"Optimal Coupling of Energy Infrastructures"

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Optimal Coupling of Energy Infrastructures

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***Abstract*— This paper presents a framework for integrated modeling and optimization of energy systems with multiple energy carriers. Based on the concept of energy hubs, a generic steady-state model for describing conversion and storage of multiple energy carriers, such as electricity, natural gas, hydrogen, or district heating, is developed and used for system optimization. Besides operational optimization of energy flows, the optimal structure of the system is investigated. Mathematically, the problems are stated as (mixed-integer) nonlinear programming problems. An example demonstrates the use and potential applications of the proposed method and highlights its features.**

I. Introduction

Nowadays common energy infrastructures such as electricity, natural gas, and local district heating systems, are mostly planned and operated independently. Motivated by different reasons, a number of recent publications suggest an integrated system view including multiple energy carriers, instead of focusing on a single carrier (see, e.g., [1]–[7]).

Various tools have been developed for the integrated analysis of energy systems employing multiple energy carriers, which are commonly denoted "multiple energy carrier systems" or "multi-carrier systems". In particular, the integrated optimization of the system structure and coordinated optimal operation were investigated. The authors of this paper contributed to the field with approaches for operational power flow optimization as well as optimization of the system structure [6], [7]. These approaches are based on the concept of "energy hubs", which can be considered functional units where multiple energy carriers are converted, stored, and dissipated [8]. This paper presents some recent achievements in modeling and optimization of energy hubs:

- Inclusion of storage in the energy hub model.
- Implementation of multi-period optimization, thereby enabling to address optimal storage utilization.
- Determination of optimal hub layouts by selecting the best-fitting elements from a set of available converter and storage devices to be placed in the hub.

In the following sections, the concept of energy hubs is shortly reviewed and a mathematical model for describing the steady-state energy flows through energy hubs is presented. This model is then used for hub optimization, both in terms of operation (optimal energy conversion and storage) and planning (optimal hub layout). An example demonstrates the basic features of the approaches.

II. The Energy Hub Concept

Ref. [8] presents the concept of energy hubs, which can be identified as units where multiple energy carriers are converted and/or stored. An energy hub represents a generalization or extension of a network node in an electrical system. Fig. 1 shows a simple example of a typical energy hub which interfaces energy infrastructures with a commercial load. Electricity and natural gas are delivered by the networks. These carriers are converted and stored within the hub in order to fulfill the requirements of the load, which demands electricity and heat.

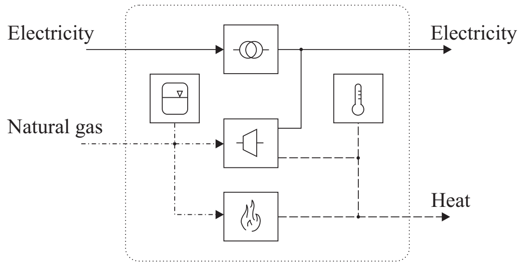

Fig. 1. Example of an energy hub for the supply of a commercial load. Electricity and natural gas are provided at the hub input; the load requires electricity and heat. The hub contains a gas tank, an electrical transformer, a gas turbine, a furnace, and a heat storage.

There are a number of examples of real facilities that can be modeled as energy hubs, for example the supply of industrial plants (steel works, paper mills), power plants (co- and trigeneration [9]), or the supply of big building complexes (shopping malls, airports). The energy hub can be regarded as a general modeling concept to describe the interactions between different energy carriers in an energy system.

Combining and coupling different energy carriers in energy hubs keeps a number of potential advantages over conventional energy supply:

- *Increased Reliability:* Energy hubs generally increase the availability of energy for the load, because it is no longer fully dependent on a single infrastructure [10].
- *Increased Load Flexibility:* Redundant paths within the hub offer a certain degree of freedom in supplying the load. Hubs can thereby substitute for unattractive energy carriers, for example at high-tariff times.
- *Synergy Effects:* Energy hubs process various energy carriers, each of which showing specific characteristics. Electricity, for example, can be transmitted over long distances with comparably low losses. Chemical carriers can be stored employing relatively simple and cheap technology.

