On the reduction of large power system models for power market simulations

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Abstract - The need for studying large scale power systems is increasing due to long distance power transfers, common power markets and planning of highly interconnected areas. Large systems are difficult to handle if not reduced. This paper focuses on reduction techniques already applied on power system models in order to achieve better calculation performance, easier handling of large amount of data and better observation of the results. In the literature there are many models for both different purposes, static or dynamic. According to the purpose of the model, different inputs and outputs criteria apply. Here, four methods are compared, two conventional, e.g. WARD, REI, and two market-based, according to their suitability for large scale reduction and development of economic models. Besides, the criteria of reducing the European electricity network are discussed and defined.

1 Introduction

NOWADAYS the studying of more realistic systems increases steadily the need for modeling bigger and more complex systems. In the case of power systems there are a lot of different models of several complexity scales, which all use different assumptions that make the comparison of results and the assessment of solutions an almost impossible task. The lack of a European benchmark economic model based on real network data creates many problems in the research community when trying to study the effects of the European electricity market and to identify a sustainable planning strategy including new investments in transmission and generation. This paper reviews several reduction techniques already published, based on nodal analysis or some typical static reduction methods, e.g. WARD and REI. The target is to define the reduction criteria and characteristics of a reduced economic model, more specifically of the European interconnected system, paying attention to the congestion points, and the power exchanges, in order to identify optimal investment plans, primarily regarding transmission.

2 Need for economic models

The liberalization, the use of cheap energy potential, e.g. from wind or solar energy, and the availability of new technologies in transmission networks expand the boundaries of power systems outside of national borders and usually outside of observation areas of single transmission system operators (TSOs). New studies regarding examination of market efficiency, maximization of generators' profits, calculation of nodal prices and maximization of consumers' benefits need accurate data based on production costs, transmission allocation capacity and consumers' behaviour of large-scale systems. This large amount of data is difficult to handle and to find. The problem becomes even larger when the target is the transmission or generation planning, where iterative processes and complex optimization techniques are applied [1]. Very often in the new framework the decision maker has to take into consideration many aspects of the power system, the society, the economy and the environment.

As proposed in [2] a single independent planning coordinator leads the investment planning process, gathers all information needed from stakeholders and interested parties, e.g. TSOs or regional operators, and provides solutions for the system development, Fig. 1. M. Ilic proposed in 1998 [3], that this is how bulk transmission investments should be decided in the new deregulated environment, while investments in transmission are very essential for market efficiency and may create advantages to few market players as compared with others.

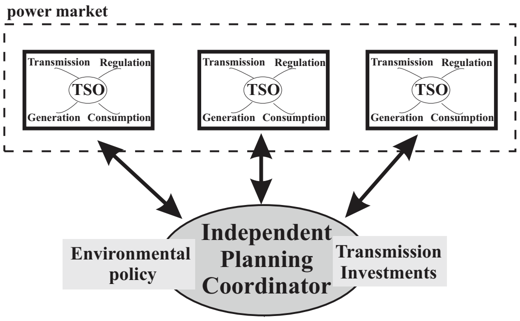

Figure 1: Planning framework

As all interested parties, regional TSOs, producers, consumers and stakeholders are active in a power market, market indicators are needed that provide sufficient information for correct decisions. For that reason economic models are needed, that are able to study areas of interest, that are affected from the final decisions and need reasonable computational effort. With economic model in this paper is meant an optimal power flow model that includes power plant types, marginal production costs, consumers behaviour, generation and transmission limits that can be used e.g. for nodal pricing calculations.

As long as production cost curves, power plants scheduling and dispatch and line capacity limits are not publicly available, economic models are basically based on assumptions. Some models for investment planning are available [4], [5] and some others are only for power flow analysis, e.g. [6]. There is not a single European benchmark model for power market and market-based planning studies.

3 Power systems network reduction

3.1 Importance of system reduction

Network reduction or equivalencing has become important for several reasons and in several different applications. In Europe the strong interconnection of high voltage networks increases the complexity of the system and the bottlenecks can often appear in the internal network of a control zone. Real time observation of the network demands very high computational effort for repeated calculations in order to provide the system with accurate price signals and security of supply. Additionally, the integration of offshore / onshore wind and solar power into the system requires power transmissions over longer distances, and it is necessary to evaluate the limitations of the transmission network. Power flow and optimal power studies for large-scale networks are needed both for operational or planning purposes.

