Coordinated Control of FACTS Devices in Power Systems for Security Enhancement

Gabriela Hug-Glanzmann, Göran Andersson
Power Systems Laboratory
ETH Zurich
Zurich, Switzerland
Email: {hug,andersson}@eeh.ee.ethz.ch

***Abstract*— FACTS devices provide the possibility to control voltages and power flows in a power system and therefore to improve the security of the system. In order to make use of this possibility, the set values of the FACTS controllers have to be chosen appropriately. A valuable option is the application of Optimal Power Flow control, where the set values are determined such that an objective function is minimized given the model of the system. But due to the large size of power systems, it is often difficult for different reasons to include the entire system into the optimization process. In this paper, sensitivity analysis is used to determine the area on which the FACTS device has considerable influence and then only this limited area is included in the Optimal Power Flow control. If there are several devices placed in the same system, the areas assigned to these devices might overlap indicating mutual influences. Therefore, a coordination of the control entities is needed in order to avoid conflicting behavior of the devices rising the issue of Multi-Area Control. Here, the method based on Approximate Newton Directions is extended for the case of overlapping areas which are determined by sensitivity analysis.**

I. INTRODUCTION

Today's power grids are driven closer to their transfer capacities due to the increased consumption and power transfers, endangering the security of the system. On the other hand, FACTS devices are a powerful technology that can solve many outstanding problems in power systems. They provide the opportunity to influence power flows and voltages and therefore are able to enhance the system security, e.g. by improving the voltage profile or increasing the transfer capacity of a system without the need of new lines [1].

In order to profit from the benefits of FACTS devices, appropriate set values for the controllers of these devices have to be determined. A possible method is the application of Optimal Power Flow control. As the main objective in this paper is to improve system security, the objective function includes the improvement of the voltage profile by minimizing voltage deviations from given reference values and the resolution of congestions by relieving overloaded lines. As economic aspect, also the minimization of active power losses is taken into account. But the need of modelling and including the entire system into the optimization process results, for realistic grids, in an optimization problem of considerable size. However, even though the influence of a FACTS device is not constrained to the bus or line where it is placed, the area in which the influence is significant is limited. Therefore, the problem size is reduced by taking only this area of influence into account in the optimization [2].

If the number of FACTS devices in a system increases, the areas of influence of several devices may overlap. Such an overlapping indicates that mutual influences among the devices exist. Hence, the control of the devices has to be coordinated, otherwise, devices with conflicting objectives might start fighting against each other. Due to the large size of realistic grids and also because of the fact that interconnected power systems often comprise several countries, or control areas, each with its own regulation entity, many studies for various applications using Multi-Area control in power systems have been carried out. But to the authors' knowledge, none of them covers the case where the areas are determined independently from each other possibly yielding overlapping areas and buses and lines which are not included in any of the areas. In this paper, the Multi-Area control method based on Approximate Newton Directions [3] is extended to these cases.

First, the models for the FACTS devices applied in this paper are given. Sect. III describes the determination of the area of influence by carrying out sensitivity analysis. In Sect. IV, the Optimal Power Flow control problem is introduced and it is explained how the Unlimited Point algorithm is used to solve it. The sensitivities are then used in Sect. V to reduce the size of the OPF problem taking into account only the area of influence of a device. In the following section, the concept of the method based on Approximate Newton Directions for non-overlapping areas is given and in Sect. VII, the extensions to this method are elaborated. Finally, results and a conclusion are given.

II. MODELLING OF FACTS DEVICES

As the intention is to improve the steady-state security, the power system as well as the FACTS devices are modelled using static equations. The FACTS devices considered in this paper are Static Var Compensators (SVC) and Thyristor Controlled Series Compensators (TCSC), since these FACTS devices, besides Phase Angle Regulators, are used most frequent in power systems [4].

An SVC is shunt-connected to a bus and influences the voltage V_{SVC} at the bus to which it is connected by injecting or absorbing reactive power Q_{SVC} [1]. This characteristic is modelled by a shunt-connected variable susceptance B_{SVC}

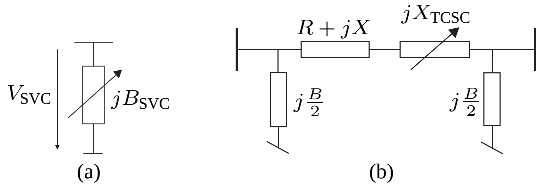

Fig. 1. (a) Model of an SVC and (b) of a TCSC.

(Fig. 1(a)) for which the injected reactive power Q_{SVC} results in

$$Q_{SVC} = -V_{SVC}^2 \cdot B_{SVC}, \tag{1}$$

As the size of the SVC is limited, a lower and an upper bound, $B_{\mathrm{SVC,min}}$ and $B_{\mathrm{SVC,max}}$, exist for the effective susceptance B_{SVC}, thus,

$$B_{\mathrm{SVC,min}} \leq B_{\mathrm{SVC}} \leq B_{\mathrm{SVC,max}}. \tag{2}$$

A TCSC is connected in series with a transmission line and is able to influence the active power flow P_{TCSC} through the line by adapting the reactance of the line [1]. Hence, the device is modelled as a variable reactance X_{TCSC} connected in series with the line, as shown in Fig. 1(b). The total reactance X_{line} of the line including the TCSC is therefore

$$X_{line} = X + X_{TCSC}, \tag{3}$$

where X is the reactance of the line itself. The effective reactance X_{TCSC} of the TCSC is limited to a range between $X_{\mathrm{TCSC,min}}$ and $X_{\mathrm{TCSC,max}}$ determined by the size of the device but also by the allowed line compensation, set to 80% capacitive and 40% inductive in this paper, resulting in

$$X_{\mathrm{TCSC,min}} \leq X_{\mathrm{TCSC}} \leq X_{\mathrm{TCSC,max}}. \tag{4}$$

III. SENSITIVITY ANALYSIS

The influence of FACTS devices is constrained to a limited area around the devices. Therefore, only this limited area has to be taken into account in the optimization process. In order to determine the area of influence of a device, sensitivity analysis is applied.

