Probabilistic N-1 security assessment incorporating Dynamic Line Ratings

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Abstract—As power systems are operated closer to their technical limits, dynamic thermal line rating (DLR) can be used to enable additional dispatching flexibility. DLR is used to dynamically adapt the ampacity of the conductors, i.e. the maximum allowed amount of current a conductor can carry without overheating, according to the meteorological conditions. This paper proposes a probabilistic N-1 security assessment method that incorporates DLR in a probabilistic constraint. To model the probability distributions of certain meteorological values we use a copula approach. We calculate the distribution of the ampacity based on the distributions of the meteorological quantities using a model of the thermal behaviour of the line and given weather forecasts. From the resulting distribution, uncertainty realizations are sampled which are needed for a scenario based methodology which is used to deal with the developed chance constraint program.

Index Terms—Probabilistic N-1 security, dynamic line rating, chance constrained optimization, copula

I. INTRODUCTION

The N-1 security criterion is a common way of ensuring a secure operation of power systems. A system is considered N-1 secure, if the failure of one component, e.g. a generator or line, does not lead to an insecure system operation. In the last years the transmission grid utilization increased in general, mainly due to load increase, intensified energy trading and the change in the production portfolio, e.g. large wind energy production away from load centers. The ENTSO-E suggests large grid reinforcements and control-based grid expansion, i.e. adding operational flexibility to the power system by increasing the controllability, as means to cope with future transmission and security needs [1].

There are different factors that limit the maximum transferable power of a transmission line, such as voltage drops, stability limits and for shorter lines up to typically 80km length mainly the thermal limit [2]. The thermal limits are often calculated under conservative meteorological conditions, e.g. high ambient temperature and no cooling from wind. By making use of information on the current meteorological conditions, the line utilization could be increased and therefore investments could be deferred or a more economical operation could become possible.

In [3], [4] projects are described, where DLR are used for a connection with a wind park. The motivation is the correlation of the energy output of the wind park and the increased cooling effect on the line with increasing wind speeds. In [5] and [6] Markov chain Monte Carlo methods are used to find the distribution of the conductor temperature based on climate parameter distributions, i.e. wind direction and velocity, solar irradiation and the ambient temperature. Further they quantify the risk of overheating in terms of an overheating probability. In [7] a framework for a probabilistic N-1 security assessment has been presented taking into account infeed uncertainty of a wind park. The uncertainty is included in a security constrained optimal power flow via a chance constraint. In order to keep the problem tractable, a scenario approach is deployed.

This paper proposes a framework for a probabilistic N-1 secure dispatch planning taking into account DLR. The DLR depend on the meteorological conditions which are only available as forecasts during the planning phase. Therefore, using the given forecasts, probability distributions of actual values are modeled using copulas [8]. The distributions take into account the temporal and spatial correlation of the forecasts and actual data. The inclusion of the meteorological distribution renders the optimization problem stochastic. In order to keep the problem tractable, we employ a scenario approach. The resulting problem can then easily be solved by prevalent numerical tools. In case studies we compare the costs of operation with the number of N-1 violations in a Monte Carlo simulation and estimate the additional grid utilization.

The paper is structured as follows: In section II the dynamic line rating model, the potential and risk are shown, section III outlines the modeling of the weather influence on the ampacity. In section IV the problem is stated and applied to a case study in section V. Section VI concludes the paper.

