Modeling the Wind Power In-feed in Germany by Data Decomposition and Time Series Analysis

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Abstract—The total wind power in-feed in Germany possesses highly predictable seasonal and diurnal patterns. The stochastic variations of wind power have consistent pseudo-cyclic patterns that are related to oscillations of air-pressure systems. The proposed model can replicate the wind power data with the hourly resolution and of time-span of years. The primary purpose of the model is for market applications such as pricing of forward contracts several years ahead. The results of the model may also be used in applications of power system operation such as reliability analysis, day-ahead scheduling, and hour-ahead scheduling.

Index Terms—Wind Power, Data Decomposition, Time Series Analysis, Power Markets, Pricing of Forward Contracts, Power Systems, Reliability Analysis, Scheduling

I. INTRODUCTION

The installed capacity of wind power in Germany has more than doubled in the past decade. By the end of 2010, the installed capacity reached 27GW [1], which is 33.75% of the maximum grid load of 80GW in the year [2]. The energy in-feed from wind power in 2010 was 37.5TWh [1], and it’s about 8% of the total grid consumption of 488TWh [2]. Germany plans to increase its onshore wind capacity to 45GW and offshore to 10GW by 2020 [1].

The substantial role of wind power in the German power grid has brought major challenges for both grid operations and power markets. Fig. 1 shows a snapshot of the situation on January 1st 2011: the maximum wind power in-feed on the day is 14GW. Due to this high wind power in-feed and because of the low consumption on the public holiday, the vertical load of transmission grid falls to a minimal level of 19GW, (where vertical load is defined as the load flowing from the high voltage transmission grid (220kV-110kV) to medium and low voltage grids [3]). This low vertical load level caused the spot price to drop to negative -40 €/MWh.

The primary purpose of this work is to build a wind power model of Germany for long-term market applications. These applications include pricing of forward contracts and other power derivatives up to several years ahead [4]. These market applications require not only a model that captures the deterministic patterns of wind power in-feed such as seasonal and diurnal patterns, but also a model that can define the uncertainties in order to take the risk into account.

Surveying literature, there has been analysis and modeling on wind power in-feed of smaller scales. They either concern the output of a single wind turbine [5], or the modeling of a wind-farm [6]. To the best of the authors’ knowledge, in this paper it is the first time stochastic modeling of wind power of power-system scale is reported. The reason for this gap of research is probably the lack of data, because, if surveying the power systems around the world, there seldom exist a time series of wind power data that is of 10GW scale and that is of time-span of several years and publicly available.

Bearing in mind the law of large numbers and central limit theorem, the wind power in-feed of Germany, which is the aggregation of the output of numerous wind turbines dispersed across Germany, should possess statistical properties that are very different from that of either a single wind turbine or a wind farm. It is worthwhile, due to both its significance to power system operation and its impact on power markets, to make a careful study on the wind power in-feed of Germany.

The main methods of the work are data decomposition and time series analysis. The idea is that wind results from the interactions of air-pressure systems of different spatial scales [7, p.8] [8], and that these factors are probably reflected and present in wind power in-feed. Therefore the wind power data could probably be separated by decomposition methods, and then modeled respectively by time series methods. Similar decomposition approach has been applied by [9] on modeling wind speed, and by [10] on modeling the spot prices of power markets.

The paper is organized as follows: Section II introduces the data and the first analysis, Section III prepares the data for further analysis by normalization and decomposition, Section IV builds the model for each data component and calibrate the models, Section V presents the overall modeling results, Section VI discusses the applications and limitations of the model, and finally Section VII concludes the paper.
II. Data and First Analysis

A. Introduction to the Data

The wind power data were collected from the websites of three German TSOs [3], which are Amprion, 50Hertz, and Tennet. EnBW was left out of the analysis because of its insignificance in wind power. Quarter-hourly data of wind power in-feed including both forecast and actual data are available on the websites. These data are updated to present time with delay of a few hours.

In our analysis we will only deal with the actual wind power data. The time series of actual wind power in-feed of the three control areas (Amprion, 50Hertz, and Tennet) were aggregated and taken as a proxy for the wind power in-feed of Germany. The original data is quarter-hourly based, it is converted to hourly data by arithmetic average. The aggregated time series starts from March 1st 2006 and ends at September 29th 2011, see Fig. 2.