A. Determination of Sensitivity Values

In general, the sensitivities K_y of variables $y(z,u)$ with respect to changes in variables u and subject to the equation system $g(z,u)$ are given by [5]

$$K_y = \frac{dy}{du} = \frac{\partial y}{\partial u} - \frac{\partial y}{\partial z} \cdot \left(\frac{\partial g}{\partial z}\right)^{-1} \cdot \frac{\partial g}{\partial u}. \tag{5}$$

Determining the sensitivities in power systems for the considered application, the state variables z and the variable y are defined as

$$\begin{aligned} z &= [V_i, \theta_i] && (6)\\ y &= [P_{ij}, Q_{ij}, V_i, \theta_i] && (7) \end{aligned}$$

where for a network with n buses for each bus $i \in \{1,\dots,n\}$, V_i and θ_i denote voltage magnitude and angle and where for all (i,j), $i \in \{1,\dots,n\}$, $j \in \{1,\dots,n\}$ for which there is a line between bus i and j, P_{ij} and Q_{ij} are the active and reactive power flows on this line at bus i. The input variable u depends on the considered device. For the SVC the sensitivity values are determined either with respect to the effective susceptance B_{SVC} or the voltage V_{SVC} and for the TCSC either with respect to the effective reactance X_{TCSC} or to the active power flow P_{TCSC}. The equality constraints $g(z,u)$ correspond to the power flow equations.

For large power systems, the usage of (5) is computationally too expensive. In such cases, the sensitivities can be determined numerically by using power flow simulations at different operating points of the FACTS device, thus,

$$K_y = \frac{dy}{du} \approx \frac{\Delta y}{\Delta u} = \frac{y_1 - y_0}{u_1 - u_0}, \tag{8}$$

where y_0, u_0 and y_1, u_1 are the system and control variables for the initial operating point and for an operating point with a slightly changed control setting, respectively.

As power systems are nonlinear systems, the sensitivity values are also dependent on the specific operating point for which the sensitivities are determined. But for all considered variables, it has been found that the sensitivities can be approximated sufficient accurately by

$$K_y = a_y \cdot u^2 + b_y \cdot u + c_y, \tag{9}$$

where the parameters a_y, b_y and c_y have to be determined appropriately. As the course of the sensitivities over the range of operating points is close to linear, the parameter a_y for the quadratic term is rather small.

B. Determination of Area of Influence

Having identified the sensitivity values, the area of influence is determined such that only buses and lines are included in the area for which the sensitivities K_y are larger than a certain limit $K_{y,\mathrm{lim}}$. But as already mentioned, the sensitivity values are also dependent on the specific operating point. Therefore, the question arises which operating point is to be taken into account and the sensitivities of which variables, thus, if for lines active or reactive power flows and for buses voltage magnitudes or angles are used. In addition, the sensitivities, e.g. for the SVC, can be determined either with respect to the effective susceptance B_{SVC} of the device or with respect to the voltage V_{SVC} at the bus where the device is placed. Investigations have shown negligible variations over the range of the device settings for

- SVC: sensitivities for reactive power flows and voltage magnitudes with respect to the voltage V_{SVC}, and for
- TCSC: sensitivities for active power flows and voltage angles with respect to the active power flow P_{TCSC} through the device.

In addition, an SVC mainly influences reactive power flows and voltage magnitudes whereas a TCSC mainly influences active power flows and voltage angles. Therefore, these sensitivities for the operating point $B_{SVC} = 0$ and $X_{TCSC} = 0$, respectively, are used to determine the area of influence.

IV. OPTIMAL POWER FLOW CONTROL

In a general OPF problem, the goal is to determine control settings such that a particular objective is minimized, taking into account equality and inequality constraints according to the model of the system. The optimal settings are then determined by solving this optimization problem by a given optimization algorithm. Here, the Unlimited Point algorithm [6] is used for this purpose.

A. Definition of Optimization Problem

For the general formulation of an optimization problem, the objective function $f(x)$, the equality and the inequality constraints, $g(x)$ and $h(x)$, have to be defined which form the optimization problem

$$\begin{aligned} \min_{x} \quad & f(x) && (10) \\ \text{s.t.} \quad & g(x) = 0 && (11) \\ & h(x) \leq 0 && (12) \end{aligned}$$

where x includes state variables z as defined in (6) as well as control variables u from now on defined as B_{SVC} for SVCs and X_{TCSC} for TCSCs and also additional slack variables used to define soft constraints.

For the Optimal Power Flow control with the goal to determine the optimal settings for FACTS devices, the equality constraints $g(x)$ correspond to the power flow equations. The inequality constraints $h(x)$ in this paper include on one hand the constraints on the FACTS device settings (2) and (4) and on the other hand constraints to prevent the lines from overloading defined as

$$|I_{ij}| \leq I_{ij,\text{lim}} + s_{ij}^2, \tag{13}$$

where $|I_{ij}|$ is the current on the line between buses i and j and $I_{ij,\text{lim}}$ is the capacity of the line. The slack variables s_{ij} are heavily penalized in the objective function such that the controller has a strong incentive to set them to zero. Like this the constraints are turned into soft constraints and it is avoided to end up with an unsolvable system.

The objective function is composed of three different components:

1) minimization of bus voltage deviations from given references to improve the voltage profile,
2) prevention of overloaded lines,
3) and minimization of active power losses,

thus,

$$\begin{aligned} f(x) = & \sum_{i=1}^{n} \omega_v \cdot (V_i - V_{i,\text{ref}})^2 + \sum_{(i,j)\in\mathcal{I}} \omega_s \cdot s_{ij}^2 \\ & + \sum_{(i,j)\in\mathcal{I}} \omega_l \cdot P_{ij,\text{loss}}(V_i, \theta_i, V_j, \theta_j), \end{aligned} \tag{14}$$

where $V_{i,\text{ref}}$ is the reference voltage for bus i, where $\mathcal{I}$ denotes all (i,j), $i \in \{1,\ldots,n\}$, $j \in \{1,\ldots,n\}$ for which there is a line between bus i and j, and where for each $(i,j) \in \mathcal{I}$, $P_{ij,\text{loss}}(\cdot)$ is the active power loss. The weighting parameters ω_v, ω_s and ω_l are chosen according to the desired importance of each term.

B. Solving the Optimization Problem

In order to find the solution of this optimization problem, the Unlimited Point algorithm introduced in [6] is applied. The resulting Lagrange function is given by

$$\mathcal{L}(x) = f(x) + \lambda^T \cdot g(x) + (\mu^2)^T \cdot h(x) \tag{15}$$

and the corresponding first-order optimality conditions by

$$\begin{aligned} \nabla\mathcal{L}(\hat{x}) &= 0 && (16) \\ g(\hat{x}) &= 0 && (17) \\ h(\hat{x}) + \hat{\epsilon}^2 &= 0 && (18) \\ \text{diag}\{\hat{\mu}\} \cdot \hat{\epsilon} &= 0 && (19) \end{aligned}$$

where ϵ are slack variables used to transform the inequality constraints into equality constraints and the hat indicates the values of the variables in the optimal point. By using the squares of ϵ and μ, the condition that the Lagrange multipliers have to be positive and that $h(x) \leq 0$ has to hold are fulfilled. The only modification to the conditions given in the original Unlimited Point algorithm is that in [6] equation (19) is formulated with squared variables μ and ϵ. The modification has been applied in order to avoid that the matrix of the linearized system becomes singular. As this equation is used to force either μ or ϵ to zero, the result is the same.

