGDF_{(L(k))} P_{G(k)}^0 \leq 0.9 P_{L(l)}^{max}. \]

To determine whether the constraints (20), (21) or the constraints (22), (23) should be enforced, we use the iterative optimization approach described below.
3.3. Iterative optimization

In a first step, we ensure that all constraints (17) with \( C_k \geq 1 \) are deleted, since the post-contingency flow is unbounded in this case. With \( C_k \in [0,1] \) for all contingencies \( k \), the iterative optimization can start. The starting point is the OPF problem given by (8) - (15), with (13), (14) replaced by (20), (21). We run the following iterative optimization:

1. Formulate the optimization with the risk-based constraints (20), (21).
2. Run the OPF
3. Check if any of the active constraints result in post-contingency line flows that are lower than \( 0.9 P_{l_{l(i)}}^{\text{max}} \).
4. If yes: Relax the particular constraint by replacing it with (22) or (23) → move to step 1.
5. If no: The optimal solution is found.

Since the optimization problem is a linear program, a constraint will never need to be changed back if it has been replaced once. The maximum number of iterations is thus finite and limited by the number of constraints which can be replaced. In practice, only a few iterations are needed. For small values of \( C_k \), more iterations are needed.

If \( C_k \) is sufficiently small, the initial OPF problem becomes infeasible. In this case, all the risk-based constraints (20), (21) are replaced by (22), (23), and the OPF is rerun. We then check whether any of active constraints can be relaxed if (22), (23) is replaced by (20) or (21). If yes, the optimization is run again, and if no, the optimal solution is found.

3.4. Choosing the risk limit

Including the risk-based constraints (20), (21) in the optimization problem allows us to control the risk level in the system according to our preferences, but also requires us to define the risk limit \( C_k \) in a reasonable way. We propose to use the solution of a deterministic SCOPF as a benchmark, and define the risk limit relative to the risk observed from the SCOPF solution. To achieve this, we evaluate the probability of secondary trip \( Pr_{l|k} \) and the corresponding risk of cascade initiation \( Risk_{l|k} \) for each outage \( k \) and each line \( l \) based on the SCOPF solution. The risk limit \( R_{\text{lim}} \) is defined relative to the maximum risk from the SCOPF as given by (24). Since the outage probabilities \( Pr_k \) can vary by orders of magnitude, we ensure that the probability limit \( C_k \) does not exceed \( C_{\text{lim}} \), the maximum \( Pr_{l|k} \) from the SCOPF solution, given by (25). The resulting limit \( C_k \) is given by (26) where the tuning parameters \( \alpha, \beta \) decide how conservative the limit on \( Pr_{l|k} \) should be relative to the SCOPF solution.

\[
R_{\text{lim}} = \max_{l,k} \left( Risk_{l|k}^{\text{SCOPF}} \right) \quad (24) \quad C_{\text{lim}} = \max_{l,k} \left( Pr_{l|k}^{\text{SCOPF}} \right) \quad (25)
\]

\[
C_k = \min \left( \frac{\alpha R_{\text{lim}}}{Pr_k}, \beta \cdot C_{\text{lim}} \right) \text{ for each contingency } k \in 1, ..., N_k \quad (26)
\]

4. Monte Carlo-based cascade simulation

To understand how a cascade would evolve after the cascade initiation (i.e., if a secondary outage takes place) and what the effect would be in terms of shed load, a Monte Carlo-based risk assessment including a full cascade simulation is implemented. For each cascade simulation, we assume that we have an initial outage \( k \) and then generate the secondary outages randomly based on the secondary outage probability \( Pr_{l|k} \). The further development of the cascade is determined by the line flows: If a line flow exceeds the capacity limit, the line is tripped. This process continues until the cascade terminates or the entire system collapses. If the system breaks up into islands, we assume that all generators participate in balancing the load/generation mismatch and that they can ramp up/down to their maximum/minimum output. If there is not enough available generation capacity, part of the load at each bus is shed. We choose to use a relatively simple cascade simulation for demonstration purposes, but the simulation could be made more sophisticated by considering, e.g., hidden failures in the protection system, frequency deviations or operator actions like redispatch [1, 9]. Although these extensions are not included here, they could be incorporated without any major changes to the method.
For each initial outage \( k \), we simulate the cascading process \( N_s \) times. After each cascade simulation \( i = 1, \ldots, N_s \), the load shed \( L_i^k \) is recorded and a new simulation is started. When \( N_s \) cascade simulations have been run, the expected load shed \( \overline{L}_k \) and the sample variance \( (s_k)^2 \) is calculated by (25) and (26).