B. First Analysis

The first impression of the data is the significant seasonal pattern. The wind power in-feed in the winter time on average is about 3GW higher than that in the summer. The seasonal variation is tracked closely by a simple sinusoidal function.

Another observation is that even though the maximum power in-feed is slightly increasing across the years, the one-year moving average of the time series is rather flat. This little increase of the general level of wind power in-feed is counterintuitive, because the installed capacity of wind power in Germany in the last five years increased on average by 8% per year [1]. The two reasons for this dampened trend might be that Germany was experiencing low wind speed in the last few years [11], and that new capacity additions were probably installed in places of less favorable wind resources than the previous installations.

The average in-feed across the years is 4.2GW, and the highest in-feed so far is about 22.5GW on February 4th 2011. In the next few years, with the higher installed capacity, and if the cycle of general wind speed turns to increase, Germany will probably see even higher wind power penetration.

III. Data Normalization and Decomposition

A. Normalization of the Data

The wind power data will hereafter be denoted as $\tilde{W}(t)$. The histogram of the data is plotted in Fig. 3a, the distribution is highly positively skewed: the majority (70%) falls below 5GW, and only 10% is larger than 10GW.

Preparing for modeling, the data has to be normalized to make their distribution approximately Gaussian, this is because the Time Series methods that are going to be applied for data analysis later on are based on a basic assumption that the random noises follow Gaussian distributions, refer to, e.g., [5].

Taking the logarithm is a popular method for normalizing data, therefore the logarithm of the data $\tilde{w}(t) = \ln \tilde{W}(t)$ was first used. The resulting histogram is plotted in Fig. 3b. It was found that the logarithm was not suitable because it over-dampened the data and changed it into a negative skewed distribution.

Preparing for further analysis the transformed wind power data $\tilde{w}(t)$ will be decomposed into several components that

\begin{table}[h]
\centering
\caption{Skewness of Wind Power Data after Transformations}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$W(t)$ & $\ln W(t)$ & $W(t)^{1/4}$ & $W(t)^{1/5}$ & $W(t)^{1/5.5}$ & $W(t)^{1/6}$ \\
\hline
1.4282 & -0.4424 & 0.1027 & 0.00043 & -0.0375 & -0.0694 \\
\hline
\end{tabular}
\end{table}

In literature [5], the power transform of square root $[\tilde{w}(t) = \tilde{W}(t)^{1/N}, N = 2]$ is found suitable for transforming the wind speed data into a balanced distribution. For the wind power data in our case, referring to TABLE I, the 5th order power transform,

$$\tilde{w}(t) = \tilde{W}(t)^{1/5}, N = 5$$

produces a mostly balanced distribution, see Fig. 3c. The time series of the transformed data is plotted in Fig. 4a. The reasons for the 5th order transform might be related to the power of wind as a cubic function of wind speed, and the nonlinear power coefficient function of wind turbines [7].
belong to different frequency bands. The idea is to decompose the data into components that are driven by different and independent physical factors, and then model each of them separately [10]. The immediately following question is whether wind speed and therefore wind power in-feed is driven by physical factors of different frequencies. If it is, then what are the frequency bands of these factors.

A first reasoning leads to that weather systems are driven by different dominant factors in different temporal and spatial scales: such as seasonal and intra-seasonal variations driven by global-scale (5000km) atmospheric systems such as North Atlantic Oscillation (NAO) [11, 12], mid-term changes of several days due to oscillations of synoptic-scale (1000km) air-pressure systems [7, 8], and the short-term variations (across hours during a day) due to the diurnal cycle of solar radiation. These rhythms of weather systems could be reasonably expected in wind speed, and therefore are reflected in the time series of wind power in-feed.