Consequently, for a better network observation and management of this huge amount of data, e.g. 5000 busses of the European electricity network and more than 7000 transmission lines, the reduction of the network to be studied is of great importance. The reduction methods are divided in static and dynamic according to the representation of the model:

Static reduction: The reduced model represents a snapshot of the system and is suitable of static analysis only. These kinds of models are appropriate for power flow calculations, for operational and planning analysis.

Dynamic reduction: The reduced model is used for analysis of dynamic effects. According to [7] the equivalent models are used for (a) large scale power system offline transient stability analysis with large disturbance, (b) large scale power system off -line dynamic stability analysis with small disturbance, (c) large scale power system on-line security assessment.

This paper refers only to static reduction methods and discusses difficulties and requirements when power market simulations and market-based planning is the purpose of the equivalent system. Thus, standard reduction techniques and market-based approaches are reviewed.

In order to create an equivalent network the user has to define the system boundaries, e.g. internal, external network and the network elements of interest. Internal is a focused detailed area, in which usually a regional utility or a TSO is interested and an external area is defined as an aggregated not detailed system. Usually it is required that the external network interacts with the internal network, as in the initial model.

4 Standard reduction techniques

Equivalencing of the external network is one of the techniques implemented to reduce computational time. Various static network equivalencing techniques have provided solutions to the problem of network size, assuming that the remote subsystems have low impact on the internal system. The typical external equivalent model uses the “REI”, “Ward” or some variations of these two basic methods, [8], [9], [10], [12], [11], [13].

These methods divide a solved load flow model of the original network into an internal (subsystem 1) and external (subsystem 2) part, Fig. 2. The goal is to represent the external system by its equivalent with a smaller number of buses and branches, while the internal system is modeled in full detail.

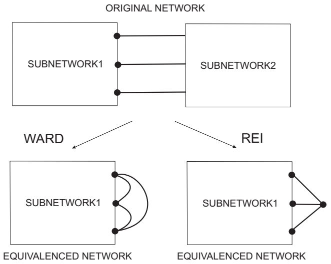

Figure 2: Network reduction with Ward and REI methods

4.1 WARD reduction method

The most used method is known as Ward reduction [14], which performs triangular reduction (Gaussian elimination) on the nodal admittance matrix of the network. The basic theory of linear network reduction is presented in [15].

The transmission system is modeled according to eq. (1) in power flow studies:

$$\mathbf{YE} = \mathbf{I} \tag{1}$$

where $\mathbf{I}$ is the nodal injection current vector, $\mathbf{E}$ is the nodal voltage vector, and $\mathbf{Y}$ is the nodal admittance matrix nxn.

If the system is divided into an internal and external part by an appropriate renumbering of nodes, this equation can be defined, so that subscript 1 denotes the internal subnetwork, that should be retained, and subscript 2 denotes the external part that should be reduced using Gaussian elimination:

$$\left(\begin{array}{c|c} \mathbf{Y_{11}} & \mathbf{Y_{12}} \\ \hline \mathbf{Y_{21}} & \mathbf{Y_{22}} \end{array}\right) \left(\begin{array}{c} \mathbf{E_1} \\ \hline \mathbf{E_2} \end{array}\right) = \left(\begin{array}{c} \mathbf{I_1} \\ \hline \mathbf{I_2} \end{array}\right) \tag{2}$$

or in expanded form:

$$\mathbf{Y_{11}E_1} + \mathbf{Y_{12}E_2} = \mathbf{I_1} \tag{3}$$

$$\mathbf{Y_{21}E_1} + \mathbf{Y_{22}E_2} = \mathbf{I_2} \tag{4}$$

This might require a reordering of the nodes by multiplication of these matrices with the permutation matrix T.