\[
\overline{L}_k = \frac{1}{N_s} \sum_{i=1}^{N_s} L_i^k \quad (25) \quad (s_k)^2 = \frac{1}{N_s-1} \sum_{i=1}^{N_s} (L_i^k - \overline{L}_k)^2 \quad (26)
\]

The number of samples needed to guarantee that the actual expected load shed \( \overline{L}_k \) lies in the range \( \overline{L}_k - \varepsilon \leq L^k \leq \overline{L}_k + \varepsilon \) with a confidence level of 99.8% is given by [13]

\[
N_s^{new} = 3.08 \frac{(s_k)^2}{\varepsilon^2} \quad (27)
\]

If \( N_s < N_s^{new} \), we run another \( N_s^{new} - N_s \) simulations before \( N_s^{new} \) is calculated again.

When all contingencies \( k \) have been simulated, the total risk of cascading in the system, measured in terms of the total expected load shed, is calculated as the sum of the expected load shed per contingency, weighted by the contingency probability:

\[
E[L_{tot}] = \sum_{k=1}^{N_K} (Pr_k \cdot \overline{L}_k) \quad (28)
\]

Below, the Monte Carlo approach is described step by step:

1. Initialization for each contingency \( k, k \in 1, \ldots, N_K \):
   1.1 Load initial system data from the OPF solution (topology, load/generation profile).
   1.2 Take element \( k \) out of operation.
   1.3 Restore load/generation balance.
   1.4 Run DC power flow.
   1.5 Calculate \( Pr_i|_k \) for every line \( i \).

2. Monte-Carlo simulation for \( n_s \in 1, \ldots, N_S \):
   2.1 Draw a uniformly distributed random number \( \rho_{(i)} \sim \mathcal{U}(0,1) \) for each line.
   2.2 Check for outages: If \( Pr_i|_k > \rho_{(i)} \), outage line \( i \).

Compute the losses for this situation:
   2.3 Check for island in the system and restore load/generation balance.
   2.4 Record shed load \( L_i \).
   2.5 Run DC power flow.
   2.6 Remove lines with \( P_{L,(i)} > P_{L_{max}} \). Are there new outages? → move to step 2.3.
   2.7 Update \( n_s = n_s + 1 \). If \( n_s < N_s \) → move to 2.1.

Check if \( N_s \) samples is enough to achieve desired accuracy:
   2.8 Calculate the mean load shed \( \overline{L}_k \) and the sample variance \( (s_k)^2 \).
   2.9 Calculate number of samples \( N_s^{new} \) needed to achieve the desired accuracy.
      If \( N_s < N_s^{new} \), set \( N_s = N_s^{new} \) and move to 2.1.
      If \( N_s \geq N_s^{new} \), save \( \overline{L}_k \).

3. If \( k < N_K \), set \( k = k + 1 \) → move to 1.1
4. If \( k = N_K \), compute total expected load shed \( E[L_{tot}] \).

5. Case study

We demonstrate the proposed method on the IEEE 118 bus system [14]. The system has \( N_L = 186 \) transmission lines and \( N_G = 54 \) generators. The load level is set to 110% of the base case load, and we assume that the generator capacity is 125% higher than the values listed in [14]. The line outage probabilities were calculated based on the line length (estimated from the line reactance) and the average outage probability for transmission lines in Germany [8]. Due to lack of reliable data for the generator outage probabilities, we only consider line outages in the calculation of the risk in this paper. In the RB-SCOPF, we thus consider the optimization problem (8)-(15), with (13) replaced by (20). For the Monte Carlo-based risk assessment, we compute the total expected load shed based on the expected load shed for the line outages \( k = 1, \ldots, N_L \).
5.1. Comparison of \( Pr_{l|k} \) and \( Risk_{l|k} \) for the SCOPF and the RB-SCOPF

We first compare \( Pr_{l|k} \) and \( Risk_{l|k} \) for the standard SCOPF and the RB-SCOPF. For the SCOPF solution, \( Risk_{l|k} \) and \( Pr_{l|k} \) are evaluated a-posteriori based on (1) and (4). The risk limits \( C_k \) for the RB-SCOPF are calculated from (26), based on the SCOPF solution. We run the RB-SCOPF with two different risk limits, computed with parameters \((\alpha, \beta) = (0.8, 1)\) and \((\alpha, \beta) = (1, 1)\), and evaluate all \( Risk_{l|k} \) and \( Pr_{l|k} \) for both solutions. We then compare the values for \( Risk_{l|k} \) and \( Pr_{l|k} \) obtained with the SCOPF and the RB-SCOPF.