To seek for these patterns of frequency, the data were taken the Fast Fourier Transform (FFT) \( f(\omega) = \sum_{t=0}^{T-1} \tilde{w}(t) \cdot e^{-i\omega t} \), where \( \omega = \frac{2\pi k}{24} \), and \( k = 0, 1, \ldots, T - 1 \). The raw spectrum \( |f(\omega)| \) and the spectral estimation [13, p.133] are plotted in Fig. 5a: the distinct spike of the highest magnitude at the low frequency range stands for the one-year seasonality (8760hrs); in the high frequency range, the modest but distinct spikes at the frequency \( \frac{1}{24} \) cycle/hour (and its harmonics) are because of the diurnal cycle of wind power in-feed; in the mid-frequency range between the period of 1 day (24hrs) and about 14 days (14×24 hrs), there is a cluster of spectrum that has no distinct periodicity, at this frequency range the physical driving factors are oscillations of 1000km-scale pressure systems; in the lower-frequency range between the period of 14 days and one-year (8760hrs) there is another cluster of spectrum of no definite periodicity, at this frequency-band the variances might be driven by intra-seasonal oscillations of global-scale atmospheric systems; furthermore, in the spectrum around the frequency of period 14 days there is a slight but distinct valley, this might be explained by that there are few physical forces oscillating at this frequency, the frequency of period 14 days therefore is selected as the cut-off frequency between the mid-frequency band and the low-frequency band.

Based on the observations on power spectrum, Finite Impulse Response (FIR) filters were designed for three frequency
bands, which are (a) frequencies of period larger than 14 days \( \omega_w < \frac{2\pi}{24 \times 14} \), (b) frequencies of period between 14 days and 1 day \( \frac{2\pi}{24} \leq \omega_d < \frac{2\pi}{24} \), and (c) frequencies of period equal to or less than 1 day \( \omega_h \geq \frac{2\pi}{24} \). The results of filtering in the frequency domain are plotted in Fig. 5b-d. The counterparts in the time domain are plotted in Fig. 4b-d. The three time series after decomposition will be denoted respectively as: low frequency data \( \tilde{w}_w(t) \), mid frequency data \( \tilde{w}_d(t) \), and high frequency \( \tilde{w}_h(t) \). Therefore it follows that

\[
\tilde{w}(t) = \tilde{w}_w(t) + \tilde{w}_d(t) + \tilde{w}_h(t) \tag{2}
\]

The results of filtering in the time domain look encouraging, refer to Fig. 4b-d. The three components obtained by decomposition, comparing them with the original time series \( \tilde{w}(t) \), behave consistently across the years. A simple sinusoidal function captures the seasonal pattern of the low frequency data, see Fig. 4b. A counter intuitive observation on the high frequency data is that the volatility in summer time is consistently higher than that in winter time, see Fig. 4d.

The histogram plots in Fig. 6 reveals the degree of uncertainty brought by each component: the mid-frequency data (day-by-day variations) has the highest standard deviation \( \sigma_d' = 0.70 \), the high frequency component (hourly variations) is of the lowest volatility \( \sigma_h' = 0.22 \), and the low frequency variations (weekly across years) only contribute a modest degree of uncertainty \( \sigma_w' = 0.60 \). Further, these three components have negligible correlations with each other, see TABLE II.

IV. MODELING AND CALIBRATION

A. Intrayear (Weekly) Model

In this section, a time series model for the low frequency data \( \tilde{w}_w(t) \) is built. Hereafter it is referred as the Intrayear Model, of which the time unit is weekly. The weekly data is obtained by sampling the low-frequency hourly data week by week.

The Intrayear Model not only has to define the seasonal pattern, Fig. 4b, but also has to model the temporal correlations of the stochastic component, which is the data that is left after taking into account the seasonal pattern. The highly predictable seasonal pattern is modeled by a sinusoidal function of period 52 weeks,

\[
f_w(t) = a_w + b_w \cdot \cos\left(\frac{2\pi}{52} t\right) + c_w \cdot \sin\left(\frac{2\pi}{52} t\right) \tag{3}
\]

The stochastic term, which is obtained by subtracting the seasonal pattern from the data \( \tilde{x}_w(t) \equiv \tilde{w}_w(t) - f_w(t) \), will be defined by a time series model. By looking at the ACF and Partial Autocorrelation Function (PACF) of the data, Fig. 7ab, it was found sufficient to model it by an AutoRegressive (AR) model of 2 lags

\[
\tilde{x}_w(t) = \alpha_{w1} \cdot \tilde{x}_w(t - \Delta t) + \alpha_{w2} \cdot \tilde{x}_w(t - 2\Delta t) + \sigma_w \cdot \tilde{\varepsilon}_w(t) \tag{4}
\]

where \( \tilde{\varepsilon}_w(t) \) is white noise of standard normal distribution.