Eq. (4) is solved for $\mathbf{E_2}$ and replaced in eq. (3), and the equivalent network with nodal admittance matrix $\mathbf{Y_{11}^{eq}}$ which contains only the buses of the internal network is obtained:

$$\mathbf{Y_{11}^{eq}E_1} = \mathbf{I_1^{eq}} \tag{5}$$

$$\mathbf{Y_{11}^{eq}} = \mathbf{Y_{11}} - \mathbf{Y_{12}Y_{22}^{-1}Y_{21}} \tag{6}$$

$$\mathbf{I_1^{eq}} = \mathbf{I_1} - \mathbf{Y_{12}Y_{22}^{-1}I_2} \tag{7}$$

Here is an example of the Gaussian elimination of any equation from the system. The equation $\mathbf{I} = \mathbf{YE}$ can be written in expanded form:

$$\begin{aligned}
&Y_{11}E_1 + \ldots + Y_{1k}E_k + \ldots + Y_{1n}E_n = I_1\\
&\vdots\\
&Y_{m1}E_1 + \ldots + Y_{mk}E_k + \ldots + Y_{mn}E_n = I_m\\
&\vdots\\
&Y_{k1}E_1 + \ldots + Y_{kk}E_k + \ldots + Y_{kn}E_n = I_k\\
&\vdots\\
&Y_{n1}E_1 + \ldots + Y_{nk}E_k + \ldots + Y_{nn}E_n = I_n
\end{aligned} \tag{8}$$

If E_k is obtained from the k^{th} equation

$$E_k = Y_{kk}^{-1}(I_k - \sum_{m\in\Omega_K} Y_{km}E_m) \tag{9}$$

and then eliminated from the remaining equations, the k^{th} equation can be deleted from the system:

$$\begin{aligned}
&Y_{11}^rE_1 + \ldots + Y_{1k}^rE_k + \ldots + Y_{1n}^rE_n = I_1^r\\
&\vdots\\
&Y_{m1}^rE_1 + \ldots + Y_{mk}^rE_k + \ldots + Y_{mn}^rE_n = I_m^r\\
&\vdots\\
&Y_{n1}^rE_1 + \ldots + Y_{nk}^rE_k + \ldots + Y_{nn}^rE_n = I_n^r
\end{aligned} \tag{10}$$

The coefficients Y_{ij}^r are modified according to equation:

$$Y_{ij}^r = Y_{ij} - \frac{Y_{ik}Y_{kj}}{Y_{kk}} \tag{11}$$

and equivalent current injections I_m^r can be calculated with equation:

$$I_m^r = I_m - \frac{Y_{mk}}{Y_{kk}}I_k \tag{12}$$

4.2 *REI reduction method*

The REI (radial equivalent independent) procedure for external equivalents was developed by Dimo [16], and it has become well-known through the work by Tinney and Powel [17]. They have shown that this reduction preserves some properties of reduced generators through an equivalent generator.

The main idea is to identify groups of similar nodes and to replace each group by one virtual node P$_E$. The power injection at this bus P_E is equal to the aggregated injections of the group of nodes that will be replaced, i.e. P_1, P_2, ..., P_N, and these N nodes become passive.

The virtual node is connected through a virtual radial network (called the REI network) to these buses as shown in Fig. **??**(b). The admittances of the REI-network branches represent the operational set-up before the reduction, so that the power flow remains the same, neglecting power losses.

With these constraints N passive nodes can be eliminated using Gaussian elimination, while initial flows and voltage angles are retained with no loss in accuracy in case of a DC power flow model. Therefore, the equivalent model in Fig. **??**(c) is exactly the same (in the sense of voltage angles and power flows) as the original model at the operating point.

4.3 *Sparsity of equivalents*

The nodal admittance matrix of the power network is typically very sparse. However, the equivalent model after network reduction is usually denser (less sparse) because the connections between retained buses have to be preserved after reduction. All nodes that were connected to the deleted node (boundary nodes) are mutually connected, whether they were originally directly connected or not.