Fig. 3 shows all non-zero \( Pr_{l|k} \) for the SCOPF and the RB-SCOPF with \( \alpha = 0.8 \) and \( \alpha = 1 \). The highest probability of secondary trip in the SCOPF solution is \( Pr_{37|8} = 0.82 \) (denoting the probability of secondary trip of line 37 following an outage of line 8). Comparing the results, we see that the RB-SCOPF increases \( Pr_{l|k} \) in some cases (e.g., for \( Pr_{41|33} \)), and decreases \( Pr_{l|k} \) in other cases (e.g., \( Pr_{159|153} = 0 \) for the RB-SCOPF with \( \alpha = 0.8 \)).

Fig. 4 shows the risk of cascade initiation \( Risk_{l|k} \), i.e., \( Pr_{l|k} \) weighted by \( Pr_k \). Comparing the \( Risk_{l|k} \) with \( Pr_{l|k} \), we see that some of the outages leading to a high \( Pr_{l|k} \) have a low risk \( Risk_{l|k} \). This is because \( Pr_k \) is low. The highest \( Risk_{l|k} \) in the SCOPF is \( Risk_{159|153}^{SCOPF} = 3.41 \times 10^{-5} \). For the RB-SCOPF with \( \alpha = 1 \), \( Risk_{159|153} = 0 \) and the highest risk is \( Risk_{41|33}^{RB-SCOPF} = 0.8 \cdot Risk_{159|153}^{SCOPF} \).

From Fig. 3 and 4, we see that there are very few outages that can lead to secondary trips, i.e., only 16 out of totally 186 possible line outages. Further, we recognize which of the lines that are influenced by each other. We see that there are several outages that might lead to an outage of line 129 (connecting bus 82 and 83), helping us to recognize line 129 as a vulnerable point in the system.

Figure 3: All outages \( k \) that lead to a non-zero probability of secondary trip \( Pr_{l|k} \) are shown on the horizontal axis, and the affected lines \( l \) are listed on the vertical axis. The color of the corresponding dots encodes the value of \( Pr_{l|k} \). Only 16 outages that are listed, because all other outages have \( Pr_{l|k} = 0 \). From the top: SCOPF, RB-SCOPF with \( \alpha = 0.8 \) and RB-SCOPF with \( \alpha = 1 \).

Figure 4: All outages \( k \) that lead to a non-zero probability of secondary trip \( Risk_{l|k} \) are shown on the horizontal axis, and the affected lines \( l \) are listed on the vertical axis. The color of the corresponding dots encodes the value of \( Risk_{l|k} \). Only 16 outages that are listed, because all other outages have \( Risk_{l|k} = 0 \). From the top: SCOPF, RB-SCOPF with \( \alpha = 0.8 \) and RB-SCOPF with \( \alpha = 1 \).
5.2. Comparison of the generation cost and the expected load shed for the SCOPF and the RB-SCOPF

Above, we demonstrated how \( \text{Risk}_{l|k} \) and \( \text{Pr}_{l|k} \) can be controlled with the RB-SCOPF. Now, we investigate how the cost of the generation dispatch is influenced by the choice of the risk limit parameter \( \alpha \), and compare the obtained generation cost with the expected load shed for each of the solutions. We then analyse the outages \( k \) that lead to a non-zero load shed in more detail to understand how the cascade evolves. Again, we use the standard SCOPF as a benchmark, and consider risk limits given by \( \beta = 1 \) and \( \alpha = \{0.5, \ldots, 1, \ldots, 1.5\} \) for the RB-SCOPF.