By applying the Newton-Raphson method, this system of nonlinear equations is first transformed into a system of linear equations, i.e.

$$\begin{pmatrix} \nabla^2\mathcal{L} & \nabla g^{\mathrm{T}} & \nabla h^{\mathrm{T}} \cdot \text{diag}(2\mu) & 0 \\ \nabla g & 0 & 0 & 0 \\ \nabla h & 0 & 0 & \text{diag}(2\epsilon) \\ 0 & 0 & \text{diag}(\mu) & \text{diag}(\epsilon) \end{pmatrix} \cdot \begin{pmatrix} \Delta x \\ \Delta\lambda \\ \Delta\mu \\ \Delta\epsilon \end{pmatrix} = -\begin{pmatrix} \nabla\mathcal{L} \\ g \\ h + \epsilon^2 \\ \text{diag}(\mu) \cdot \epsilon \end{pmatrix}. \tag{20}$$

Starting with initial values $x^{(0)}$, $\lambda^{(0)}$, $\mu^{(0)}$ and $\epsilon^{(0)}$ and iteration step $k = 1$, the variables are updated by

$$\begin{aligned} x^{(k)} &= x^{(k-1)} + \Delta x^{(k)} && (21) \\ \lambda^{(k)} &= \lambda^{(k-1)} + \Delta\lambda^{(k)} && (22) \\ \mu^{(k)} &= \mu^{(k-1)} + \Delta\mu^{(k)} && (23) \\ \epsilon^{(k)} &= \epsilon^{(k-1)} + \Delta\epsilon^{(k)} && (24) \end{aligned}$$

until the changes are smaller than a pre-defined threshold yielding the optimal values $\hat{x}$, $\hat{\lambda}$, $\hat{\mu}$ and $\hat{\epsilon}$.

V. LIMITED AREA CONTROL

In the optimization problem defined in Sect. IV-A, the entire grid of the system is taken into account to determine the optimal settings for the FACTS devices. For large power systems, the computational effort to solve this problem is tremendous and also the data of the entire system is often not available. Therefore, the limited area control is derived where only the area of influence of the device is taken into account in the optimization process.

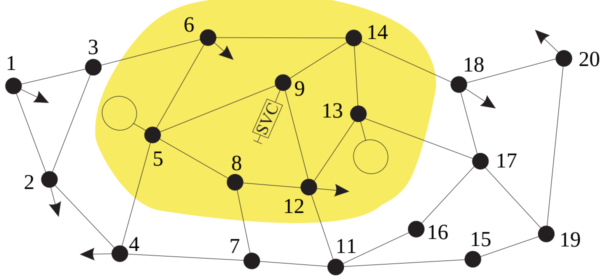

Fig. 2. Illustration of the limited area control

In Fig. 2, an example of a grid with an SVC at bus 9 and a possible area of influence is shown. For buses which are at the border of the area and for which there exist adjacent buses not included in the area, the power flow equations cannot be set up because the voltage magnitude and angle of the buses outside of the area are not included in the state variables of the optimization problem any more. For such buses, e.g. bus 8, the voltage magnitude and angle are determined using the sensitivity values to approximate the influence of the FACTS device yielding

$$V_j = V_{j,0} + K_{V_j} \cdot u, \qquad (25)$$

$$\theta_j = \theta_{j,0} + K_{\theta_j} \cdot u, \qquad (26)$$

where $V_{j,0}$ is the voltage magnitude if the set point of the FACTS device is zero and K_{V_j} is the sensitivity of the voltage magnitude at bus j with respect to u. For the voltage angle the notation is accordant.

It was already discussed in Sect. III that the sensitivities with respect to the effective susceptance and reactance of SVC and TCSC vary significantly over the range of the device settings but can be approximated as a function in B_{SVC} and X_{TCSC} by a second-order polynomial given in (9). In order to have the best possible approximation of the voltage magnitude and angle, the average between the sensitivity value at the initial point $B_{SVC} = 0$ or $X_{TCSC} = 0$ and the set point for u is taken to approximate the influence of the FACTS device. Hence, the sensitivities K_{V_j} and K_{θ_j} are given by

$$K_{V_j} = \frac{1}{2} \cdot a_{V_j} \cdot u^2 + \frac{1}{2} \cdot b_{V_j} \cdot u + c_{V_j}, \qquad (27)$$

$$K_{\theta_j} = \frac{1}{2} \cdot a_{\theta_j} \cdot u^2 + \frac{1}{2} \cdot b_{\theta_j} \cdot u + c_{\theta_j}. \qquad (28)$$

Having several FACTS devices placed in the same system, the areas of these devices may overlap indicating mutual influences among and necessitating a coordination between them. Such a coordination is achieved by multi-area control.

VI. MULTI AREA CONTROL

In multi-area control, the overall problem defined in (10)-(12) is decomposed into several subproblems, each associated with an area, and solved in an iterative and coordinated procedure. There exist various decomposition techniques which have been proposed and applied in the literature. Each of these techniques has its advantages and disadvantages. Comparisons of a range of decomposition techniques can be found in [7], [8], [9].

In this paper, the method based on Approximate Newton Directions proposed in [3] and applied for OPF in [10] is used. In this method, each bus and the associated variables are assigned to one distinct area p. These variables are so-called local variables and are the decision variables of area p. All other variables in the system are external variables for area p and are assumed to be fixed to the values obtained from the other areas.

Concerning constraints, two types are distinguished: constraints which include only local variables and constraints which include local as well as external variables, e.g. the power flow constraints associated to a bus at the border of area p. The latter ones are called interconnecting constraints and are included as regular hard constraints in area p and as soft constraints in the neighboring area m for which the used external variables are decision variables; i.e. these constraints are included in the objective function of area m penalized by the corresponding Lagrange multiplier given from area p. Regarding the objective function, the overall objective function is assigned completely to each area with external variables fixed. The values for external variables and Lagrange multipliers are communicated among the areas after each iteration step.