So far, the energy hub concept has been used for greenfield design studies [8], the characterization of trigeneration plants [9], and the conception of fuel cell systems [11].

III. Modeling

In this section, a steady-state model for describing energy flows through energy hubs is presented. Models for energy conversion and storage are integrated into a generic energy hub

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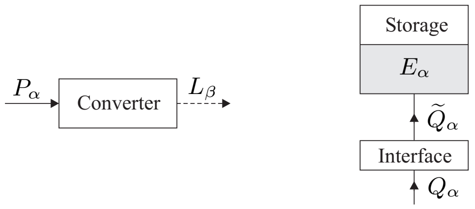

(a) Converter element with steady-state power input P_α and output L_β.

(b) Storage element exchanging the power Q_α; internal power $\widetilde{Q}_\alpha$, stored energy E_α.

Fig. 2. Converter and storage models.

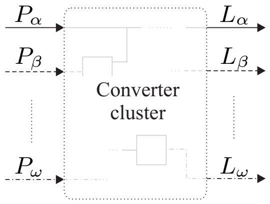

Fig. 3. Converter cluster with multiple in- and outputs.

model, which can be applied to any configuration of converter and storage elements. The model is based on the following assumptions and simplifications:

- The system is considered to be in steady-state.
- Within energy hubs, losses occur only in converter and storage elements.
- If not mentioned explicitly, unidirectional power flow from the input to the output of the converters is assumed.
- Power flow through the hub is characterized through energy, power, and efficiency only.

A. Converter Model

Consider a converter device as indicated in Fig. 2(a) that converts an input energy carrier α into another carrier β. Input and output power flows can be considered to be coupled:

$$L_\beta = c_{\alpha\beta} P_\alpha \tag{1}$$

where P_α and L_β are the steady-state power in- and outputs, respectively. $c_{\alpha\beta}$ is the *converter coupling factor* which defines the coupling between input and output power flow. In the simplest case, this factor corresponds to the converter's steady-state energy efficiency, which can be assumed constant or as a function of the converted power.

Converter elements or combinations of different energy converters may have multiple in- and outputs. A general model covering multiple inputs and outputs can be stated according to Fig. 3. Summarizing all power inputs and outputs in vectors $\mathbf{P}$ and $\mathbf{L}$, respectively, enables the formulation of multi-input multi-output conversion analogous to (1):

$$\underbrace{\begin{bmatrix} L_\alpha \\ \vdots \\ L_\omega \end{bmatrix}}_{\mathbf{L}} = \underbrace{\begin{bmatrix} c_{\alpha\alpha} & \cdots & c_{\omega\alpha} \\ \vdots & \ddots & \vdots \\ c_{\alpha\omega} & \cdots & c_{\omega\omega} \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} P_\alpha \\ \vdots \\ P_\omega \end{bmatrix}}_{\mathbf{P}} \tag{2}$$

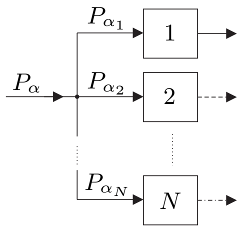

Fig. 4. Dispatch of total input power P_α to converters $1, 2, \ldots, N$.

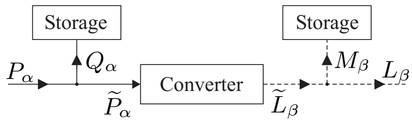

Fig. 5. $\alpha\beta$-converter with α-storage at the input and β-storage at the output.

The *converter coupling matrix* $\mathbf{C}$ describes the mapping of the powers from the input to the output of the converter cluster. The entries of the coupling matrix are converter coupling factors as defined in (1). Each coupling factor relates one particular input to a certain output. The coupling matrix can be assumed constant or as a function of the powers.

