On Using Reduced Networks for Distributed DC Power Flow

Emil Iggland, Student Member, IEEE and Göran Andersson, Fellow, IEEE

Abstract—The last decades have seen a big change in the European system of electric transmission grids. Liberalization, the influx of Information and Communications Technology and a strong growth in cross-border trading have changed the realities which grid operators face. Due to the rise in interaction between Transmission System Operator areas, primarily in terms of increased energy trading, there is an increased dependency on the state of the neighboring systems. In order to have a good estimate of the situation in the own area, the Transmission System Operator needs to have an idea what is happening in the other areas. This can be achieved either with full data sharing, or with decentralized calculation methods. There may be situations where complete data sharing is not applicable as Transmission System Operators are not willing, or able, to share the data. This paper presents a reduced decentralized calculation method which improves on previously proposed methods by speeding up calculation time. An iterative procedure is used to find the solution which all participants find suitable, under consideration of their own objectives. The new method is compared to the centralized, full-data, solution and a distributed method where the full system model is used.

Index Terms—DC power flow, decentralization, equivalent networks, multi-area systems, network reduction, Ward method

I. INTRODUCTION

In inter-connected electric power systems different Transmission System Operators (TSOs) are collectively responsible for the operation of the individual power systems. Traditionally there was little interaction between the power systems as the tie-lines were primarily designed, and used, for the exchange of power in emergency situations. Due to the relatively small amount of power which flowed across the tie-lines the power networks could be considered as independent and as being, de-facto, islanded. Along with the increase in power-demand, a European drive towards more integrated markets, the liberalization of the European power markets and increased in-feed of geographically concentrated renewable energy sources this previous state is no longer given. The TSOs gave a cooperative role in operating the total grid.

Two main paths are envisioned for the future operation of the European electricity grip. These fall into the categories of 'super-TSO', where a single instance has control over the entire system. This would entail the TSOs transferring their tasks and knowledge and responsibility to a common entity. The second path is a co-operative model where the TSOs retain their autonomy and control, but increase the amount of data-sharing which is performed. The increase in information-sharing increases the area over which the TSO has visibility. The increase in this visibility range, also known as the observability area, does not mean an increase in the area which can be influenced, nor does it mean an increase in the area of responsibility.

These two distinctions are shown schematically in Fig. 1, where the circles indicate the area of responsibility. The dashed line indicates the super-TSO being responsible for all sub-areas.

The move towards a super-TSO would require changes which are not realistic at the moment, including reduced autonomy. As such a cooperative solution is more likely to be implemented.

A number of problems have to be solved in order for a completely cooperative solution, including but not limited to, sharing of control reserves, a knowledge of in-feed in neighboring countries, an intimate knowledge of the rules and regulations governing the operation of the neighboring systems. While a majority of these issues are of a regulatory nature, there are purely technical solutions to some of the.

This paper presents a method with which the solution to the power flow problem can be solved in a distributed manner. In contrast to previous methods, the one proposed in this paper uses a reduced system model to reflect the influence of the external nodes.

The paper is organized as follows: section II outlines the new proposal, sections III and IV evaluate the proposed method on a test system and present the results. Finally section V summarizes the finding and presents an outlook to the future.

II. PROBLEM FORMULATION

One of the classical problems of power system operation is the solution of the power flow equation. The power flow
equation describes the distribution of power flows across the components of the power systems as a function of the net nodal power in-feed, the network topology and the component properties. The solution of the power flow problem is usually a straightforward problem when all information is known. Owing to their position the TSO in a system operating in an islanded mode has this information. In the case where there are multiple TSOs operating portions of an interconnected system this is no longer the case. The case where there are multiple TSOs operating an interconnected system will be designated as multi-area operation.