The concept of the method based on Approximate Newton Directions emerges from the first-order optimality conditions for an optimization problem as given in (16)-(19). Applying the described decomposition, the first-order optimality conditions for all areas combined are equivalent to the first-order optimality conditions for the overall problem [3]. As with the Unlimited Point algorithm the inequality constraints are transformed into equality constraints, the following description will focus only on equality constraints.

The procedure to set up the subproblems is as follows:

1) Determine which buses, and thus which variables, are included in which area.
2) Assign the overall objective function to each area p and define the external variables as fixed.
3) For each bus i in each area p, set up the power flow equality constraints and include them into the constraint set of area p.
4) Determine for each constraint whether it is an interconnecting constraint $g_{p,\text{int}}(x_p, x_{p_e})$ which involves local variables x_p as well as external variables x_{p_e} or whether it is a constraint $g_p(x_p)$ only using local variables.
5) Include $g_{p,\text{int}}(x_p, x_{p_e})$ in each area m as a soft constraint in the objective function for which a variable x_m appears as external variable in this equation.

For M areas, the subproblem to be solved by area $p \in \{1, \ldots, M\}$ with decision variables x_p at a particular iteration step is therefore given by

$$\min_{x_p} \; f(x_p, \overline{x}_{p_e}) + (\overline{\lambda}_{p_e,\text{int}})^{\mathrm{T}} g_{p_e,\text{int}}(x_p, \overline{x}_{p_e}) \qquad (29)$$

subject to

$$g_{p,\text{int}}(x_p, \overline{x}_{p_e}) = 0 \qquad (30)$$

$$g_p(x_p) = 0 \qquad (31)$$

where the subscript p denotes local variables and equations associated with buses in area p. The subscript p_e is used accordingly for external variables and equations associated with external buses. The bar and subscript p_e notation for a variable, e.g., $\bar{v}_{p_\mathrm{e}}$, indicates that the value of v is set to the value determined for v in the previous iteration. The subscript int indicates interconnecting constraints that include local variables as well as external variables. The variables $\lambda_{p_\mathrm{e},\mathrm{int}}$ are the Lagrange multipliers for the interconnecting constraints $g_{p_\mathrm{e},\mathrm{int}}$ included as soft constraints in area p and as hard constraints in the area where the bus to which this constraint is associated to is located.

Using this problem setup for each area, the iterative optimization process is started. Instead of solving the subproblems until optimality only the first Newton-Raphson step is applied. Then the updated variables are exchanged and used in the next iteration. The outline of the scheme is therefore as follows:

1) Each area p initializes its variables x_p and $\lambda_{p,\mathrm{int}}$ by setting the variables for voltage magnitudes and angles and the manipulated variable to the current steady-state values and the Lagrangian multipliers to a common value, e.g. to 0.1. Iteration counter l is set to 1.
2) Given $\overline{x}_{p_\mathrm{e}}$ and $\overline{\lambda}_{p_\mathrm{e},\mathrm{int}}$ from the initialization or iteration step $l-1$, each area $p \in \{1,\ldots,M\}$ determines in parallel with the other areas the first iteration step of the Newton method for its subproblem given by equations (29)-(31) to obtain $\Delta x_p^{(l)}$ and $\Delta\lambda_{p,\mathrm{int}}^{(l)}$.
3) Each area updates its variables by $x_p^{(l)} = x_p^{(l-1)} + \Delta x_p^{(l)}$ and $\lambda_{p,\mathrm{int}}^{(l)} = \lambda_{p,\mathrm{int}}^{(l-1)} + \Delta\lambda_{p,\mathrm{int}}^{(l)}$.
4) The areas exchange the requested values resulting from their optimization problem with their neighbors.
5) Unless a stopping condition is satisfied, e.g., the absolute changes in all variables from step $l-1$ to l are smaller than a pre-defined threshold, the next iteration is started by increasing l and going back to step 2.

The second step of this scheme actually corresponds to solving

$$C^{(l)} \cdot \Delta^{(l)} = d^{(l)} \tag{32}$$

where $C^{(l)}$ contains on its diagonal the system matrices according to (20) for the subproblems of all areas $1\ldots M$, the vector $\Delta^{(l)}$ is the update vector for the system variables and Lagrange multipliers and $d^{(l)}$ is the composition of all righthand vectors as given in (20), both ordered by areas $1\ldots M$.

The condition for convergence to the optimal point is derived in [3] and is given by

$$\hat{\rho} = \rho\left(I - \hat{C}^{-1} \cdot \hat{C}_*\right) < 1 \tag{33}$$

where $\rho(A)$ is the spectral radius of matrix A. The matrix $\hat{C}$ corresponds to C at the optimal point. The matrix $\hat{C}_*$ is the system matrix for the overall problem in the optimal point, rows and columns ordered such that

$$C_*^{(l)} \cdot \Delta_*^{(l)} = d^{(l)} \tag{34}$$

gives the Newton-Raphson step at iteration l for the overall system, thus, C_* is built according to C. Therefore, the block diagonals of $\hat{C}_*$ and $\hat{C}$ are equal but the off-diagonal elements of $\hat{C}_*$ are non-zero whereas these element are zero in $\hat{C}$. For further explanations, refer to [10].

If this condition is not fulfilled, additionally a Preconditioned Conjugate Gradient method has to be applied in order to reach local convergence. As the optimization problem is possibly non-convex and non-linear, the Generalized Minimal Residual method (GMRES) [11] is chosen in [3] with $C^{(l)}$ as pre-conditioner.

The GMRES algorithm is used to solve a linear system of equations, e.g. determining the first Newton-Raphson step of the overall problem by solving (34), without having to take the inverse of the system matrix. The exact solution is reached after the number of steps equal to the number of variables in the system. But if an appropriate pre-conditioner is used, already after a few steps a quite accurate solution is obtained. In the considered application, the goal is to bring the solution $\Delta^{(l)}$ closer to $\Delta_*^{(l)}$.