In contrast to single-input single-output converters, the converter coupling factors are in general no longer equal to energy efficiencies when considering multiple in- and outputs. Since the total input of an energy carrier may split up to several converters, so-called *dispatch factors* are introduced that define the dispatch of the total power input to the elements converting this carrier. Fig. 4 outlines this concept. The total input flow P_α splits up to N converter devices. The dispatch factors ν_{α_k} specify how much of the total input power P_α flows into converter k:

$$P_{\alpha_k} = \nu_{\alpha_k} P_\alpha \tag{3}$$

where $k = 1, 2, \ldots, N$. Due to conservation of power, the dispatch factors must fulfill the following requirements:

$$0 \leq \nu_{\alpha_k} \leq 1 \quad \forall\alpha, \forall k \tag{4a}$$

$$\sum_k \nu_{\alpha_k} = 1 \quad \forall\alpha \tag{4b}$$

B. Storage Model

Consider the storage model in Fig. 2(b) which comprises a storage interface and an ideal storage. The relation between power exchange Q_α and internally stored energy E_α is stated as follows:

$$\widetilde{Q}_\alpha = e_\alpha Q_\alpha = \mathrm{d}E_\alpha/\mathrm{d}t \approx \Delta E_\alpha/\Delta t \triangleq \dot{E}_\alpha \tag{5}$$

where

$$e_\alpha = \begin{cases} e_\alpha^+ & \text{if } Q_\alpha \geq 0 \quad \text{(charging/standby)} \\ 1/e_\alpha^- & \text{else} \quad \text{(discharging)} \end{cases} \tag{6}$$

e_α can be considered the storage efficiency, including the efficiency of the storage interface, which converts the energy carrier exchanged with the system into the carrier stored internally.[1]

[1]We use the same subscript α for power and energy, even if the storage may convert one energy carrier into another one.

Energy hubs may contain storage elements at their input and output sides, and between converters connecting in- and outputs (see Fig. 5). The power flowing into the converter equals the total hub input P_α minus the storage power Q_α. At the output of the converter, the sum of the load L_β and storage flow M_β is provided:

$$\widetilde{P}_\alpha = P_\alpha - Q_\alpha \tag{7a}$$

$$\widetilde{L}_\beta = L_\beta + M_\beta \tag{7b}$$

The power flow coupling between the input and the output of the hub is then described by

$$[\mathbf{L} + \mathbf{M}] = \mathbf{C}\ [\mathbf{P} - \mathbf{Q}] \tag{8}$$

where $\mathbf{M}$ keeps all output side storage powers, and $\mathbf{Q}$ contains the input side storage flows. All storage influence can be summarized in an equivalent storage flow vector related to the output side. Assuming a constant coupling matrix $\mathbf{C}$, superposition can be applied and the equivalent storage flows can be stated as

$$\mathbf{M}^{\text{eq}} = \mathbf{C}\,\mathbf{Q} + \mathbf{M} \tag{9}$$

Now (8) can be written as

$$\mathbf{L} = \mathbf{C}\,\mathbf{P} - \mathbf{M}^{\text{eq}} \tag{10}$$

This equation enables the transformation of storage flows between inputs and outputs of converters.

With (5) and (9) we can calculate the equivalent storage power flows $\mathbf{M}^{\text{eq}}$ directly from the storage energy derivatives:

$$\underbrace{\begin{bmatrix} M_\alpha^{\text{eq}} \\ \vdots \\ M_\omega^{\text{eq}} \end{bmatrix}}_{\mathbf{M}^{\text{eq}}} = \underbrace{\begin{bmatrix} s_{\alpha\alpha} & \cdots & s_{\omega\alpha} \\ \vdots & \ddots & \vdots \\ s_{\alpha\omega} & \cdots & s_{\omega\omega} \end{bmatrix}}_{\mathbf{S}} \underbrace{\begin{bmatrix} \dot{E}_\alpha \\ \vdots \\ \dot{E}_\omega \end{bmatrix}}_{\dot{\mathbf{E}}} \tag{11}$$

The *storage coupling matrix* $\mathbf{S}$ describes how changes of the storage energies affect the hub output flows. In other words, it maps all storage energy derivatives into equivalent output-side flows. The entries of the storage coupling matrix are called *storage coupling factors*.