II. DYNAMIC LINE RATING MODEL

A. Modeling of Steady-State Thermal Line Behaviour

In literature there exist different models to estimate the temperature of the conductor [9],[10]. We use the IEEE model for this paper [11]. For steady-state operation, the basic principle is to calculate the conductor temperature such that cooling effects are balanced by heating effects. The heating is caused by irradiation of the sun and current flows. The cooling effects are radiated heat losses and (forced) convection, e.g. wind. For the steady-state case, the power balance can be written as $q_c + q_r = q_s + I^2 \cdot R$. $q_c$ is the convection heat loss rate. It mainly depends on the wind speed and the attack angle, i.e. the angle between the wind direction and the line direction.
$q_c$ is the radiated heat loss rate, which mainly depends on the difference between the conductor temperature and the ambient temperature, $q_c$ is the rate of solar heat gain by solar irradiation and $I$ is the current, $R$ the temperature dependent resistance of the conductor. Note, that the steady-state equation is independent of the line length, i.e. the powers per unit length are considered. In order to calculate the ampacity for given meteorological conditions, the maximum conductor temperature $T$ is assumed. For the selection of $T$ the thermal limits of the conductor, the allowed sag of the line as well as the degradation should be considered. For a given maximum temperature, the ampacity is calculated as:

$$I_{\text{max}}(T) = \sqrt{\frac{q_c(T) + q_r(T) - q_s}{R(T)}}$$

(1)

B. Influence of meteorological quantities on line ampacity

This part demonstrates the influence of the meteorological quantities solar irradiation, ambient temperature and wind velocity and direction on the conductor ampacity.

Throughout this paper, for the base case a static thermal line rating is used. It is calculated using the conservative assumptions of an ambient temperature of $40^\circ\text{C}$, an irradiation of $900\text{W/m}^2$ and a wind speed of $0.5\text{ m/s}$ flowing with an $22.5^\circ$ angle to the line. The assumptions are made such that for the given meteorological data the probability of an overloading is less than $0.5\%$. In practice, these assumptions vary depending on the prevalent climate. Further, the line parameters of a "Drake" 26/7 ACSR line [11] are assumed throughout this paper.

Fig. 1 shows the influences of meteorological quantities on the ampacity by varying only one variable from the base case. The influence is substantial, especially the additional cooling due to higher wind speeds. In some cases, the ampacity is more than twice as the base case. The next step is to investigate the influences on the ampacity.  

C. Potential and Risk

As the static limits are calculated under conservative assumptions, the effective transmission capacity given the meteorological conditions is expected to be higher in almost every case. The left plot of Fig. 2 displays the ascending theoretical ampacities calculated using realistic meteorological conditions. The values are normalized to the ampacity of the base case. It can be observed that the potential is significant. It should however be noted, that meteorological conditions are local quantities and the ampacity corresponds to the worst case along the whole line. Thus, for a successful implementation of DLR, a reasonable amount of measurement stations would have to be placed along the line. For simplicity, we assume that the given meteorological data represents the worst case along the line.  

As the dispatch planning is performed some time before the operation, only the meteorological forecasts are available. The forecasts are not perfect, thus the ampacity can not be estimated perfectly. In the right plot of Fig. 2 the distribution of the ampacity error, i.e. the difference of the ampacity calculation with forecasted1 and actual data for different forecast horizons is displayed. It can be seen that the forecasted ampacity can differ quite significantly from the actual but reduces for smaller forecast periods. As deviations from the forecasts pose an operational risk, the forecast errors are included in the N-1 calculation. Their modeling is described in the next section.

III. MODELING OF THE INFLUENCE OF THE WEATHER

The meteorological data exhibits strong correlations, both temporal, i.e. between forecast and actual values, and spatial, i.e. measurements of different weather stations. The goal is to find the probability distribution of every meteorological quantity considered when its forecast is known. We consider not only the local forecast of one quantity but also the forecasts of stations with high correlation. Thus we capture the additional information on the local quantities given by surrounding stations. In order to model the dependence of multiple random variables, i.e. the actual value, its forecast and corresponding forecasts from other stations, multivariate probability distributions have to be approximated. We therefore employ a copula approach [8]. Copulas have been used extensively in risk management and finance [12],[13], but also in weather research [14].  

A copula is a multivariate distribution with uniform marginal distributions, which describes the dependence between random variables. A joint probability distribution can be expressed as its marginal distributions and their dependence given by the copula. There are different families of copulas.