In order to simplify notation, C_*, C and d are used for $C_*^{(l)}$, $C^{(l)}$ and $d^{(l)}$ in the following algorithm. The update $\Delta^{(l)}$ determined by the areas is chosen as initial point. The GMRES algorithm is then given by:

```
1  $\tilde{\Delta}^{(0)} := \Delta^{(l)}$
2  $r := C^{-1} \cdot \left(d - C_* \cdot \tilde{\Delta}^{(0)}\right)$
3  $w_1 := r/||r||$
4  $z_1 := (w_1, r)$
5  $q := 0$
6  while $||C_* \cdot \tilde{\Delta}^{(q)} - d|| > t_q \cdot ||d||$
7      $q := q + 1$
8      $r := C^{-1} \cdot C_* \cdot w_q$
9      for $i := 1$ to $q$ do
10         $h_{i,q} := (w_i, r)$
11         $r := r - h_{i,q} \cdot w_i$
12     end
13     $h_{q+1,q} := ||r||$
14     $w_{q+1} := r/h_{q+1,q}$
15     for $i := 1$ to $q - 1$
16         $h_{i,q} := \beta_{i+1} \cdot h_{i,q} + \gamma_{i+1} \cdot h_{i+1,q}$
17         $h_{i+1,q} := -\gamma_{i+1} \cdot h_{i,q} + \beta_{i+1} \cdot h_{i+1,q}$
18     end
19     $\alpha := \sqrt{h_{q,q}^2 + h_{q+1,q}^2}$
20     $\gamma_{q+1} := h_{q+1,q}/\alpha;\ \ \beta_{q+1} := h_{q,q}/\alpha;\ \ h_{q,q} := \alpha$
21     $z_{q+1} := -\gamma_{q+1} \cdot z_q;\ \ z_q := \beta_{q+1} \cdot z_q$
22     $v_q := z_q/h_{q,q}$
23     for $i := q - 1$ down to 1 do
24         $v_i := \left(z_i - \sum_{j=i+1}^{q} h_{i,j} \cdot v_j\right)/h_{i,i}$
25     end
26     $\tilde{\Delta}^{(q)} := \tilde{\Delta}^{(0)} + \sum_{i=1}^{q} v_i \cdot w_i$
27 end
```

The only parameter for which a coordination among the areas is needed is the residual vector r determined in row 8. As $C^{(l)}$ is block diagonal, the inverse of this matrix consists of the inverses of each block, each determined by the corresponding area. Each area is responsible for a distinct part of the vector and is able to set the vector together if it gets the rest of the vector from the other areas. Therefore, at each iteration step within the GMRES algorithm communication is needed only in row 8.

An overview over the entire iteration scheme is shown in

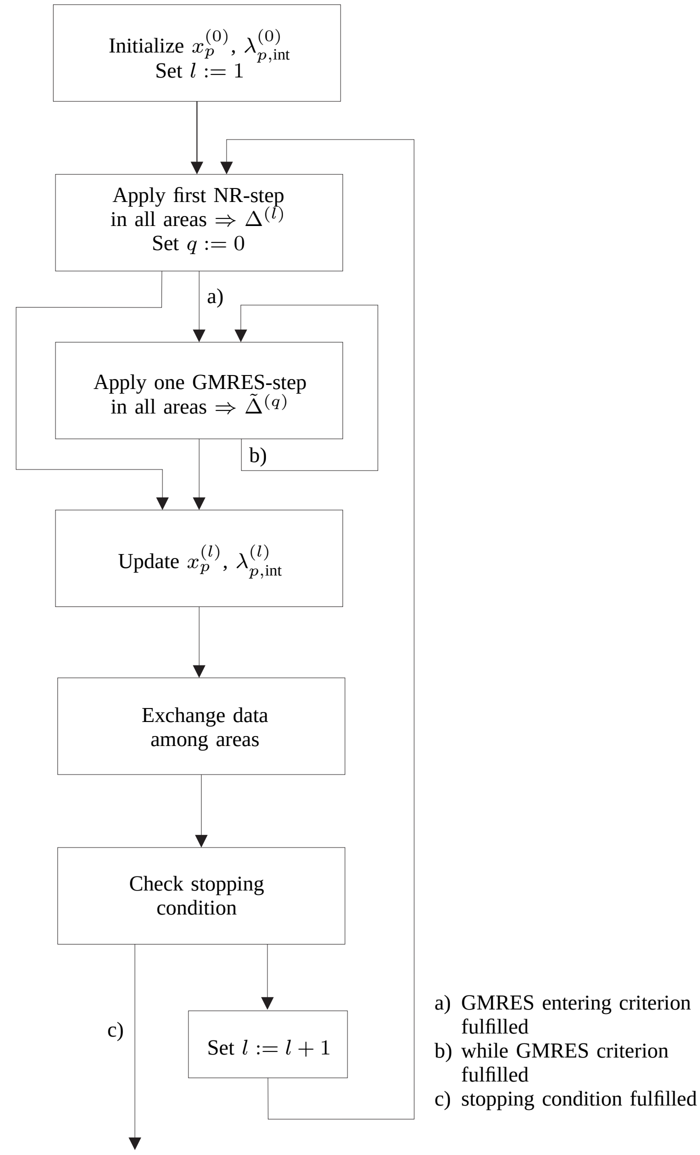

Fig. 3. Flow chart for the optimization scheme.

Fig. 3. Applying the Approximate Newton Direction method in combination with the GMRES method, the solution for many optimization problems in power systems can be found in a distributed way.

VII. EXTENSIONS

In the described method, it is assumed that each bus is included in exactly one area, i.e. the areas are non-overlapping. For the application described in the previous sections, this method has to be extended for the case where the areas are overlapping, thus, some buses are included in more than one area, and where there exist buses not included in any of the areas.

In case of overlapping areas, there are buses which cannot uniquely be assigned to one particular area but are common to at least two areas. This raises questions on how to treat the common variables, how to decompose the objective function and how to include constraints concerning the common part of the areas (see also [12]).

A. Decision Variables

From the point of view of a particular area p four types of buses can be determined:

1) local buses: buses which are only included in area p;
2) common buses: buses included in area p and some other area m, hence, located in the common area;
3) external buses: buses not included in area p but included in some other area m;
4) omitted buses: buses of the entire system not included in any of the M control areas.

The terms local, common, external and omitted are also used for the variables associated with the respective buses. The local variables for area p are denoted by x_p, the common variables by x_{p_c}, and the external variables by x_{p_e}.

The decision variables of area p include the local as well as the common variables whereas the external variables are assumed fixed and given by the corresponding area. After each outer iteration step l, the required values of common and external variables are exchanged among the areas. As will be explained in the following, the omitted variables do not appear in any constraint.

B. Constraints

In a power system, two real valued power flow equations are associated to each bus. These constraints correspond either to active power balance, reactive power balance, fixed voltage magnitude or fixed voltage angle setting. Taking the areas in Fig. 4 as an example, it is obvious that for the gray colored buses, the areas need omitted variables to set up the active or reactive power balances. But other than for external variables, these omitted variables are not decision variables of any area and therefore cannot be assumed to be decided by an other area. Hence, the power flow equations for these buses have to be adjusted.