In order to get a reduced model that is easier to analyze than the original one it is necessary to preserve its sparsity. High number of new branches can decrease the effect of nodes reduction on the sparsity of the model and the impedance of new branches can sometimes be very high leading to numerical problems. Different groups of nodes influence the sparsity of the network if they were deleted. In [18] the authors present how the number of new branches can be minimized if the group of nodes for elimination, is divided into disconnected subnetworks or into subnetworks that are connected only through overlapping border nodes. The main idea is either to preserve some non-essential nodes in the new model, or to delete them in some order so that the sparsity of the equivalent network is retained. However, the optimal group of nodes that minimizes the density of the network is not defined.

5 Market-based reduction techniques

The creation of large power markets expanding through several control areas, requires new reduction methods focusing on economic criteria and generation participation. Typical methods use either reduction based on locational marginal prices [19], or power transfer distribution factors (PTDFs) [20]. This two methods will be reviewed in the following section.

5.1 LMP based reduction method

The basic idea of the LMP based method is to create clusters, e.g. aggregated areas, based on locational marginal prices. In order to avoid creating clusters where congested lines exist, only nodes with similar nodal prices are aggregated. For the aggregation the REI method is used. As the power flows from low price to high price nodes and since REI does not differentiate between different generation technologies, equivalent marginal production cost curves are developed that produce the same flows [21]. A selected group of generators produces P_{tot} aggregated optimal actual power at a cost of C_{opt} marginal costs. Defining lower and upper production limits for this group, P_{tot} is divided in an equally spaced data set of d data points, where $P_{tot,1} = P_{tot}^{min}$ and $P_{tot,d} = P_{tot}^{max}$. By means of an economic dispatch within the group of generators each $P_{tot,i}$, $i = 1, ..., d$ is associated with a cost $C_{opt,i}$. An equivalent quadratic costs curve where $P_{G_{eq}}$ and C_{eq} are the equivalent generator output and costs, respectively, eq. (13), is assumed.

$$C_{eq} = a_{eq} P_{G_{eq}}^2 + b_{eq} P_{G_{eq}} + c_{eq} \tag{13}$$

The coefficients a$_{eq}$, b$_{eq}$ and c$_{eq}$, are calculated using linear regression in order to find the values that fit the set of data d the best from eq. (14).

$$\begin{bmatrix} \sum_{i=1}^{d} P_{tot,i}^2 & \sum_{i=1}^{d} P_{tot,i}^3 & \sum_{i=1}^{d} P_{tot,i}^4 \\ \sum_{i=1}^{d} P_{tot,i} & \sum_{i=1}^{d} P_{tot,i}^2 & \sum_{i=1}^{d} P_{tot,i}^3 \\ d & \sum_{i=1}^{d} P_{tot,i} & \sum_{i=1}^{d} P_{tot,i}^2 \end{bmatrix} \times \begin{bmatrix} a_{eq} \\ b_{eq} \\ c_{eq} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{d} P_{tot,i}^2 C_{opt,i} \\ \sum_{i=1}^{d} P_{tot,i} C_{opt,i} \\ \sum_{i=1}^{d} C_{opt,i} \end{bmatrix} \tag{14}$$

In order to calculate the initial nodal prices an optimal power flow (OPF) calculation for clustering is required, which means that the detailed initial system has to be known. For large scale systems and for systems that extend within more than one control area, this calculation might not be possible due to lack of information.

5.2 PTDF based reduction method

When the transmission network and its impact on power exchanges is to be studied, other indicators are often used. These are the power transfer distribution factors (PTDFs) that indicate how, in terms of percentage power flow, a bilateral transfer of a specific amount of power influences the rest of the transmission network. From the definition eq. (15), [22], it is obvious that the calculated values are highly dependent of the transmission lines and generation capability:

$$\frac{\Delta P_n}{\Delta T_{i,i'}} = PTDF_{i,i',n} \tag{15}$$

where $\Delta T_{i,i'}$ refers to the shift in power injections nodes i and i' and ΔP_n is the power flow through line n.

Thus, a reduction based on PTDFs is, depending on the actual configuration of the system, as in case of LMP method application.