The generation cost decreases with increasing \( \alpha \), as shown in Fig. 5. This is because a higher value for \( \alpha \) leads to higher values of \( C_k \), which leads to a relaxation of (20). For high risk limits, the cost decrease saturates. In this case, the \( C_k \) of all the active constraints are bounded by the max. probability of secondary trip from the SCOPF, \( \text{Pr}_{l|k} = 0.82 \). For \( \alpha = 1 \) (i.e., when the risk limit in the RB-SCOPF is equal to the maximum risk of the SCOPF), the cost of the RB-SCOPF is lower than the cost of the SCOPF. Fig. 6 shows the generation cost plotted against the expected load shed of the SCOPF and each of the RB-SCOPF solutions. In most cases, a higher generation cost (meaning a lower value of \( \alpha \)) leads to a lower load shed. This decrease is however not monotonous. There is one case with an unexpected high load shed, the case where \( \alpha = 0.8 \). To explain this high load shed, we investigate how the cascade evolves in more detail.

Fig. 7 shows the expected load shed for the line outages with non-zero \( \text{Pr}_{l|k} \). For some of the outages, the cascade stops before it leads to any load shed. For outages of the lines 97 and 144-150, the expected load shed is relatively similar, and the differences are due to different values for \( \text{Pr}_{l|k} \). For outage 153, the differences in expected load shed are however quite large, and the expected load shed is high particularly for low values of \( \alpha \), e.g., for \( \alpha = 0.8 \). When analysing the situation after the outage of line 153 in more detail, we see that there are two lines with \( \text{Pr}_{l|k} > 0 \) and thus four possible outage combinations that can occur.
The four combinations and their corresponding load shed are listed in Table I. Notably, case II with secondary trip of line 129 leads to 37.4 MW load shed, while case IV with a secondary trip of both 129 and 159 leads to 0 MW of shed load. Assuming that the secondary outage of line 129 and line 159 are independent events, the probability of case II is given by

\[ P_{\text{Case II}} = Pr_{129|153} \cdot (1 - Pr_{159|153}) \]

A high value for \( Pr_{159|153} \) decreases \( Pr_{\text{Case II}} \), and thus decreases the expected load shed of the initial outage 153. From Fig. 3, we see that \( Pr_{159|153} = 0 \) when \( \alpha = 0.8 \). The high expected load shed is due to a high probability of case II.

This analysis illustrates that it is not possible to control the expected amount of load shed only by controlling the risk of cascade initiation. A lower risk limit might increase the expected load shed in some cases, in particular when there are several possible secondary outages involved.

Table 1 Possible developments of the cascade after the outage of line 153.

<table>
<thead>
<tr>
<th>Case</th>
<th>Secondary outage</th>
<th>Development of cascade</th>
<th>Load shed</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-</td>
<td>No further outages</td>
<td>0 MW</td>
</tr>
<tr>
<td>II</td>
<td>129</td>
<td>Cascading outage, system breaks into islands</td>
<td>37.4 MW</td>
</tr>
<tr>
<td>III</td>
<td>159</td>
<td>No further outages</td>
<td>0 MW</td>
</tr>
<tr>
<td>IV</td>
<td>129, 159</td>
<td>Loss of large generator at bus 99, no further outages</td>
<td>0 MW</td>
</tr>
</tbody>
</table>

6. Conclusion

This paper proposes a new, risk-based security measure, which aims at reflecting the risk of cascading events in a risk-based optimal power flow (OPF) formulation. The risk-based security criterion is based on the probability of cascade initiation, i.e., the risk that one initial N-1 contingency can lead to secondary outages. The probability of secondary trips was modelled as a function of both the post-contingency line flow and the change in line flow from the pre- to the post-contingency state. Based on the proposed criterion, the RB-SCOPF was formulated, which allows us to control the risk of cascade initiation through the use of risk-based N-1 constraints. Further, the risk-based security criterion was implemented in a Monte Carlo-based simulation framework, which allows us to compute the expected load shed for a given generation dispatch.

The RB-SCOPF and the Monte Carlo based assessment is applied to a case study for the IEEE 118 bus system, and compared with the outcome of a standard SCOPF. We show that the RB-SCOPF can effectively reduce the risk of cascade initiation, but that the reduction in risk comes at an increased cost. Based on the Monte Carlo simulation, we see that in most cases are able to reduce the expected load shed by reducing the risk limit in the RB-SCOPF. However, dependent on how the cascade actually evolves, a lower risk limit might in some cases increase the total expected load shed, illustrating the fact that it is not possible to control the expected amount of load shed by only controlling the risk of cascade initiation.

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BIBLIOGRAPHY