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1 Data from 11 stations in Switzerland from August 2011 until August 2012. The data contains measurements of wind, irradiation and ambient temperature every 3h and forecasts for the next 24h starting every 3h. Wind angle is not measured, therefore we assume $22.5^\circ$ for our calculations.
We demonstrate the concept for a normal copula.

Let $X$ be a vector of random variables which follows a multivariate distribution where the entry $X_i$, $i \in [1,m]$, is a random variable with distribution function $F_{X_i}$ and probability density function $f_{X_i}$. The transformation $U_i = F_{X_i}(X_i) \sim \mathcal{U}(0,1)$ leads to an uniformly distributed $U_i$ in $[0,1]$. Using the transformation $Z_i = F_{X_i}^{-1}(F_{X_i}(X_i)) \sim \mathcal{N}(0,1)$ leads to a standard normally distributed random variable $Z_i$. A $m$-dimensional normal copula can thus be written as $C_X(u_1, \ldots, u_m) = F_{X_1}(u_1) \cdots F_{X_m}(u_m)$ with $\Sigma$ being the covariance matrix, which describes the dependance of the random variables[14]. The corresponding density function is denoted with $f_X(u_1, \ldots, u_m)$. The joint probability density of the actual realization varies as is shown in Fig. 3. The more red the higher is the density.

Using historic data we can estimate the density functions and fit the copula using the maximum likelihood method. Samples from a multivariate normal distribution, i.e. the copula, can easily be created and can then be transformed to the desired samples of $X$ using (2). Fig. 3 shows exemplarily the densities of wind speed forecasts and actual values. The samples are generated using a copula and exhibit a correlation and mean close to the one of the measured data. The diagonal pattern indicates a strong correlation between forecast and actual value. Thus for a given forecast value the marginal distributions of the actual realization varies as is shown in Fig. 4 for three different forecasts.

We assign every transmission line to a weather station and build a copula for every of the considered meteorological values and every geographical location.

IV. PROBLEM FORMULATION

To evaluate the potential of DLR we study the case of computing the optimal generation dispatch which ensures the N-1 security criterion. Toward this direction we formulate an optimization problem with probabilistic constraints to capture the fact that the line rating constraints depend on certain meteorological uncertainties. We adopt an approach similar to [7] and show how DLR can be incorporated in the developed framework. To ensure tractability of the resulting chance constrained program, we follow a scenario based methodology.

A. Problem set-up

Consider a power network comprising of $N_G$ generators, $N_L$ loads, $N_I$ lines and $N_b$ buses. For the subsequent derivations, a DC power flow is employed [15]. The power injection vector is given by $P = I_G P_G - I_L P_L$, where $P_G \in \mathbb{R}^{N_G}$ is the active power production of the generating units and $P_L \in \mathbb{R}^{N_L}$ represents the loads. Matrices $I_G$ and $I_L$ are of appropriate dimension and map the generator and the load indices to the power injection vector that corresponds to all buses. Their $(i,j)$ element is “1” if generator (respectively load) $j$ is connected to bus $i$ and zero otherwise.

By algebraic manipulations of the DC power flow equations, which involve the elimination of the reference angle, it can be shown that the power flow of the lines $P_l \in \mathbb{R}^{N_I}$ depend linearly on the power injection vector $P \in \mathbb{R}^{N_G}$. Namely, $P_l = K P$, where $K \in \mathbb{R}^{N_l \times N_b}$ is a matrix with constant entries that depends on the parameters and the topology of the network. For a detailed derivation of this linear representation the reader is referred to [7]. Note that matrix $K$ is equivalent with the notion of the Power Transfer Distribution Factors (PTDF) [16].