The general DC power flow approximation is shown in equation (1), where $B$ is the grid susceptance matrix, $P$ the vector of net injections and $\Theta$ is the vector of node angles.

$$ B \cdot \Theta = P \quad (1) $$

In multi-area operation the grid consists of a number of sub-grids. Hence the grid model also consists of sub-models. Equation (2) indicates the totality of these sub-models for a three area situation. $\Theta_1$ indicated the node angle vector belonging to area 1, analogously for areas 2 and 3. The matrices $B_{1,1}$ and $B_{1,3}$ model the dependence of area 1 on area 2 and area 3 respectively.

$$ \begin{pmatrix} B_{1,1} & B_{1,2} & B_{1,3} \\ B_{2,1} & B_{2,2} & B_{2,3} \\ B_{3,1} & B_{3,2} & B_{3,3} \end{pmatrix} \begin{pmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{pmatrix} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} \quad (2) $$

The central power flow problem, as shown in equation (1), is solvable even for large systems. A number of papers describing a splitting of the power flow model into multiple areas have been published. [1]-[4]. All these methods use iterative procedures to solve the decoupled power flow equations.

The solution to the DC power flow problem, can only be cleanly split into parallel portions when there is no coupling between certain parts of the system. In general the splitting of the system into subsystems introduces errors. The stronger the coupling between the area, the greater the introduced error will be. Examples of areas with strong coupling include the France - Switzerland - Italy triangle.

The central solution of the DC power flow problem can grow big with increasing system size. However two factors alleviate this problem, the first being the good solution methods. The second is that, in the case where the TSOs has full knowledge of the neighboring system, and thus of the effects on their own system, appropriate reductions of external networks can be made, thus reducing the size of the external network model. In the distributed calculation this is not applicable. Without in-depth knowledge of the other system, a fixed model must be used. As in previous papers a single-depth model of the external networks is considered. Lines which are incident to both networks are modeled as a real bus in the own network, followed by the line connection, and an additional bus representing that in the external grid. Fig. 2 indicates this modeling approach. Nodes denoted $m_i$ and $n_i$ are real nodes in the areas of TSOs $m$ and $n$ respectively. Each border node is thus included in two models. The solid lines indicate cross-border lines, and the dashed lines indicate intra-area lines, i.e. the remainder of the grid. Nodes $m_i$ and $n_i$ are additional, or virtual, nodes in areas $n$ and $m$ respectively. This model means that all affected TSOs have the same model of the interconnection.

The general formulation of the decentralized solution approaches is as follows:

1) Model the own system - including the first external nodes. These nodes are termed local and remote respectively. This model is called the known system.
2) Solve the load flow problem for the known system.
3) Communicate the solution of remote and local nodes to the neighboring systems.
4) Receive the solutions from the neighboring systems.
5) Determine a new solution which is a compromise between the two solutions, as explained below.
6) GOTO 2 until convergence

Steps 3 and 4 are interchangeable in order, and can not be assumed to occur sequentially in time. As there is no control over the actions of the neighboring systems, data may be exchanged at any time, allowing for multiple updates of external data between two iteration steps, or an update being missed. It should be noted that a data-exchange solution is necessary for this approach to work. For inter-connected areas this is generally the case, and only minor changes would be required to allow this method to work.

The determination which is the new compromise solution is performed as an optimization problem formulated in equation (3).

$$ \min_x f(x) = [c_{\theta}, c_T, c_p] \cdot [\Delta \theta, \Delta T, \Delta p]^T \quad (3) $$

subject to

$$ x \in [-\pi, \pi] $$

$$ \sum \Delta P = 0 $$

The decision variable $x$ in this optimization problem is the angles of the border nodes. These angles are selected such that the mismatch between the own solution, the neighbors solution and the decision variable solution is minimized. Three factors are penalized: border angle mismatch, denoted $\Delta \theta$, transmission mismatch, denoted $\Delta T$ and nodal power mismatch, denoted $\Delta p$. Each factor is assigned an individual cost, $c_{\theta}, c_T$ and $c_p$ respectively. The angular mismatch describes the difference between the local angles and those transmitted by the neighboring TSOs, the transmission mismatch describes the difference between the power flowing across interconnect calculated locally and that transmitted by the other TSOs. The nodal power mismatch is the given by the net power generation in the local nodes. The angular mismatch is calculated as the angle of the local node minus the value given by the remote system. The reference angle must be treated specially. In the central solution of the DC power flow an pre-defined node is allocated a fixed angle. In the decentralized approach this can either be solved by some form of leader-election, or, as done in this paper, by assigning one node in one area a predefined angle. Since this additional constraint only appears in one of the areas, it is not included in the formulation of eq. (3).
As the objective of the optimization problem is not to find an economical solution, the absolute magnitude of the costs does not play a crucial role, rather their relationship with each other. Only a low number of constraints is needed as the dispatch has already been performed. One needs to ensure that the angles do not exceed the full rotation, and that the change in generation is zero, as the area is already at zero net generation.