C. Complete Energy Hub Model

Based on the previous equations, the flows through an energy hub are modeled by the following relation:

$$\mathbf{L} = \mathbf{C}\,\mathbf{P} - \mathbf{S}\,\dot{\mathbf{E}} \tag{12}$$

This fundamental equation is the basis for operational and structural hub optimization. The following example demonstrates how this model is derived for a given combination of converter and storage elements.

D. Example

Consider the hub shown in Fig. 1 which contains three converters and two storage devices: transformer, gas turbine, and heat exchanger; gas tank and hot water storage. The transformer's steady-state energy efficiency is denoted $\eta_{\text{ee}}^{\text{T}}$. The gas turbine is characterized by its gas-electric and gas-heat efficiencies $\eta_{\text{ge}}^{\text{GT}}$ and $\eta_{\text{gh}}^{\text{GT}}$, respectively. The furnace operates with an efficiency $\eta_{\text{gh}}^{\text{F}}$. Efficiencies of the storage devices are e_{g} and e_{h}, for the gas tank and the heat storage, respectively. The electrical connection between input and output is assumed to be lossless.

The converters (transformer, gas turbine, furnace) are described by the following matrix:

$$\mathbf{C} = \begin{bmatrix} \eta_{\text{ee}}^{\text{T}} & \nu\eta_{\text{ge}}^{\text{GT}} \\ 0 & \nu\eta_{\text{gh}}^{\text{GT}} + (1-\nu)\eta_{\text{gh}}^{\text{F}} \end{bmatrix} \tag{13}$$

where the dispatch factor ν defines which part of the total input P_{g} is converted by the gas turbine. The storage elements exchange the powers

$$Q_{\text{g}} = \frac{\dot{E}_{\text{g}}}{e_{\text{g}}}; \quad M_{\text{h}} = \frac{\dot{E}_{\text{h}}}{e_{\text{h}}} \tag{14}$$

Now (8) can be formulated:

$$\begin{bmatrix} L_{\text{e}} \\ L_{\text{h}} + \frac{\dot{E}_{\text{h}}}{e_{\text{h}}} \end{bmatrix} = \begin{bmatrix} \eta_{\text{ee}}^{\text{T}} & \nu\eta_{\text{ge}}^{\text{GT}} \\ 0 & \nu\eta_{\text{gh}}^{\text{GT}} + (1-\nu)\eta_{\text{gh}}^{\text{F}} \end{bmatrix} \begin{bmatrix} P_{\text{e}} \\ P_{\text{g}} - \frac{\dot{E}_{\text{g}}}{e_{\text{g}}} \end{bmatrix} \tag{15}$$

Alternatively, the hub can be described according to (12). Therefore all storage flows have to be transformed to equivalent output-side flows:

$$M_{\text{e}}^{\text{eq}} = \frac{\nu\eta_{\text{ge}}^{\text{GT}}}{e_{\text{g}}}\dot{E}_{\text{g}} \tag{16a}$$

$$M_{\text{h}}^{\text{eq}} = \frac{\nu\eta_{\text{gh}}^{\text{GT}} + (1-\nu)\eta_{\text{gh}}^{\text{F}}}{e_{\text{g}}}\dot{E}_{\text{g}} + \frac{1}{e_{\text{h}}}\dot{E}_{\text{h}} \tag{16b}$$

From these equations, the storage coupling matrix can be extracted:

$$\mathbf{S} = \begin{bmatrix} \frac{\nu\eta_{\text{ge}}^{\text{GT}}}{e_{\text{g}}} & 0 \\ \frac{\nu\eta_{\text{gh}}^{\text{GT}} + (1-\nu)\eta_{\text{gh}}^{\text{F}}}{e_{\text{g}}} & \frac{1}{e_{\text{h}}} \end{bmatrix} \tag{17}$$