As seen from the PACF, the autocorrelation of the first lag \( \alpha_{w1} \) is positive, and that of the second lag \( \alpha_{w2} \) is negative. This pattern, as it will be seen, is consistent in both the mid- and high-frequency data. This autocorrelation pattern of positive first lag and second negative lag produces a pseudo-cyclic behavior that is probably general in weather systems. The pseudo-cyclic pattern of weather data is related to oscillations of atmospheric systems of different spatial scales [14, p.8], [8].

The overall model for the low frequency data is therefore written as the combination of the deterministic and the stochastic models as,

\[
\tilde{w}_w(t) = f_w(t) + \tilde{x}_w(t) \tag{5}
\]

For estimating the model parameters, the model is derived into the form of AR(2) as

\[
\tilde{w}_w(t) = \alpha_{w1} \cdot \tilde{w}_w(t - \Delta t) + \alpha_{w2} \cdot \tilde{w}_w(t - 2\Delta t) + [f_w(t) - \alpha_{w1} \cdot f_w(t - \Delta t) - \alpha_{w2} \cdot f_w(t - 2\Delta t)] + \sigma_w \cdot \tilde{\varepsilon}_w(t) \tag{6}
\]
The problem of estimating the parameters of (6) is a nonlinear least-square optimization problem, and solved by the nonlinear regression routine ‘nlin’ in MATLAB. The results are shown in Fig. 8 and TABLE III. The ACF of the residuals, which is the noise data that is left after calibrating the model, indicates that the temporal correlations of the data have been adequately modeled, see Fig. 7c, and the histogram of the residuals is sufficiently normally distributed, see Fig. 7d.

The parameters of the autoregressive terms $\alpha_{w1} = 0.2829$ and $\alpha_{w2} = -0.1647$ [14, p.100] indicate that the stochastic term $\tilde{x}_w(t)$ has a pseudo-cyclic pattern of period approximately five weeks. This pseudo-periodicity of 5 weeks might be related to intra-seasonal oscillations of global-scale atmospheric systems such as NAO [11], [12]. Further explanations (and thus might its possible validation) concerning this pseudo-periodicity belong to the field of Atmospheric Science, and are beyond the circle of engineering knowledge of the authors.

### B. Intraweek (Half-daily) Model

The mid-frequency data $\tilde{w}_d(t)$ is of frequency $\frac{2\pi}{24 \times 12} \leq \omega_d < \frac{2\pi}{24}$. The hourly data are sampled at the rate of every 12 hours, therefore the time unit of the model is half-daily.

To identify the suitable time series model the ACF and PACF of the data are again referred, see Fig. 9ab. The ACF indicates a periodic behavior. The PACF, which is significant for the positive 1st lag and the negative 2nd lag and then persistently smaller for higher order lags, implies a pseudo-cyclic pattern. The ACFs again suggest an AR(2) model as,

$$\tilde{w}_d(t) = \alpha_{d0} + \alpha_{d1} \cdot \tilde{w}_d(t - \Delta t) + \alpha_{d2} \cdot \tilde{w}_d(t - 2\Delta t) + \sigma_d \cdot \tilde{\varepsilon}_d(t)$$  \hspace{1cm} (7)

The problem of estimating the parameters is a simple ordinary least square problem, and is solved by the standard MATLAB function ‘regress’. The results are shown in TABLE IV.

The ACF of the resulting residuals suggests that the simple AR(2) model is an adequate filter for the temporal correlations, see Fig. 9c. The autoregressive parameters $\alpha_{d1}$, $\alpha_{d2}$ [14, p.100] indicates a pseudo-cycle of about 3.75 days. This period of 3.75 days is consistent to the findings of [8] that there is a peak in the power spectrum of wind speed data at the period of about four days.