As the trans-border flows are included in the problem, the constraints can be simple. The angles are not allowed to pass beyond the full circle, and the sum of net generation in all areas must be zero - cross-border transfers included.

The fact that both areas also take into account the remainder of their network ensures that the power flow equations are fulfilled in the remainder of the system. The general suitability of this approach is shown in [1]–[4]. The decentralized solution is not trivial, and requires substantial time to solve. Two approaches to calculation-time reduction can be considered. In contrast to the method where the node angles are transmitted, the TSOs have no information on which angles are not known a-priori. The operator \( A \otimes a \) denotes the removal of element \( a \) from the vector, or matrix, \( A \), thereby reducing the dimensions of \( A \). The \( i \)-th element of a vector \( A \) is denoted \( A(i) \), with the corresponding notation for matrices being \( A(i,j) \) for the \( j \)-th column in the \( i \)-th row, the entire row or column being denoted by ‘:’. The elements of interest which are returned by this algorithm is the reduced system model, \( B_{eq} \cdot \Theta_{eq} = P_{eq} \).

Algorithm 1 Gaussian Reduction

\[
\begin{align*}
\Gamma^m_L &= \Omega^m_L \\
B^m_{eq} &= B^m \\
p^m_{eq} &= p^m \\
\Theta^m_{eq} &= \Theta^m \\
\text{while } i = \Gamma^m_L \text{(1)} &\text{ do} \\
\text{for } j \in \Omega^m_L \otimes i &\text{ do} \\
\text{for } k \in \Omega^m_L \otimes i &\text{ do} \\
B_{eq}(j,k) &= B_{eq}(j,k) - \frac{B_{eq}(j,k)}{P_{eq}(i,i)} \\
\text{end for} \\
p_{eq}(j) &= P_{eq}(j) - \frac{B_{eq}(j,i)}{P_{eq}(i,i)} P_{eq}(i) \\
\text{end for} \\
p_{eq} &= P_{eq} \otimes p_{eq} \\
B_{eq} &= B_{eq} \otimes B_{eq} \\
\Theta_{eq} &= \Theta_{eq} \otimes \Theta_{eq} \\
\Gamma_L &= \Gamma_L \otimes i \\
\text{if } \dim(\Gamma^m_L) = 0 &\text{ then} \\
\text{BREAK} \\
\text{end if} \\
\text{end while}
\]

In contrast to the method where the node angles are transmitted, the TSOs have no information on which angles were selected by the neighboring system, thus there exists no reference angle. As such their solution systems may be rotated versus each other. This process can be thought of as a form of leader-election.

In order to align the systems with each other a secondary optimization problem is solved. The systems are rotated until the difference between the systems is minimal, as expressed in equation (5), with \( \Theta_1, \Theta_2 \) being the node angle vectors of the local and corresponding remote border nodes respectively.
A simple addition of a linear offset to all the area angles does not change the flows within the areas, as it affects all nodes equally. As the first cross-border node is included, the cross-border flows are not changed either. This problem is equivalent to solving an estimation problem, as given by equation (6), where $\alpha_i$ are the angles in the local system, $\beta_i$ are the angles in the remote system, and $x$ is the linear offset which is added to the local angles.

$$\min_{x \in [-\pi, \pi]} \{ (\Theta_l + x) - \Theta_r \}$$  

$$\min_x (\sum_i |\alpha_i - \beta_i + x|)$$

As with the power flow portion of the problem, a further data exchange must be implemented. This is outlined in Algorithm 2. The areas exchange their selected angles, calculate which new updated angle gives the least total difference, and then update their own angles to this value. In order to avoid oscillations a damping factor $\gamma_d$ is introduced.

