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Controlled Islanding with VSC-HVDC Links

Master Thesis
PSL 1401

EEH – Power Systems Laboratory
Swiss Federal Institute of Technology (ETH) Zurich

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Zurich, 29th of August 2014
Acknowledgements

This work was conducted at Power Systems Laboratory, within the Department of Information Technology and Electrical Engineering at the Federal Institute of Technology Zurich (ETH Zurich).

I would like to express my sincere thanks to Markus Imhof for his commitment to supervise me in this thesis. I appreciated his insights even though often there were thousands of kilometres and several time-zones between us. I would also like to thank Maria Vrakopoulou and Spyros Chatzivasileiadis for their valuable advices. Moreover, I feel deeply grateful to Prof. Dr. Göran Andersson, head of the Power Systems Laboratory at ETH Zurich, for hosting me as a student researcher in his lab and guiding my academic studies as a mentor. Furthermore, I would like to thank the entire team at Power Systems Laboratory for having an open ear for all kinds of issues.

Additionally, I would like to thank my supervisors at e-netz ag: Thomas Ingold and Thomas Marti, for providing the flexibility that allowed to combine this thesis with the employment at their engineering company.

During the last year, I was very glad to have my dear friends Markus, Marvin, Mitchel, Dimitris, Alexandra and another Mitchel. We will, definitely, know each other forever. I would like to thank you all, especially Marie Reinecke Fuchs from Dresden for replacing this thesis in being the middle point of my life.

Finally, I would like to extend my thanks to my always supportive family, especially my mother, without whom my entire academic studies and this work would not have been possible.
Abstract

Controlled islanding has been widely discussed in the literature as a novel concept for the power system protection. However, the islanding can result in unbalanced islands where the power mismatch will often cause an instability that can lead either to a subsequent load shedding or generator tripping within the islands.

Controlled VSC-HVDC links can mitigate the negative consequences of the controlled islanding by allowing active power transmission between the islands. Moreover, VSCs can assist in controlling the voltage within the islands.

This thesis shows that the controlled VSC-HVDC links can support the controlled islanding by stabilising the islands in terms of frequency and voltage. For this purpose, we extend the existing K-means clustering based controlled islanding algorithm [KCVA10] so that it can provide an islanding solution in which the islands are interconnected by the VSC-HVDC links. Moreover, we introduce a simple control scheme for the VSC-HVDC links that allows to use them for stabilising the islands. We simulate the controlled islanding to show the effectiveness of the proposed algorithm and the control scheme.
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1 Introduction

Electricity networks have become the main infrastructure for our society since the last century. Every economical activity in a modern country presupposes areawide availability of electrical energy supplied by the power plants and transmitted to where it is consumed. Well-being of the population and economical prosperity of countries is tightly bounded to constant electricity supply and consequences of an interruption might be horrendous. Thus, it is essential to ensure permanent operation of the power system providing constant energy supply to the consumers.

However, in the recent past, wide area power outages did occur leaving enormous economical damage behind. The North-American blackout in 2003 was the largest, affecting more than 50 million people. Over 63 GW of load, corresponding to approximately 11% of total consumption in the North-American power system, were cut down from the supply for several hours [ADF05]. The cost of that power outage was estimated to lie between 4 and 6 billion US-dollar [Fai04]. In the same year, two other major blackouts occurred in Europe. Each of them resulted in a loss of more than 6 GW of load and had also a considerable financial cost [ADF05, Val13]. We have taken those historic examples as a motivation, and dedicated this study to the prevention of such wide area power outages.

1.1 Background

Analysing the blackouts occurred in the past, it is noticeable, that all of those have more in common than just a severe financial cost. As discussed in relevant studies [PKT06] [BPRT05], most major outages are initiated by a disturbance, which can be either a single or unrelated multiple events such as faults of power system components or a relay misoperation. After a disturbance, a general sequence of events leads to the uncontrolled cascading outages, followed by an eventual collapse of the entire power system. If adequate automatic and manual control actions are not taken, a power system instability will lead to a blackout, culminating in cascading outages of power.

It is clear, that for a large, wide area system like electrical grid, an absolute reliability and security is unachievable, as it is impossible to eliminate factors like the acts of nature or a human error. Nonetheless, by taking preventive and active security measures on the power system, it is possible to reduce the risk that a failure or a misoperation will result in cascading outages. In terms of prevention of failures - prioritised replacement of older components have to be undertaken. Additionally, regular testing, evaluation and maintenance in order to directly improve the system reliability, can significantly reduce the possibility of misoperation.
In terms of active security measures, many established technologies can ensure stable operation of the grid and help mitigating the risk of uncontrolled cascading outages followed by a wide-area blackout. These are technologies such as coordinated emergency control including under frequency load shedding, flexible alternating current transmission using FACTS and HVDC, as well as an online dynamic security assessment along-with real-time monitoring and control [PKT06].