### C. Intraday (Hourly) Model

The model for the high frequency data $\tilde{w}_h(t)$ will be referred as the Intraday Model. The first inspection of the power spectrum of the data suggests significant diurnal patterns, refer to Fig. 5d. The time series plot shows considerable seasonal variations, in which the data in the summer time is more volatile than these in the winter time, see Fig. 4d. The data therefore is classified by months, and one model is calibrated for each month.

The deterministic part of diurnal cycle is modeled as a Fourier series up to the 9th order. The 9th order is determined by counting the distinct spikes of the power spectrum, refer to Fig. 5d,

$$f_h(t) = \alpha_h + \sum_{k=1}^{9} \left[ b_{hk} \cdot \cos\left(\frac{2\pi k}{24} t\right) + c_{hk} \cdot \sin\left(\frac{2\pi k}{24} t\right) \right]$$  \hspace{1cm} (8)

The diurnal patterns are estimated using the least square method, see the results in Fig. 10: the diurnal cycles in summer are more pronounced than these in winter, similar seasonal variations of the diurnal cycle were also observed in a Danish wind farm [15]; another observation is that the summer cycle peaks in the afternoon while the winter cycle has a slight peak at night.

For the stochastic term, which is the data that is left after taking into account the deterministic diurnal pattern, $\tilde{x}_h(t) \equiv \tilde{w}_h(t) - f_h(t)$, the ACF and PACF of the data suggest the same pattern as these of the low frequency and mid-frequency data. January and July are shown for example in Fig. 11. Therefore again an AR(2) is considered suitable,

$$\tilde{x}_h(t) = \alpha_{h1} \cdot \tilde{x}_h(t - \Delta t) + \alpha_{h2} \cdot \tilde{x}_h(t - 2\Delta t) + \sigma_h \cdot \tilde{\varepsilon}_h(t)$$  \hspace{1cm} (9)

The overall model for the high frequency data therefore is,

$$\tilde{w}_h(t) = f_h(t) + \tilde{x}_h(t)$$  \hspace{1cm} (10)
which written in the form of an AR(2) model is
\[
\tilde{w}_h(t) = \alpha_{h1} \cdot \tilde{w}_h(t - \Delta t) + \alpha_{h2} \cdot \tilde{w}_h(t - 2\Delta t) \\
+ [f_h(t) - \alpha_{h1} \cdot f_h(t - \Delta t) - \alpha_{h2} \cdot f_h(t - 2\Delta t)] \\
+ \sigma_h \cdot \tilde{\epsilon}_h(t)
\]
(11)

The parameters of (11) is estimated by nonlinear regression solvers, see the results in TABLE V.

The parameters \(\alpha_{h1}\) and \(\alpha_{h2}\) [14, p.100] suggest that the stochastic term has a pseudo-cycle of approximately 14hrs. This periodicity of 14hrs is consistent to the semi-diurnal cycle (12hrs) of wind speed as reported in [7, p.8] [8]. Concerning the volatility term \(\sigma_h\), the point to note is that the volatilities of the summer are twice of these in the winter. By examining the ACF and histograms of the residuals, Fig. 11cdgh, it is reasonable to conclude that the simple AR(2) model has accomplished an adequate job.

Another point to notice is that the \(R^2\) is much higher (0.9) for the high frequency data than for both the low frequency (0.3) and mid-frequency (0.4) cases. The direct implication is that the hour-ahead prediction might be done accurately by only using the historical wind in-feed data, and that the hour-ahead forecast is much easier to predict than the cases for day-ahead or week-ahead. This could be explained by the reasoning that the hourly variations of wind power are driven by local effects that have little spatial correlations across Germany, refer to, e.g., [16, p.22]. According to the law of large numbers, these local and independent variations are averaged out when aggregating the large numbers, as \(\sigma \propto \frac{1}{\sqrt{N}}\).

V. SIMULATION AND OUT-OF-SAMPLE RESULTS

A. Simulation of the Overall Model

In this section the performance of the overall modeling result is evaluated. In Section IV, the data till the end of 2009 were used to calibrate the models, and the data of 2010 and 2011 were deliberately left for out-of-sample tests. The fact is that whether the 2010 and 2011 data is included or not in calibrating the models only leads to negligible differences in the resulting parameters. This fact validates the models from another perspective.