**Algorithm 2 Linear Fitting**

```
while $\Delta \Theta \geq \epsilon$ do
  exchange_data()
  $\min_x (\Theta_l + x - \Theta_r)$
  set_angles( $\Theta_l + \gamma_d \cdot x$ )
end while
```

III. DESCRIPTION OF THE TEST SYSTEM

The test system which is used is the IEEE RTS-96 test system as described by [7]. This test system consists of three copies of a smaller test system. The separation between TSO responsibility areas is performed along the borders of the smaller test systems. Nodes in the range 100-199 are allocated to TSO1, nodes in the range 200-299 are allocated to TSO2 and nodes in the range 300-399 are allocated to TSO3. The optional DC line between nodes 113 and 316 is omitted. A simplified schematic structure of the system is shown in Fig. 3. TSO1 is connected to TSO2 with three lines. TSO3 is connected to TSO1 and TSO2 with one line each.

The DC power flow is purely real, and corresponds to the real portion of the initial loads and generations. The generation is scaled linearly to fit the load. The net generation of each area, including exports, is zero.

The test system specification is fed into an object-oriented Matlab-model, with each TSO-model being run on a separate computation node. In order to enable background message sending and receiving a Java add-on is employed to allow multi-threaded behavior and network communication.

IV. RESULTS

In order to evaluate the suitability of the method, the results of the decentralized and centralized methods are compared. As the centralized solution has all knowledge, it is the correct solution. As a second benchmark, the full decentralized solution is used. It is known from literature that the results of the full decentralized solution are very close to the central solution.

A. Accuracy

The results of this simplified distributed DC-PF is compared to a variant of the methods proposed by [11]–[4] which uses the full neighboring system in order to calculate the angles. The referenced methods were adapted to be as close as possible to the proposed method. This means that all referenced to generation costs were neglected. It is assumed that the TSOs have already performed dispatching action, either by their own optimization problem, or by a market mechanism.

For the DC power flow approximation the critical element is the node angles. The power flow in the lines is given as a linear transformation of the angles, expressed in equation (7), where the flow $F_{i,j}$ in the line from node $i$ to node $j$ is determined by the difference in node angles $\theta_i$ and $\theta_j$, and the line impedance $X_{i,j}$. As such the discussion of the accuracy of the results will only be made on terms of the node angles and not the line flows.

$$F_{i,j} = \frac{1}{X_{i,j}} \cdot (\theta_i - \theta_j)$$

The results for the area operated by TSO2 can be seen in Fig. 4, where the solid line represents the results of the reduced decentralized solution, the dotted line represents the full decentralized solution and the dashed line represents the centralized solution. The results are shown for TSO2, the remaining two areas show similar results. As can be clearly seen there is little discrepancy between the full decentralized and the centralized method. The results shown in this figure are exceptionally good, generally the results are of a slightly lower standard. Due to a discrepancy in the timing, the solution is not fully deterministic. Depending on the instantaneous loading of the computational nodes, there is a difference in the time taken, this leads to a difference in the order in which the optimization problems are solved, and hence may give slightly different answers.

It can clearly be seen from the figure that the match between the reduced decentralized solution is not as good as the full solution. For use in daily operation, the reduced method does not seem to provide sufficient accuracy in order to be valuable as a tool, the errors are too large to be used. However, in the case when a large number of scenarios have to be evaluated, the improvement in calculation speed, as discussed in IV-B may prove to be more important.
The values for the costs $c_\theta, c_T, c_p$ were the same in both the reduced and the full case, $[10^3, 1, 1]$. The mismatch between border angles was much more heavily penalized than the other factors, this gives good match at the borders, but relatively low performance in the center of the system.

In order to remove some of the error incurred by the use of the reduced method, the nodal power mismatch can be more heavily penalized. Taking a weighting of $[1, 1, 10^3]$, meaning that node mismatch is much more heavily penalized, the results are different. They are shown in Fig. 5. Here it can be seen that the quality of the results is improved compared to those show in Fig. 4. The angles given by the reduced model follow the general curve of the centralized solution, with slight deviations at some nodes.

It is interesting to note that the coefficients which lead to a good solution for the full model do not lead to a good solution for the reduced model. Further research into the appropriate selection of these coefficients is necessary.