In order to find the voltage magnitudes and angles at these buses the formula (25) and (26) derived in Sect. V are used. For buses included only in area p, only the influence of the FACTS device of this area has to be taken into account, resulting in

$$V_j = V_{j,0} + \frac{1}{2} a_{V_j,p} \cdot u_p^3 + \frac{1}{2} b_{V_j,p} \cdot u_p^2 + c_{V_j,p} \cdot u_p, \tag{35}$$

$$\theta_j = \theta_{j,0} + \frac{1}{2} a_{\theta_j,p} \cdot u_p^3 + \frac{1}{2} b_{\theta_j,p} \cdot u_p^2 + c_{\theta_j,p} \cdot u_p. \tag{36}$$

In the case where a bus which is connected to omitted buses is included in the common area, this indicates that the devices of both areas have significant influence on voltage magnitude and angle at this bus. Thus, the corresponding equations are

$$\begin{aligned} V_j = {} & V_{j,0} + \frac{1}{2} a_{V_j,p} \cdot u_p^3 + \frac{1}{2} b_{V_j,p} \cdot u_p^2 + c_{V_j,p} \cdot u_p + \\ & + \frac{1}{2} a_{V_j,m} \cdot u_m^3 + \frac{1}{2} b_{V_j,m} \cdot u_m^2 + c_{V_j,m} \cdot u_m, \end{aligned} \tag{37}$$

$$\begin{aligned} \theta_j = {} & \theta_{j,0} + \frac{1}{2} a_{\theta_j,p} \cdot u_p^3 + \frac{1}{2} b_{\theta_j,p} \cdot u_p^2 + c_{\theta_j,p} \cdot u_p + \\ & + \frac{1}{2} a_{\theta_j,m} \cdot u_m^3 + \frac{1}{2} b_{\theta_j,m} \cdot u_m^2 + c_{\theta_j,m} \cdot u_m, \end{aligned} \tag{38}$$

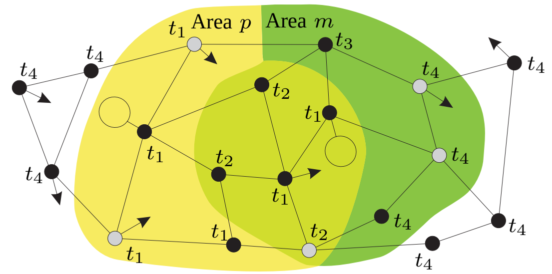

Fig. 4. Illustration of different bus types.

TABLE I

TREATMENT OF CONSTRAINTS ASSOCIATED WITH DIFFERENT BUS TYPES FOR AREA p

type	bus location	how to deal with constraints
t_1	local, common	$g_{t_1}(x_p, \overline{x}_{p_\mathrm{e}}, x_{p_\mathrm{c}})$
t_2	common	$g_{t_2}(x_p, \overline{x}_{p_\mathrm{e}}, x_{p_\mathrm{c}})$ $f_p = \cdots + \overline{\lambda}_{p_\mathrm{c}} g_{t_2}(x_p, \overline{x}_{p_\mathrm{e}}, \overline{x}_{p_\mathrm{c}})$
t_3	external	$f_p = \cdots + \overline{\lambda}_{p_\mathrm{e}} g_{t_3}(x_p, \overline{x}_{p_\mathrm{e}}, \overline{x}_{p_\mathrm{c}})$
t_4	external, omitted	-

where u_m is an external variable for area p and vice-versa.

Now it is possible to set up power flow equations for each bus using only local, external and common variables. As there are constraints involving common variables, which is not the case in the basic method, it has to be discussed on how such constraints have to be treated.

The first step is to distinguish between four different types of buses depending on where they are located and to which buses they are connected. The bus types t_1-t_4 can be described as:

1) bus type t_1: all local buses and in addition, common buses that are not connected to any local bus;
2) bus type t_2: common buses that are connected to at least one local bus;
3) bus type t_3: external buses that are connected to at least one local bus;
4) bus type t_4: omitted buses as well as external buses that are connected to only external, common and omitted buses.

In Fig. 4, an illustration of these types from the view point of area p is given. Depending on the bus type, the constraints associated to the bus are treated differently in the problem formulation. In general, each area includes all constraints associated to buses included in the area as hard constraints with external variables fixed. If an area m uses local variables of area p to formulate a hard constraint, this constraint has to be included in the objective function of area p as soft constraint with external and common variables fixed and weighted with the Lagrange multiplier of area m for this constraint. Table I gives an overview on how the constraints are taken into account.

Thus, for a particular area, its constraints of type t_1 and t_2 are included as hard constraints with external variables fixed, while its constraints of t_2 and t_3 are included as soft constraints in the objective function with external and common variables fixed. Constraints associated with buses of type t_4 are not taken into account in the considered area.

TABLE II

DECOMPOSITION OF THE OBJECTIVE FUNCTION FOR AREA p

term in overall $f(\cdot)$	how to include in $f_p(\cdot)$
$f_1(x_p)$	$f_p(\cdot) = \cdots + f_1(x_p)$
$f_2(x_p, x_{p_\mathrm{c}})$	$f_p(\cdot) = \cdots + f_2(x_p, x_{p_\mathrm{c}})$
$f_3(x_{p_\mathrm{e}}, x_{p_\mathrm{c}})$	$f_p(\cdot) = \cdots + 0$
$f_4(x_{p_\mathrm{c}})$	$f_p(\cdot) = \cdots + 1/\sigma \cdot f_4(x_{p_\mathrm{c}})$
$f_5(x_p, x_{p_\mathrm{e}})$	$f_p(\cdot) = \cdots + f_5(x_p, \overline{x}_{p_\mathrm{e}})$

C. *Objective Function*

The overall objective function consists of terms which from the view point of a given area involve local, common, external and/or omitted variables. Terms including omitted variables are discarded as the fact that the corresponding buses are not included in any area implies that the FACTS devices do not have significant influence on these terms. This yields a reduced objective function f_{red}. Terms including only local and/or external variables, can be treated as within the basic method but terms involving common variables need an additional consideration. Coming forth from the first-order optimality conditions, the reduced overall objective function f_{red} has to be decomposed such that it holds that the gradient of this reduced overall objective function is equal to the sum of the gradients of the objective functions of the areas, i.e.

$$\frac{\partial f_{red}}{\partial x} = \sum_{p=1}^{M} \frac{\partial f_p}{\partial x}. \qquad (39)$$

Table II shows how terms depending on local, common and external variables, and combinations of these are taken into account in the decomposed system from the view point of area p fulfilling criteria (39). The number of areas that include the common variables appearing in the considered term is denoted by σ.