Finally, (12) can be stated:

$$\begin{bmatrix} L_{\text{e}} & L_{\text{h}} \end{bmatrix}^{\text{T}} = \mathbf{C} \begin{bmatrix} P_{\text{e}} & P_{\text{g}} \end{bmatrix}^{\text{T}} - \mathbf{S} \begin{bmatrix} \dot{E}_{\text{g}} & \dot{E}_{\text{h}} \end{bmatrix}^{\text{T}} \tag{18}$$

IV. Optimization

Various optimization problems addressing the operation as well as the structure of energy hubs are of interest. In the following, we discuss two of these problems based on the following assumptions and simplifications:

- The loads at the hub outputs are inelastic, i.e., they consume constant power within the time period considered.
- Penalties related to the individual energy carriers and system components are independent and separable from each other.

A. Optimal Hub Operation

Considering a given hub structure, questions related to its optimal utilization arise, for example how much of which energy carrier should be consumed, and how should energy be converted and stored internally? Consider for example the energy hub in Fig. 1. The heat demand can either be met by converting natural gas in the furnace or in the gas turbine. The electricity load is supplied redundantly as well. An approach

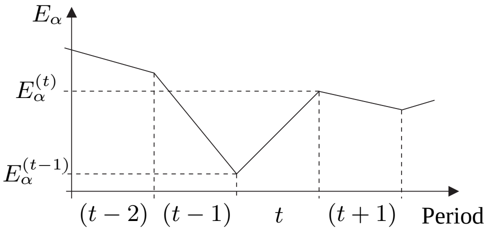

Fig. 6. Storage energy E_α at different time periods t. $E_\alpha^{(t)}$ refers to the energy stored at the end of period t.

for optimally dispatching the converters within an energy hub for a single snapshot of the load is presented in [6]. Including also the storage elements requires the implementation of multi-period optimization, because it is pointless to consider storage utilization at a single time instant. The approach from [6] is now extended by including storage and considering multiple time periods. The optimization problem is stated as a nonlinear constrained problem comprising an objective function, equality and inequality constraints [12].

The storage elements are basically modeled as outlined in Sec. III-B. When storage elements are present in the hub, the objective function may also depend on the stored energies E_α, besides the power consumption P_α and dispatch factors ν_{α_k}. For the multi-period optimization, the total objective corresponds to the sum of the objectives of each period:

$$\mathcal{F} = \sum_{t=1}^{N_t} \mathcal{F}^{(t)} \left(P_\alpha^{(t)}, \nu_{\alpha_k}^{(t)}, E_\alpha^{(t)} \right) \tag{19}$$

where N_t is the number of time periods considered, and $\mathcal{F}$ is the scalar-valued objective function.

An equality constraint is given by the hub power flow equation (12). This equation contains the storage energy derivatives $\dot{E}_\alpha$. As indicated in Fig. 6, the change of storage energy within a period t is

$$\dot{E}_\alpha^{(t)} = E_\alpha^{(t)} - E_\alpha^{(t-1)} + E_\alpha^{\text{stb}} \tag{20}$$

Here $E_\alpha^{(t)}$ denotes the storage energy at the end of period t; E_α^{stb} represents the standby energy losses of the α-storage per period ($E_\alpha^{\text{stb}} \geq 0$). Merging (12) and (20) yields the hub flow requirement for each period:

$$\mathbf{L}^{(t)} = \mathbf{C}^{(t)}\mathbf{P}^{(t)} - \mathbf{S}^{(t)} \left[\mathbf{E}^{(t)} - \mathbf{E}^{(t-1)} + \mathbf{E}^{\text{stb}} \right] \quad \forall t \tag{21}$$

In order to obtain sustainable storage utilization, another equality constraint can be included in the problem formulation which requires that the storage energies at the end of the last period of the studied time interval are equal to the initial energies:[2]

$$E_\alpha^{(0)} = E_\alpha^{(N_t)} \quad \forall \alpha \tag{22}$$

Inequality constraints are given by power and energy limits of the converter and storage elements:

$$\underline{P}_{\alpha_k} \leq \nu_{\alpha_k} P_\alpha^{(t)} \leq \overline{P}_{\alpha_k} \qquad \forall t, \forall \alpha \tag{23a}$$

$$\underline{Q}_\alpha \leq Q_\alpha^{(t)} \leq \overline{Q}_\alpha \qquad \forall t, \forall \alpha \tag{23b}$$

$$\underline{M}_\beta \leq M_\beta^{(t)} \leq \overline{M}_\beta \qquad \forall t, \forall \beta \tag{23c}$$

$$\underline{E}_\alpha \leq E_\alpha^{(t)} \leq \overline{E}_\alpha \qquad \forall t, \forall \alpha \tag{23d}$$

The characteristics of the dispatch factors (4) represent additional inequalities.