A modern power system is often operated under high load conditions. This is a consequence of constantly increasing power demand and higher cross-border and long distance energy exchanges due to the market liberalisation. Together with the backwardness of infrastructure expansion, this leads to a situation where safety margins have been progressively reduced over the last years, driving the system closer to its stability limits [KCVA10]. To mitigate for the increasing risk of a common disturbance driving the grid into a wide area blackout, novel security measures have to be discussed.

As such, **Intentional Controlled Islanding** (ICI) is widely investigated [LL06]. Its concept foresees the splitting of the power system into subnetworks in case of an emergency. The subnetworks (also called islands) are isolated power systems, and not necessarily synchronised with each other. The purpose of the controlled islanding is to contain the occurred disturbance within one of the islands so that the survival of the other islands is ensured. Once stabilised, the islands can be synchronised and reconnected, restoring the normal operational state of the grid.

A negative consequence of the controlled islanding, is that after the separation, a power imbalance can occur within the islands. While some islands might have more power generation than consumption, the others will not have enough electricity to supply the loads which are present within the island boundaries. In this context, the first are often called generation-rich islands and, given an initial power mismatch, will have to quickly reduce their electricity production. The latter are regarded to as load-rich islands and will have to either instantly increase the electricity production or reduce the consumption in order to restore power equilibrium. If, however, the imbalance persists, system frequency will progressively deviate from the nominal equilibrium frequency $\omega_0$.

The missing power will be instantaneously compensated by the kinetic energy stored in the turbines that participate in electricity production. A frequency deviation $\Delta\omega = |\omega - \omega_0|$ immediately means a lower quality of delivered electricity, because much of the consuming equipment is optimised in such a way, that it operates best when the power system frequency is at its nominal value $\omega_0$.

The most important example of such equipment are the generators. As it is shown in [KBL94], an electrical frequency deviation inevitably changes the mechanical frequency, characterising the rotation of the shaft connecting the generators with the turbine. This leads to vibrations which can permanently damage the machine and is, therefore, adverted by disconnecting the generator from the grid if the frequency deviation becomes to big. Indeed, after this, the mismatch becomes even bigger, evoking further increase of $\Delta\omega$. As a result, a sequence of outages might take place leading to a total blackout [Val13].
1.2 Motivation

There are two mechanisms installed in a power system to mitigate the consequences of a mismatch between electricity consumption and production, hereby, preventing the frequency deviation. First mechanism is the, so called, frequency control, and it consists of primary, secondary and tertiary control. Primary frequency control and secondary frequency control include constantly active automatic controllers which regulate the amount of power produced by the turbines and maintain the short-term power balance. In a longer term (several minutes), the tertiary control is manually activated to release the faster primary and secondary control reserves. Though the automatic controllers are active during the normal operation of the grid, they might be inept to compensate a larger mismatch before the frequency falls down to a critical threshold due to the dynamics of involved turbines. Moreover, the amount of additional power reserve might be insufficient, which can lead to an emergency situation.

To protect the power system from such situation, there is another mechanism called Under-Frequency Load Shedding (UFLS). In contrast to the frequency control, it is used only in case of an emergency, disconnecting an amount of load that is predefined by the UFLS-scheme if there is a severe power imbalance in the system.

After controlled islanding, frequency control will often be insufficient to compensate the mismatch. Thus, UFLS will often be necessary to stabilise the load-rich islands. This is very undesirable, as it implies a potential impact on both demand and supply side. Due to the load shedding, the consumers will experience a power outage and the producers will not be selling any electricity to them. At the same time, the generation-rich islands will have to counteract exactly the opposite situation. They will have to quickly reduce the power production by tripping some of the present generators.

The HVDC links can serve to connect the islands so that an excessive power can be transmitted to the load-rich ones. Nevertheless, eventual rotor angle instabilities will still be isolated as those can not be transferred through an HVDC connection. As a result, less or none load shedding or generator tripping will be necessary after a controlled islanding.

HVDC links have been increasingly applied in the power systems all over the world. Most of the recent installation use the Voltage Source Converters (VSC) as those become liable for commercial applications. VSC-HVDC has several advantages for the network stability and its main feature is an independent active and reactive power control [FAD09]. VSC-HVDC links can, therefore, not only be used for power transmission, but also to control the voltage.

They also seem to be very useful in context of controlled islanding. If the power system can be split so that the islands are interconnected by the present VSC-HVDC links, then they can be stabilised more effectively. While active power transmission will be used to stabilise the frequency, the reactive power control by the VSCs can be used to stabilise the voltage in the islands. Hereby, the negative consequences of controlled islanding, such as UFLS and voltage instabilities can be mitigated. This might be an additional argument in the discussion about the configuration of a modern power system.