The target is to define the equivalent reactances of the equivalent transmission lines in order to retain the original PTDF values. Aggregated zones, that are connected through possible or actual congestion corridors, are created and according to the approximation of the injection shift factors of the aggregation areas, the line reactances are calculated. PTDFs are here represented by injection shift factors (ISF), ψ, for any transaction between two nodes i and i', assuming that one of the nodes is the slack bus, eq. (16):

$$PTDF_{i,i',n} = \psi_n^i - \psi_n^{i'} \tag{16}$$

The optimization problem when the final topology of the reduced system is known is described in eq. (17):

$$min_{x_b} \{norm[\Psi - x_b^{-1} A^T (A x_b^{-1} A^T)^{-1} \hat{A}]\} \tag{17}$$

where Ψ consists of the zonal injection factors for power exchanges from all non-slack busses to the slack bus, x_b is the unknown matrix of reactances and A the node-branch incidence matrix of each power exchange and $\hat{A}$ is the node-transaction incidence matrix.

This method requires also an optimal power flow, usually a linearized DC-OPF, for the calculation of LMPs, for identification of congestion paths and generation dispatch. Hence, a detailed initial network representation is needed. Although the result is more accurate than with WARD method regarding PTDF values, the accuracy always depends on the topology of the network and operating point.

6 Test reduction of complete model

The WARD method was tested on the complete IEEE 300-bus and the whole UCTE network, considering it as one area for the calculation of a DC power flow, without defining external and internal areas. The reduction was based on gradual elimination of nodes according to their connectivity to other nodes. Gradual elimination means that end-nodes are first deleted while their injection is moved to the node it was connected to, then nodes with two transmission connections are merged, three transmission connections etc. This is represented by the reduction level on x-axes.

In Fig. 4 the calculation time of the IEEE 300-bus and the UCTE network is presented. For the IEEE model the calculation time except for reduction level two is decreasing, which is not the case for the UCTE model. This happens probably because of high density of the admittance matrix in reduced UCTE model. Nevertheless, the measurement is not very accurate as Matlab measures CPU time which differs within different computational systems
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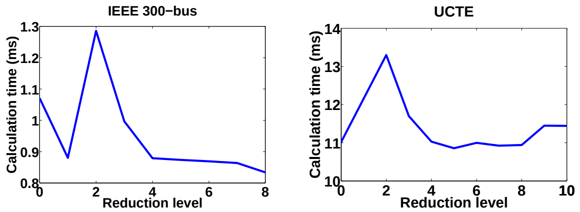

Figure 4: DC power flow average calculation time of the IEEE (left) and UCTE (right) network for different reduction levels

As shown in Fig. 5 and Fig. 6 the total number of nodes decreases constantly, however the total number of lines increases after a certain reduction level for both models. The same effect has been also observed in Powerworld, when creating gradually smaller and smaller internal networks the number of lines tend to increase.

This means that in WARD method there is an optimal number of reduced transmission lines, different for different models.

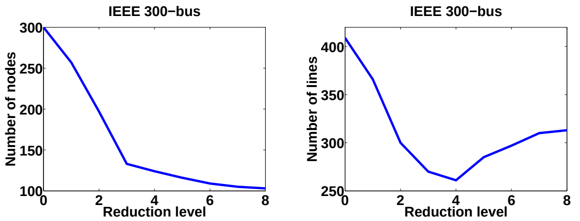

Figure 5: Number of nodes (left) and number of lines (right) for different reduction levels of the IEEE-300 bus network model using WARD reduction

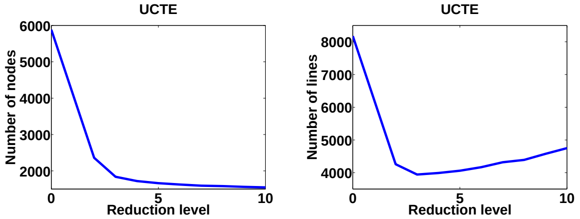

Figure 6: Number of nodes (left) and number of lines (right) for different reduction levels of the UCTE network using WARD reduction

This effect was not observed applying REI reduction as the number of lines was decreasing with decreasing number of nodes, Fig. 7. Accordingly, the calculation time was sinking as well. The major problem with REI is that generators are not differentiated during the aggregation.