D. *Extended Procedure*

Having defined how constraints and objective function are formulated in the overlapping case, the adapted procedure to set up the subproblems is as follows:

1) Determine which buses, and thus which variables, are included in which area and distinguish between local, common, external and omitted variables.
2) Define the objective function of each area p including the terms of the reduced overall objective function as defined in Table II.
3) For each bus i in each area p, set up the power flow equality constraints and include them into the constraint set of area p.
4) Determine the type $t_1, \ldots, t_4$ for each bus.
5) For buses of type t_2 and t_3 include the constraints associated with these buses according to Table I as soft constraints into the objective function.

The scheme for the optimization process stays the same as for the non-overlapping case including the application of the GMRES algorithm.

It has to be noted, that the case with the limited areas does not converge exactly to the solution of the overall optimization because with the approximations of the influences of the FACTS devices on the voltage magnitudes and angles at the border buses some errors are introduced and terms on which the devices have little influence, i.e. terms concerning omitted values, are neglected. But as has been shown in [2], these errors are small if the areas have an appropriate size.

VIII. RESULTS

The described method is applied to the control of FACTS devices in the IEEE 57 bus grid. In the following, three different test cases are considered: two cases with SVCs and one with TCSCs. The parameters in the objective function (14) are assigned to $\omega_s = 10$, $\omega_l = 0.2$ and $\omega_v = 1$ such that the terms concerning voltage deviations and active power losses contribute roughly the same to the objective function value and that overloadings of lines are penalized heavily. The starting point $x^{(0)}$ for the iteration process is set to the base case where no FACTS device is in operation.

A. Test Case 1

In the first test case, an SVC is placed at bus 14 and another SVC at bus 29. The areas of influence for the two devices are shown in Fig. 5. The limits of the lines are chosen such that no line is overloaded when the setting of the FACTS devices are set to zero. Therefore, the focus lies on improving the voltage profile and decreasing the active power losses.

The courses of the effective susceptances B_{SVC} over the outer iterations l are shown in Fig. 6. Already after a few iteration steps, the SVC settings converge to their final values. As the spectral radius is 0.84, the Preconditioned Conjugate Gradient method is not needed and therefore no additional inner iterations q have to be carried out.

The performance of the limited multi-area control is determined by comparing the resulting value for the overall objective function $f(\cdot)$ and the FACTS device settings B_{SVC} with the base case when no FACTS device is in use and with the results for the optimal control where the entire grid is taken into account in the optimization. In Table III, these values are given for this first test case. The small deviations in device settings and consequently also in the objective function value between the limited multi-area and the optimal case result from the fact that by using approximations for the voltage magnitudes and angles at the border of the areas, some errors are introduced.

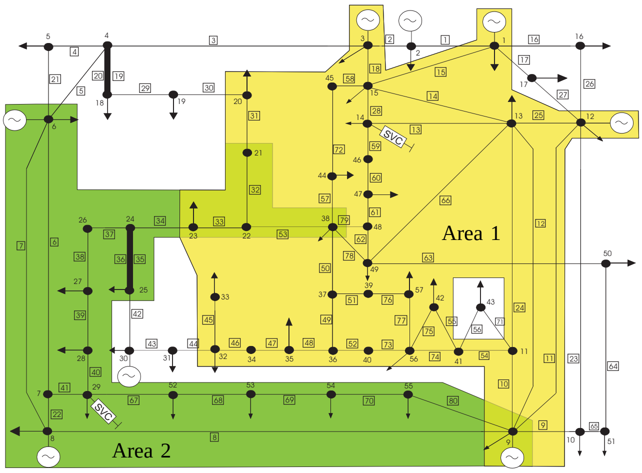

Fig. 5. Areas of influence for SVCs at buses 14 and 29.

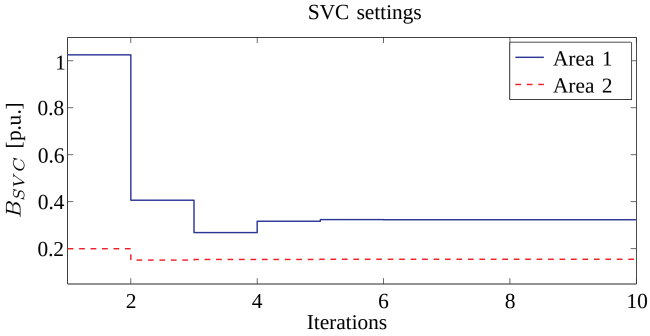

Fig. 6. Course of the effective susceptances B_{SVC} of areas 1 and 2 for test case 1.

TABLE III

OBJECTIVE FUNCTION VALUES AND EFFECTIVE SUSCEPTANCES OF SVCS FOR TEST CASE 1.

	Reference	Optimal	Limited
$f(\cdot)$	0.09536	0.09206	0.09208
B_{SVC_1}	0.0	0.33664	0.32316
B_{SVC_2}	0.0	0.17801	0.15512

B. Test Case 2

The second test case differs from the first case just by the values for the line limits. The line limit for line 33 is chosen such that it is slightly overloaded in the base case. This problem now yields a spectral radius of 1.49. Hence, some inner iterations q, i.e. GMRES iterations have to be carried out. In Fig. 7, the course of the SVC settings over the outer iterations l are shown. Additionally, 62 additional inner iterations of the GMRES algorithm are necessary in total to make the settings converge to their final values.

The resulting values for the objective function and the effective susceptances of the SVCs are listed in Table IV. In the second row, the values of the objective function term concerning the line loadings are given. In the optimal as well

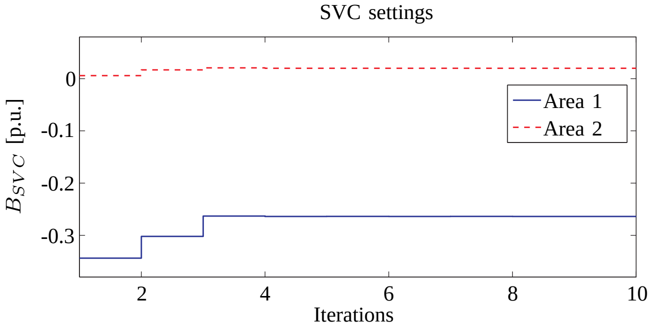

Fig. 7. Course of the effective susceptances B_{SVC} of areas 1 and 2 for test case 2.

TABLE IV

OBJECTIVE FUNCTION VALUES AND EFFECTIVE SUSCEPTANCES OF SVCS FOR TEST CASE 2.