For the N-1 security analysis we take into account any single outage involving the tripping of a line, load or generator. The number of possible outages can be written as $N_{out} = N_G + N_L + N_I$. Any generator or load outage will result into a generation-load mismatch; to compensate for this mismatch, the remaining generating units should modify their production providing secondary frequency control reserves. This is achieved by distributing the resulting mismatch to the generator units according to a distribution vector $d \in \mathbb{R}^{N_G}$. The elements of $d$ show the percentage of the power mismatch with which each generator should increase or decrease its production. The new operating point of the generators is given by $P_G - d P_{mismatch}$, where $P_{mismatch}$ is the generation-load mismatch that may occur due to a component outage. Under this representation the term $d P_{mismatch}$ plays the role of the reserved power.
B. Chance constrained problem formulation

In this subsection we formulate a probabilistically N-1 secure dispatch for the generating units while taking into account the DLR constraints. We introduce the superscript $i$ at the variables defined in the previous section to emphasize their dependency on the specific outage $i = 0, \ldots, N_{\text{out}}$, where the “0” index is the case without outages. Let $C_1$ and $C_2$ be matrices of appropriate dimension that encode the linear and quadratic production cost, respectively. The resulting optimization problem is given by

$$
\min_{P_G \in \mathbb{R}^{N_G}} \left( C_1^T P_G + P_G^T C_2 P_G \right)
$$

subject to

1) Power balance constraint:

$$
\mathbb{1}_{1 \times N_s} (I_G P_G - I_L P_L) = 0,
$$

2) Generation constraints: for all $i = 0, \ldots, N_{\text{out}}$

$$
P_{\text{min}}^i \leq P_G^i - d^i P_{\text{mismatch}}^i \leq P_{\text{max}}^i.
$$

This constraint encodes our requirement that the scheduled generation dispatch plus the reserve contribution will not result in a new operating point outside the generation limits.

3) Line rating constraints:

$$
P \left( \delta \in \mathbb{R}^{N_s} \mid |K_i P_i| \leq P_{\text{DLR}}^i (\delta) \text{ for } i = 0, \ldots, N_{\text{out}} \right) \geq 1 - \epsilon
$$

where probability is meant with respect to the probability distribution of the uncertainty vector $\delta \in \mathbb{R}^{N_s} \mathbb{R}_{m}$ which includes certain meteorological uncertainties. Specifically, $N_m$ is the number of different meteorological uncertainties and $N_s$ is the number of stations in the sense of different locations that we have these kind of measurements. In our case we considered wind, irradiation and ambient temperature uncertainty (i.e $N_m = 3$ and $N_s = 11$ stations). The power injection vector is defined as $P_i = I_G (P_G^i - d^i P_{\text{mismatch}}^i) - I_L P_L$, as discussed in the previous subsection, and $P_{\text{DLR}}^i (\cdot) : \mathbb{R}^{N_s} \mathbb{R}_{m} \to \mathbb{R}^{N_l}$ denotes the uncertainty dependent line rating limit. The uncertainty function $P_{\text{DLR}}^i (\cdot)$ follows the distribution of the ampacity calculated using (1) based on the probability distributions of the meteorological uncertainties $\delta$.

C. Scenario based methodology

Problem (3)-(6) is a quadratic program with probabilistic constraints. Specifically, the chance constraint (6) implies that for any line rating (uncertain parameter), the lines, both in the base and the N-1 case, do not get overloaded with probability at least $1 - \epsilon$. To obtain a solution for this problem, we need to deal with the chance constraint while ensuring tractability of the resulting formulation. For this purpose, we use the so called scenario approach [17], where the the chance constraint is substituted with a finite number of hard constraints, each of them corresponding to a different realization of the uncertainty vector $\delta$. Moreover, it is shown in [18] that if the number of scenarios $N$, that one needs to generate, is selected according to

$$
N \geq \frac{1}{\epsilon} \left( \ln \frac{1}{\beta} + N_d \right),
$$

then with confidence of at least $1 - \beta$ (here: 99.9%) the optimal solution of the resulting scenario program satisfies the line rating constraints with probability $1 - \epsilon$. Variable $N_d$ denotes the number of decision variables, which for this case is equal to the number of the generating units whose dispatch we seek to determine (i.e. $N_d = N_G$).