1.3 Research Objective

The objective of this study is to show that the VSC-HVDC links can support controlled islanding by stabilising the islands. To do so, following problems have to be solved:

**Controlled Islanding Algorithm:** The algorithm calculating the island boundaries has to consider the existing VSC-HVDC links so that after islanding, the islands are connected by those links. For this purpose, first, a literature study of existing controlled islanding approaches has to be done. Afterwards, the algorithm corresponding to one of the approaches has to be extended, so it can consider the VSC-HVDC links while determining the island boundaries.

**VSC-HVDC Control:** Given that the islands are interconnected by the VSC-HVDC links, a control scheme has to be developed. It has to allow using the links to help stabilising the islands. Hereby, both, active and reactive power control of the VSCs should be considered.

**Simulation:** The controlled islanding has to be simulated in various test systems using the extended controlled islanding algorithm to compute the island boundaries and the developed control strategy to stabilise the islands.

1.4 Outline

To show how we have fulfilled the research objective, we start with the Chapter 2, where we provide the necessary theoretical background. This Chapter includes an extensive literature review of existing controlled islanding approaches (Section 2.1). Having picked an approach developed in our research group [KCVA10], in Sections 2.2 and 2.3 we provide theoretical knowledge necessary to understand the controlled islanding algorithm which we extend in the next Chapter.

In Chapter 3, we explain the methods we use to fulfil the research objective. First, we extend the controlled islanding algorithm so that it can account for the existing VSC-HVDC links. This is described in Section 3.1. In the subsequent Section 3.2, we develop the VSC-HVDC control scheme which is used to help stabilising the islands. At last, in Section 3.3 we describe how we have simulated the controlled islanding with the extended algorithm and the developed control scheme. Hereby we define various cases for the simulation.

In Chapter 4, we provide the simulation results obtained for various cases that were described in Section 3.3. For each case, we provide a short description. At the same time, we explain why the results fulfil the research objectives drawing some intermediate conclusions.

The final conclusion, is then drawn in Chapter 5. In this last chapter, we also provide an outlook for the future work.
2 Theory

We will summarise the theoretical knowledge necessary for our study on controlled islanding in this Chapter. At first, we will provide an extensive literature review of the existing controlled islanding approaches (also called ICI-approaches) in Section 2.1. Having chosen an ICI-approach based on slow coherency grouping and K-means clustering developed in our research group [KCVA10], we provide the theoretical background on slow coherency grouping in Section 2.2 and on K-means clustering in Section 2.3. This knowledge will be necessary to understand the controlled islanding algorithm (ICI-algorithm) which we will extend in the next Chapter.

2.1 Controlled Islanding Approaches

An Intentional Controlled Islanding Approach has to provide a set of transmission lines which need to be cut, in order to split the power system into electrically isolated islands. This (cut-)set is called islanding solution and ideally, it has to satisfy a large number of constraints such as:

- load-generation balance
- transient stability
- generator coherency
- voltage stability
- availability of transmission line

Finding a perfect islanding solution results, due to the combinatorial explosion of the search space in a, so called, \textit{NP-hard problem} (i.e. a problem that is not necessarily solvable in polynomial time) that might be too complicated to solve [SZL03]. Instead, ICI-approaches consider islanding as an optimisation problem taking into account only some of the constraints above at a time. Depending on the formulation of the objective function and the considered constraints, there are various ICI-approaches proposed in the literature, which we review in this section.

In general, ICI-approaches can be classified by the two main types of the objective function used to find the optimal solution:

\textbf{Minimal Power Imbalance}: find an islanding solution so that the load-generation imbalance within the island is minimised.
Minimal Power-Flow Disruption: find an islanding solution so that the change of the power flow pattern within the system after the islanding is minimised.

Minimal power imbalance is used to find a solution where the resulting islands have similar active power production and consumption. Herewith, the amount of load shedding, eventually, required after the islanding is also minimised.

Minimal power-flow disruption is used to find a solution where islanding would cause minimal changes in the active power flow. Herewith, the eventual machine swings following the islanding are minimised improving transient stability of the islands. Additionally, the risk to overload the transmission lines, by significantly disrupting the power flow, is also minimised [Hen80].