Optimal operation of the hub can now be formulated in the following way:[3]

Minimize the objective function (19)
subject to the

- hub equation (21);
- storage energy constraints (22);
- power and energy limits (23);
- dispatch factor properties (4).

In general, this formulation represents a nonlinear constrained optimization problem. The solvability of the problem depends on the actual equations used. When the objective function is convex and all constraints are expressed as linear equations, then the solution space is convex and the global optimum can be determined using numerical methods. Otherwise numerical methods can be used to search for a solution within the feasible region, but global optimality cannot be guaranteed.

B. *Optimal Hub Layout*

When designing energy hubs, a limited number of converter and storage elements is available. Different combinations of elements will result in different hub characteristics. In order to obtain the desired hub performance, optimization can be employed. The problem is to find the optimal selection of elements to be placed in an energy hub from a given set of converter and storage elements. Available elements can be characterized by technical, environmental, and economic parameters. The performance of the hub may depend not only on its elements but also on the situation outside of the hub, i.e., the supplying infrastructures and the loads. For example, certain elements will perform well in low load situations, while others may be advantageous when loads are high. The supply infrastructures may be characterized by time-dependent energy prices, which is another aspect to consider. Utilization of a certain element may be profitable as long as electricity is cheaper than natural gas. Also the combination of elements is important. Certain devices may perform well only when they are used together with others. These considerations yield the conclusion that the whole system (supply infrastructures, energy hub, and loads) has to be considered when determining the optimal hub layout. A multi-period optimization is performed for given loads and energy prices. The optimization model comprises on/off-type variables assigned to each available element; this integer variable I_k

[2]This is an intuitive assumption which could be reconsidered against the background of a liberalized market environment. Under certain circumstances, it could be reasonable to exploit the storage more or less, ending up with lower or higher storage energy at the end of the last period.

[3]A multi-period approach could also include constraints related to the change of a quantity between two periods, such as ramping, minimum up- and downtimes, etc. [13]. For the sake of simplicity, such limits are not considered, but can be included in the approach.

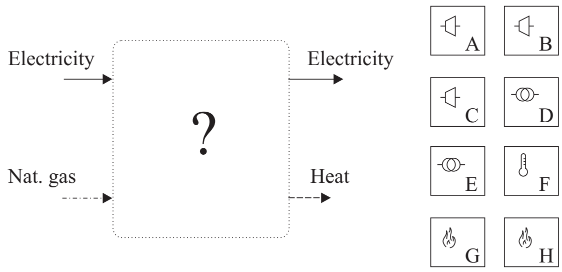

Fig. 7. Hub to be filled with available elements A, B, ..., H.

represents the decision whether element k is used in the hub layout or not:

$$I_k = \begin{cases} 1 & \text{if element } k \text{ is used} \\ 0 & \text{else} \end{cases} \tag{24}$$

For economic optimization, the objective function should penalize the cost for energy, operation, maintenance, and installation. Installation cost are fixed cost which occur if the element is used for the hub. Maintenance and operation cost are normally dependent on the operation of the device. The cost for energy depend on the operation of the hub as well, which is determined by the loads. It can be concluded that the objective function generally depends on the decision variables as well as on the energy and power quantities related to the hub and its elements:

$$\mathcal{F} = \sum_{t=1}^{N_t} \mathcal{F}^{(t)} \left(P_\alpha^{(t)}, \nu_{\alpha_k}^{(t)}, E_\alpha^{(t)}, I_k \right) \tag{25}$$

Equality constraints are given by the hub model (21) and the storage requirement (22). Converter and storage coupling matrices contain the decision variables I_k, where every entry dedicated to an element k is multiplied by I_k. Also restrictions related to the individual elements include these integer variables.