Using the models from Section IV, a sample path of hourly wind power in-feed across a one-year horizon is constructed by simulation. Separately one path is simulated for each frequency, see Fig. 12, thereafter the three simulated components are aggregated and taken the 5th power,

\[
\tilde{W}(t) = \tilde{w}(t)^N = [\tilde{w}_w(t) + \tilde{w}_d(t) + \tilde{w}_h(t)]^N, N = 5
\]
(12)

The first inspection of the simulation result is encouraging, see Fig. 13. The temporal variations of the simulated time series resemble these of actual data to a large extent. The seasonal and diurnal variations are also well represented. The seasonal variation of uncertainty is also well reflected. (The corresponding red curves of the expected mean and standard deviation in Fig. 13 were calculated analytically: the mean is the 5th raw moment of the normal distribution of the overall model \(\tilde{w}(t)\), and the standard deviation is the square root of the sum of the variances of the separate models.)

For out-of-sample inspection, another simulation path is...
Fig. 12. Simulation of the Component Models: (a) intrayear model $\tilde{w}_w(t)$, (b) intraweek model $\tilde{w}_d(t)$, and (c) intraday model $\tilde{w}_h(t)$

Fig. 13. Overall Result of the Simulation: simulated one-year sample path of German wind power in-feed, the expected mean values, and the confidence interval of one standard deviation

Fig. 14. Out-of-sample Evaluation of the Overall Model: the sample path of actual German wind power in-feed in 2010 is compared with a simulated sample path, (a) whole year, (b) winter, and (c) summer.

<table>
<thead>
<tr>
<th>TABLE VI</th>
<th>CENTRAL MOMENTS OF THE ACTUAL AND SIMULATED SAMPLE PATHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual(2007-2010)</td>
<td>4327</td>
</tr>
<tr>
<td>Simulation(4 paths)</td>
<td>4213</td>
</tr>
</tbody>
</table>

plotted against the actual data of 2010, see Fig. 14a. The winter and summer months of 2010 are plotted respectively in Fig. 14bc.

B. Histograms of the Simulations and the Actual

Besides the visual inspections, to validate the overall model it is necessary to examine whether the simulation produces empirical probability distributions that are similar to these of the actual data.

The comparisons are shown in Fig. 15 for both the transformed data (left column) and the original scale (right column). The empirical distributions of the actual data, for each year from 2007-2010, were plotted against those of 4 simulated one-year paths. The upper graphs are the Empirical Probability Density Function (EPDF) plots, and the lower graphs are the Empirical Cumulative Density Function (ECDF) plots. Overall the statistical distributions of the simulations resemble closely those of the actual data.

In terms of quantitative measures, see TABLE VI. The first two central moments of the distributions match closely; for the higher order moments like skewness and kurtosis, the performance as seen is worse. The reason for why the skewness and kurtosis is not a close match is that the simulations tend to produce values that are close to or even exceed the installed capacity of 27GW, while the maximum of the actual data is around 22GW. This issue will be covered briefly in the next discussion section.

VI. DISCUSSIONS ON APPLICATIONS AND LIMITATIONS

The primary purpose of the model is for long-term market applications, such as pricing of forward contracts up to several years ahead and constructing the Hourly Price Forward Curve (HPFC) [4]. These applications require a wind power model that not only takes into account the predictable seasonal patterns and diurnal cycles, but also considers the extent of uncertainty of wind power in-feed in different time scales. The proposed model satisfies these two basic requirements.

Admittedly, for these applications, there are other even longer-term factors to consider, for example: the trend of wind speed in the scale of many years [11], the further increase of the installed wind capacity, and the increase of the share of the offshore wind farms. These factors might be taken into account by extending and revising the first results of this work.
The residual data in constructing the simulations. The close low frequency data as discussed before, or by bootstrapping were encountered quite often. This issue might be improved simulated values that exceed the installed capacity of 27GW ability of the wind power in-feed exceeding the overall in-

In the winter time seem to be larger than these in the summer variations might be used to determine the amount of additional reserve capacity that is needed due to wind power in-feed.

Concerning the limitations of the models, for the low frequency data, Fig. 4b, it is noticed that the weekly variations might be used to improve the accuracy of both the daily and hourly variations could be used to improve the modeling of uncertainty for both the daily and hourly


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