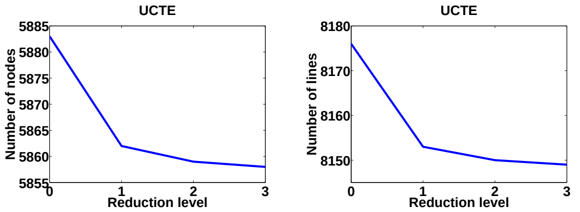

Figure 7: Number of nodes (left) and number of lines (right) for different reduction levels of the UCTE network using REI reduction

As aforestated, LMP and PTDF methods were not possible to test due to lack of OPF data for the initial systems.

7 Reduction criteria for power market and planning simulations

Firstly, as the target is to study large-scale networks, e.g. the European electricity network, the network elements should be categorized according to their importance and influence on other elements. Thus, for instance in transmission planning the distribution network need not to be modelled. However, the aggregated loads should be correctly distributed in the 380kV - 400kV network, so that similar loading conditions are simulated. Due to lack of information for the OPF calculation, the network has to be divided in areas of interest like in standard reduction methods using an internal and external part. Although, these areas need not be limited by national borders.

As the economic models must provide accurate market indicators the generators of the studied area should remain unchanged, or at least have the same marginal cost characteristics, like in LMP method, so as to produce the same power exchanges. In order to check for congestions and critical paths between and within areas, interconnections and lines with large transmission capacities should not be eliminated. This means that some nodes outside of the main focus area will remain unchanged and new transmission lines are created. The major difficulty is that usually the transmission capacity of the equivalent lines is unknown, however approximations based on typical values and experience might help to overcome this problem. Assumptions are to be met also when the initial system data are not sufficient for economic analysis. However, it is simpler to make assumptions for a small area than for the whole system. In this case, the LMP and PTDF methods are disqualified, because of lack of data needed beforehand, as explained before.

Based on the previous analysis and review, it seems like a combination of WARD and LMP method would be the most appropriate. However, as the marginal production costs are missing, the idea is first to create the optimal topology and then based on assumptions to develop the economic model. A procedure that was applied in Powerworld for the UCTE model follows:

1. Identify the area of interest including neighboring networks, that might be split into other subareas.
2. Start a gradual reduction beginning from outside, keeping generators and interconnectors.
3. Reduce once again equivalencing the generators outside the area of interest.
4. In case that the voltage level of 380kV is of interest, eliminate the nodes at 220kV and 110kV system level.

So for different focus areas different models are developed. In case that the focus is Germany, Switzerland and the neighbor countries Italy, France, Austria, it creates a system of 460 busses, 162 generators (32 in Switzerland and 110 in Germany), 331 loads, 98 tie-lines and a total of 1334 transmission lines. This system is easier to handle and gives promising results, which is not in the scope of this paper.

8 Summary

Summarizing, all methods use some form of aggregation, either in clusters or in external network. Standard equivalents are suitable for power flow calculations, however generation type differences, congested lines and nodal price differences are totally neglected. Thus even if the initial cost data for generators are available, the final OPF model will be completely different.

In the contrary, market-based methods focus more on congestions and costs characteristics, however the equivalent network depends on the actual configuration of the system. Additionally, it is required that information for costs data is available before the reduction. However, for large scale systems it is almost impossible to know the marginal costs of production in all control areas. Additionally, the calculation of nodal prices for the original system might not be realistic because of computational limitations.

WARD and REI are more topology oriented, while LMP and PTDFs are more based on actual generation and consumption levels.

The most appropriate approach seems to be a modified WARD combined with one of the market-based methods, however assumptions are always needed. The aggregated transmission or generation planner although is responsible for the whole system, can focus in areas affected from the decision, without losing much of accuracy.

The table 8 characterizes the four methods according to the reduction criteria mentioned before:

Table 1: Characterization of methods according to reduction criteria for economic models

	WARD	REI	LMP	PTDF
Generators	-	-	x	x
Interconnections	x	-	x	x
Congestions - Trans.limits	-	-	x	x
Loading conditions	x	-	x	x
Large-scale systems	x	x	-	-
OPF calculation	-	-	x	x

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