### TABLE I

<table>
<thead>
<tr>
<th></th>
<th>Full Decentralized</th>
<th>Reduced Decentralized</th>
<th>Speedup Factor (approx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSO1</td>
<td>1297</td>
<td>107</td>
<td>12</td>
</tr>
<tr>
<td>TSO2</td>
<td>1427</td>
<td>81</td>
<td>20</td>
</tr>
<tr>
<td>TSO3</td>
<td>342</td>
<td>112</td>
<td>1.8</td>
</tr>
</tbody>
</table>

### B. Speed up

The main benefit of the proposed method is not an increase in accuracy. Rather accuracy is sacrificed for speed. For the three areas under consideration, the full decentralized method is quite slow. Table I shows the average run-time taken to solve the reduced and the full decentralized problem respectively, for five different runs. The runs were performed on Intel Core2Duo-6600 processors, running 64-bit GNU/Linux. The comparison to the solution of the simple DC power flow problem for the centralized approach is not considered here.

Two interesting facts which may make this method useful should be noted. The first is that for the reduced decentralized solution all run-times are of the same order of magnitude, in the order of 1 - 3 minutes. For the full solution there is a much bigger variability between the areas. In the case of TSO3 it should be noted that the run-time for the reduced decentralized solution is not as strongly reduced as for the other cases. The fact that TSO3 only has one interconnection with each of the other TSOs, might be an explanation for this fact. As during each iteration information is requested from the other TSOs the approximate time until updated information is returned is similar. The solution of the DC power flow is approximately as time consuming as the Gaussian reduction. Due to the structure of the interconnection of the reduced network models received by TSO3 are trivial, and thus do not provide much improvement. Conversely the size of the optimization problem which is solved by TSO3 is much smaller, thus reducing the calculation time relative to TSO1 and TSO2.

What is interesting to note is that the runtime is similar for all three TSOs when using the reduced method, something which is not the case when using the full method. This is attributed to the simpler optimization problem which has to be solved. TSO1 and TSO2 have double the number of variable angles to solve compared with TSO3, which would explain why the problem takes longer to solve.

### V. CONCLUSION

This paper has presented a simplified distributed power flow calculation method. This method can be used by TSOs when they desire to have a co-operative solution to the power flow problem, but are not willing to fully exchange data.

The method presented does not provide as accurate results as the complete solution. There are discrepancies between the results. It is beneficial that the calculations are much faster. The solution time is greatly reduced, by up to a factor of 10. The method proposed is currently not as accurate as the full decentralized methods, but it is expected that through suitable choice of the cost coefficients comparable accuracy can be achieved.
Data exchange is reduced, more confidentiality can be maintained. This enables TSOs to keep their confidential data within their own institution. Since only a reduced model of the network must be transmitted, the sending TSO has the possibility to influence the results of the neighboring areas. It this method is used in extension to the classical, 1-node deep, model of neighboring systems this influence can be minimized.

It is shown that the proposed method can be applied. Due to the differences in results, the proposed method should not be used as the sole calculation method during daily operation where a high level of detail is needed. Prudent use of this method might be for cooperative analysis of system changes, where the actual change can, for what ever reason, not be transmitted. Other situations where the proposed method can be useful include contingency screening and scenario analysis.

ACKNOWLEDGEMENT

Financial support from swisselectricresearch, project ‘Security of Multi-Area Power Systems’ is gratefully acknowledged.

REFERENCES


Emil Iggland (M’11) was born in Linköping, Sweden. He received his BSc and MSc in Electrical Engineering from ETH Zürich, Switzerland in 2008 and 2009 respectively. He is currently a researcher at ETH Zürich, working towards a PhD. His research interest include system security, the operation of multi-area power system.

Göran Andersson (M’86, SM’91, F’97) was born in Malmö, Sweden. He obtained his M.S. and Ph.D. degree from the University of Lund in 1975 and 1980, respectively. In 1980 he joined the HVDC division of ASEA, now ABB, in Ludvika, Sweden, and in 1986 he was appointed full professor in electric power systems at the Royal Institute of Technology (KTH), Stockholm, Sweden. Since 2000 he has been a full professor in electric power systems at the Swiss Federal Institute of Technology (ETH), Zurich. His research interests include power system dynamics and control, power markets and future energy systems. Göran Andersson is a fellow of the Royal Swedish Academy of Sciences, and of the Royal Swedish Academy of Engineering Sciences. He is Editor-in-Chief of IET Proceeding Generation, Transmission and Distribution, and the recipient of the IEEE PES Outstanding Power Educator Award 2007 and of the George Montefiore International Award 2010.