The optimal hub layout problem is then basically defined as the operational problem in Sec. IV-A, but the model equations include the decision variables I_k. If $I_k = 1$, then element k is considered in the model equations; if $I_k = 0$, the related quantities, equations, and constraints vanish.

The problem of optimal hub layout represents a mixed-integer nonlinear constrained problem [12]. Both the objective function as well as the constraints may include nonlinearities and discontinuities, which results in a nonconvex solution space. Numerical solvers can be used to find a solution, but it cannot be ensured that the global optimum has been achieved.

V. Example

Fig. 7 shows an empty hub and a set of available elements. The hub is connected to electricity and natural gas networks at the input side, and it has to deliver electricity and heat to a commercial load at the output. All elements are characterized by different ratings, efficiencies, and installation cost, see Tab. I and II. The loads to be supplied at the output side and the energy prices at the input side of the hub are shown in Fig. 8.

The first three potential hub elements A, B, and C are different combined heat and power technologies. Compared with B and C, option A shows the highest efficiencies and lowest installation cost, but also the lowest rating. Option B has the highest rating, but most of the gas is converted into heat instead of more expensive electricity. Option C has the highest investment cost, but both rating and efficiencies are high as well. Elements D and E are electrical transformers. D is smaller than E, less efficient, but also cheaper. The converters G and H represent gas furnaces with different properties. Device G is smaller than H but more efficient, whereas installation costs are equal. Also a thermal storage element could be used for the hub.

TABLE I

Data of Converter Elements.

Converter element k	Max. input power in pu	Efficiencies in % el.	th.	$\sum$	Installation cost b_k in 1000 mu
A	10	43	43	86	100
B	20	25	55	80	250
C	15	32	53	85	300
D	7	97	–	97	30
E	10	98	–	98	40
G	7	–	75	75	40
H	10	–	80	80	40

TABLE II

Data of Storage Element F.

Charge efficiency in %	90
Discharge efficiency in %	90
Min./max. power in pu	$-3/3$
Min./max. energy in pu	0.5/10
Standby losses in pu	0.2
Installation cost b_F in 1000 mu	15

The investment cost for the hub should be depreciated within 10 years. During this time, the hub is intended to meet the given 12-hour load requirement 365 days a year (beyond these 12 hours, the load can be neglected). Incorporating the installation cost of the devices into a 12-hour optimization requires to relate the installation cost to one 12-hour load cycle. This can be done by transforming the total installation cost:

$$b'_k = \frac{b_k}{T_\text{d} \cdot N_\text{c}} \tag{26}$$

where T_d is the depreciation time (in years) and N_c is the number of load cycles the hub has to perform within one year (in cycles per year). In this example we have $T_\text{d} = 10\,\text{a}$ and $N_\text{c} = 365\ \text{a}^{-1}$. The total cost for one 12-hour load cycle are modeled as the cost for energy plus the transformed installation cost of the devices used for the hub:

$$\text{TC} = \sum_{t=1}^{12} \left(a_\text{e}^t P_\text{e}^t + a_\text{g}^t P_\text{g}^t \right) + \sum_k b'_k \cdot I_k \tag{27}$$

The questions to be answered are (a) which elements should be used in the hub, and (b) how should these elements be operated in order to obtain minimum total cost TC? The problem is stated as outlined in Sec. IV-B, with the additional constraint that only one element of each category can be used (max. one CHP, one transformer, one furnace). Implementation is done in Matlab [14] using the solver “minlpBB” from Tomlab [15].