The only prerequisite to inherit the probabilistic guarantees offered by the scenario approach is the initial problem to be convex with respect to the decision variables. We are able to apply the scenario approach since the underlying optimization problem is convex. In our case, this is trivially satisfied, since the objective function is quadratic and the constraints are linear with respect to $P_G$. Applying this procedure, results in an optimization problem where the set of constraints inside the probabilistic constraint are imposed for every scenario according to (7). However, the line rating constraints exhibit a specific structure: they are of the form $AP_G + b \leq f(\delta)$, where $f(\cdot)$ is a “stacked” version of the uncertainty vector $P_{\text{DLR}}$ and $A, b$ are of appropriate dimensions with constant entries. We can thus pre-generate $N$ scenarios according to (7), use them to construct $f(\delta_k)$ for $k = 1, \ldots, N$ and enforce the line rating constraints only for the $\min_k f(\delta_k)$, where the minimum here is supposed to be element-wise. That way the size of the resulting problem does not depend on the number of scenarios that we have to generate. The resulting problem can be solved efficiently using existing numerical tools [19].

V. Case Studies

The case studies focus on three aspects: The operational costs which are expected to be lower due to the increased dispatch flexibility, the security in terms of the number of insecure instances and the grid utilization. The simulations are run for 50 different case, where each case consists of forecasted and actual values. The samples needed for the scenario approach are randomly selected from samples generated with the copula models. The numbers are 259 for $\epsilon = 10\%$, 517 for $\epsilon = 5\%$, 2582 for $\epsilon = 1\%$. We use the IEEE 30-bus system for our purposes, where the load and generation capacities are scaled with a factor $k > 1$ so as to push the grid to its limits. Fig. 5 displays the average operational costs and the average of insecure instances for different $\epsilon$ and different prediction horizons. We determine the number of insecure instances by checking the number of violations of the N-1 security criterion for 5000 samples for every case. For comparison, also the deterministic cases (using only the forecasted ampacity), the case with static line limits and with the actual possible ampacity are plotted. Fig. 6 displays the median, the $25^{th}$ and $75^{th}$ percentiles of the highest feasible $k$ normalized to the factor of the case with static line ratings. Feasible means, that a N-1 secure grid operation is possible.
Fig. 5. Operational costs and number of insecure instances for different \( \epsilon \) and different forecast periods.

Fig. 6. 25\textsuperscript{th} and 75\textsuperscript{th} percentiles of the scaling factors \( k \) for loads and generator capacities for different DLR setups and actual transmission limits compared with the static line rating.

We summarize the observations in the following:

- If the security is increased, i.e. \( \epsilon \) is reduced, the cost of operation increase and the number of insecure instances decrease as the limiting scenario becomes more conservative. \( \epsilon \) can thus be seen as trade-off parameter between risk and costs.
- The operation cost using the ampacity calculated only with the forecast (no probability distribution) are lower when using the chance constraint. On the other hand, the insecure instances increase drastically. The highest average costs are incurred for the static line rating and the number of insecure instances are below 0.5\% of all cases.
- Fig. 6 suggests, that DLR enables a higher grid utilization in most of the cases. The maximum grid utilization is achieved for perfectly known meteorological conditions. This however would require a suitable online monitoring as presented in [20].

VI. CONCLUSION

In this paper we presented a method for the N-1 security assessment which incorporates dynamic line ratings. The probability distributions of the ampacities of the transmission lines are estimated based on multivariate probability distributions of temporally and spatially correlated meteorological influences using a copula approach. The ampacity distributions are included in the N-1 security optimization as a chance constraint. The problem is kept tractable by deploying a scenario approach. The results of a case study suggest a potential for increased utilization of the transmission grid while maintaining a comparable risk level.

Future work would consist in a combination of the proposed method with uncertain infeed, e.g. wind power production.

Further, the potential of increased grid utilization should be quantitatively estimated.

REFERENCES