	Reference	Optimal	Limited
$f(\cdot)$	0.11293	0.09821	0.09855
$\sum_{ij\in\mathcal{I}} \omega_s \cdot s_{ij}^2$	0.01757	0.0	0.0
B_{SVC_1}	0.0	-0.26981	-0.26388
B_{SVC_2}	0.0	0.04565	0.01998

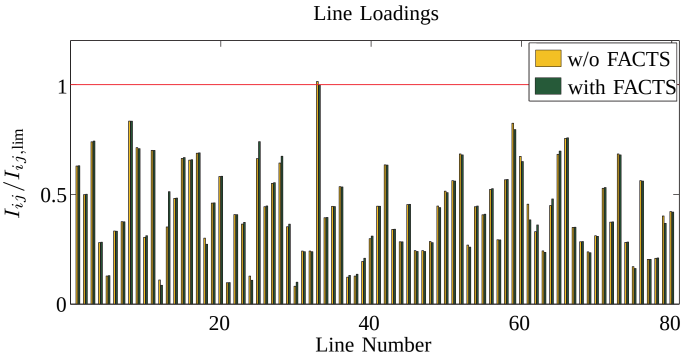

Fig. 8. Line loadings without and with SVCs for test case 2.

as in the limited mulit-area control all line loadings are brought below 100 %. In Fig. 8, these line loadings are shown for the reference and the limited multi-area case, thus, the overloaded line 33 is relieved and brought below its limit.

C. Test Case 3

In the third test case, two TCSCs are placed in the IEEE 57 bus grid, i.e. in lines 22 and 72. The areas of influence determined by sensitivity analysis are shown in Fig. 9. The line limits are chosen such that lines 6 and 60 are overloaded in the base case, when the FACTS devices are out of operation.

Figure 10 shows the evolution of the FACTS device settings. As the spectral radius for this problem is lower than 1, the Preconditioned Conjugate Gradient method is not needed and therefore no inner iterations are carried out. After only a few steps the FACTS device settings converge to their final values.

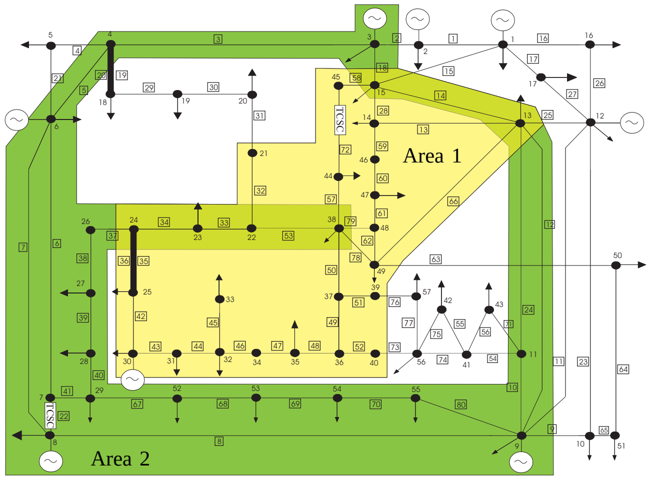

Fig. 9. Areas of influence for TCSCs in lines 22 and 72.

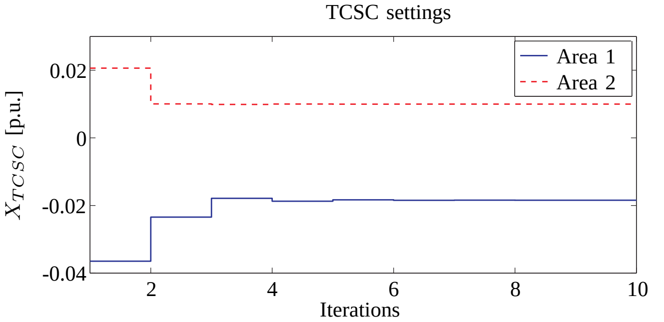

Fig. 10. Course of the effective reactance X_{TCSC} of areas 1 and 2 for test case 3.

TABLE V

OBJECTIVE FUNCTION VALUES AND EFFECTIVE REACTANCES OF TCSCS FOR TEST CASE 3.

	Reference	Optimal	Limited
$f(\cdot)$	0.32843	0.09542	0.09713
$\sum_{ij\in\mathcal{I}} \omega_s \cdot s_{ij}^2$	0.23306	0.0	0.00171
X_{TCSC_1}	0.0	-0.01756	-0.01844
X_{TCSC_2}	0.0	0.01003	0.00997

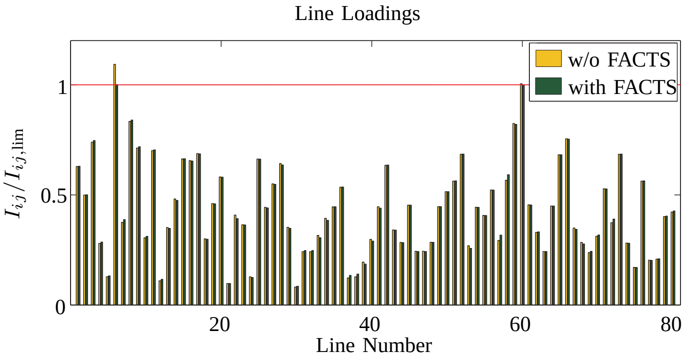

Fig. 11. Line loadings without and with TCSCs for test case 3.

In Table V, the objective function values and the FACTS device settings for the base, the optimal and the limited multi-area case are listed. The TCSC settings in the limited multi-area control are close to the values determined by the optimal control and the same also hold for the objective function value. The reason for the deviations again lies in the approximations of the voltage magnitudes and angles at the border of the areas using sensitivity values. Consequently, also small errors in the determination of the line loadings are introduced. The controller in the limited multi-area case sets the effective reactances such that using these approximations the line loadings are all below 100% but in reality line 6 is still slightly overloaded. Thus, the term corresponding to the line loadings, given in the second row of Table V, is not exactly zero. But looking at the line loadings for the base case and for the limited multi-area control as shown in Fig. 11, the line loadings are reduced significantly.

IX. CONCLUSION

In this paper, a control for FACTS devices is developed which is also feasible for large power systems. The Optimal

Power Flow control is applied to a limited area including only buses and lines on which the device has considerable influence. The method based on Approximate Newton Directions with additional Preconditioned Conjugate Gradient method is extended for the case where the areas are independently determined by sensitivity analysis, which may lead to overlapping areas as well as situations where part of the grid is entirely left out from the optimization.

The simulation results show fast convergence for the device settings to the final values. The control performance of the limited multi-area case is close to the performance of the optimal control taking the entire grid into account.

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