The answer to the above question (a) is shown in Fig. 9. Elements A, E, and F should be used. The CHP unit A is selected due to its high power-to-heat ratio and low installation cost. Transformer E is used instead of D due to higher efficiency and

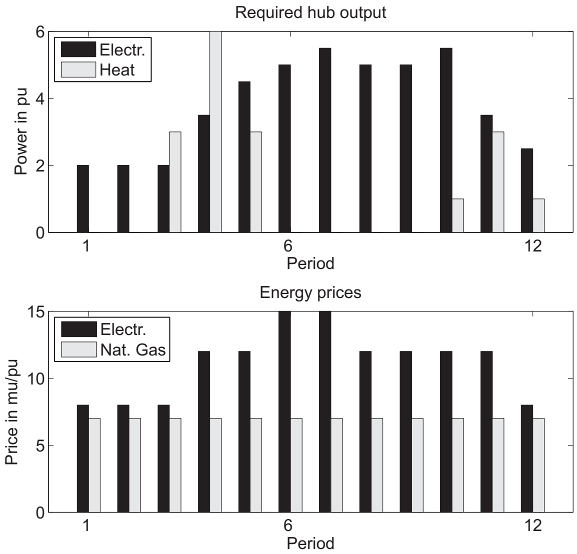

Fig. 8. Assumed load profiles and energy prices for 12 periods. Power in per unit (pu), energy prices in monetary units (mu).

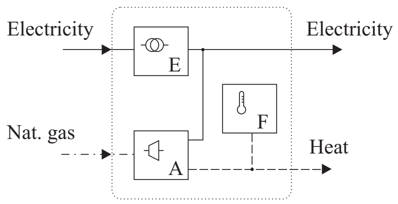

Fig. 9. Optimal hub layout with elements A, E, and F.

rating, although the installation cost are significantly higher. The storage F is needed in order to enable CHP operation; it compensates the heat generated by the CHP when there is no heat required by the load. However, a detailed explanation of the results is difficult due to the complexity of the problem.

The result of the operational optimization problem (b) is shown in Fig. 10. The hub consumes and converts high amounts of natural gas whenever loads and prices are high. The upper plot in Fig. 10 shows that the hub consumes no electricity from the grids in theses periods. The storage is mainly discharged during the first heat load peak and recharged afterwards during the electricity price peak.

VI. Summary and Outlook

The energy hub concept provides a flexible and comprehensive framework for modeling and optimization of multi-carrier energy systems. Various analysis tools have been developed based on this approach. This paper introduces multi-period optimal dispatch of energy hubs with converter and storage elements, as well as the determination of optimal hub structures. The approaches can be used in the system planning phase as well as for operation scheduling. Interactions and synergies among the different energy carriers and technologies can be identified and analyzed.

Current and future work is dedicated to dynamic modeling and analysis of multi-carrier energy systems, as well as the integration of transportation systems into the hub model.

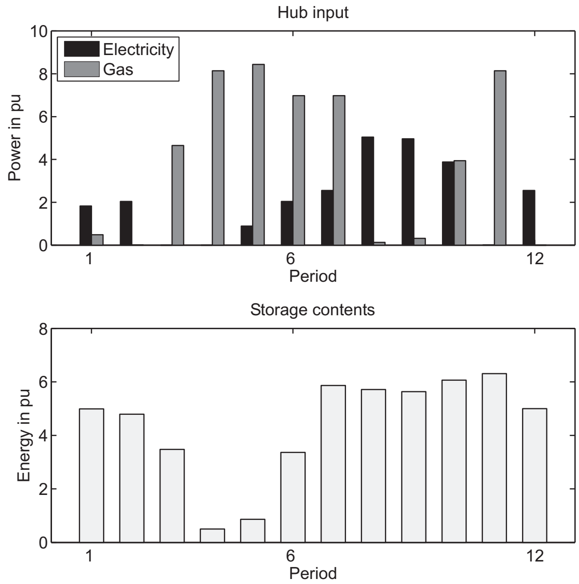

Fig. 10. Optimal hub input powers and storage energy, both in per unit (pu).

Acknowledgements

The authors would like to thank ABB, Areva, Siemens, and the Swiss Federal Office of Energy for supporting the research project Vision of Future Energy Networks, as well as the other members of the project team for inspiring discussions.

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