Independent of the used objective function, there are two obvious requirements imposed on an ICI-approach. In order for controlled islanding to be successful, it has to be done quickly and, at the same time, yield stable islands considering the operational state of the system. To obtain stable islands, different aspects have to be considered. Power system stability has been defined in [KPA+04], and is classified as presented in Figure 2.1. The three categories - rotor angle stability, frequency stability and voltage stability impose various constraints on the islanding solution. In order to ensure the rotor angle stability, the islands should comprise coherent generators which are approximately synchronous. For the frequency stability, the active power mismatch of the islands has to be minimised while for the voltage stability the reactive power balance needs to be considered. While the time frame of interest in stability studies lies within a few seconds, it is essential to conduct the islanding before the disturbance can spread throughout the system [KPA+04]. Therefore, an ICI-approach...
must be computationally efficient in finding an islanding solution under a sufficient number of constraints, promoting stability of the islands.

To find an islanding solution, the power system buses have to be clustered so that the system can be split into islands. For this purpose, the grid with its \( n \) buses and \( l \) transmission lines can be represented as an undirected graph

\[
G := (\mathcal{V}, \mathcal{E})
\]

where

\[
\mathcal{V} := \{v_1, \ldots, v_n\}
\]

is the set of nodes (vertices) representing the buses and

\[
\mathcal{E} := \{e_{ij} | i, j = 1, \ldots, n\}
\]

is the set of edges representing the transmission lines. The graph entirely reflects the topology of the grid and can be extended to the weighted undirected graph

\[
G := (\mathcal{V}, \mathcal{E}, \mathcal{W})
\]

by defining a set

\[
\mathcal{W} := \{w_{ij} | w_{ij} = w(e_{ij}), \forall e_{ij} \in \mathcal{E}\}
\]

where each element \( w_{ij} \) is the weight assigned to the edge \( e_{ij} \) for \( \forall i, j = 1, \ldots, n \). A weight can represent various system characteristics such as the transmission line length or the power flow through the line. The graph then can be entirely described by the weighted adjacency matrix defined as an \( n \times n \) matrix \( A = [a_{ij}] \) where

\[
a_{ij} = \begin{cases} 
  w_{ij} & \text{if there is a line between buses } i \text{ and } j \text{ i.e., } e_{ij} \in \mathcal{E} \\ 
  0 & \text{otherwise}
\end{cases}
\]

If the grid is represented as a graph, then a \( K \)-way clustering \( C := \{C_1, \ldots, C_k\} \) separates its nodes into \( K \) clusters \( C_k \) so that

\[
\forall v_i \in \mathcal{V} \quad \exists C_k \subset \mathcal{V} : v_i \in C_k
\]

with \( i = 1, \ldots, n \) and \( k = 1, \ldots, K \) while

\[
\bigcup_{1}^{K} C_k = \mathcal{V}; \quad \bigcap_{1}^{K} C_k = \emptyset.
\]

The clustering can be described by a partitioning matrix \( P = [p_{ik}] \) with

\[
p_{ik} = \begin{cases} 
  1 & \text{if the node } i \text{ is assigned to the cluster } C_k \\ 
  0 & \text{otherwise}
\end{cases}
\]
While the rows of $\mathbf{P}$ correspond to the $n$ nodes, its columns represent the $K$-clusters which the nodes are assigned to. At the same time, a clustering defines the cut-set $Q(C) \subseteq \mathcal{E}$ as a set of edges whose end-nodes belong to the different clusters. By cutting the corresponding transmission lines during the islanding, the power system buses are split between the islands determined by the clustering $C$.

Finding an islanding solution constitutes a system splitting problem which, following the terminology of the graph-theory, can be formulated as follows:

**System Splitting Problem**: For a given graph $G$ and a desired number of clusters $K$, find an optimal clustering $C$ so that a predefined objective function is minimised.

Naturally, graph partitioning techniques from other fields of research can be applied also to an electrical grid. Finding an optimal clustering can also be seen as an optimisation problem which allows the usage of conventional optimisation techniques.

For example, simulated annealing approach was one of the first propositions for the power system splitting [IS90]. There, finding an optimal clustering is seen as a combinatorial problem. Each node, one after another, is assigned to one of the clusters minimising the objective function. Commonly for simulated annealing, the choice of the objective function is almost unlimited, which can be used to fulfil as many of the constraints named above as it appears reasonable. However, simulated annealing approach often provides only a local optimal solution. In order to reliably provide the global optimum, the computation (annealing) time has to be dramatically increased even for the improved mean-field annealing version of the approach [VDBMI90].

There also have been other approaches to the power system splitting that used conventional graph partitioning techniques [YRA93,MQ92]. Same as simulated annealing approach, they were developed to partition the network for the parallel computing purposes of power system research. This application does not have to consider the state of an operating system, and it does not require computational efficiency necessary for a real-time system splitting. However, for the controlled islanding, online performed specialised approaches had to be developed.

**Spectral Clustering**

An ICI-approach based on the spectral graph theory has been proposed specifically for the controlled islanding [LRJL05]. Spectral clustering was originally developed to allow quick islanding for the power infrastructure defence purposes. This approach analyses the eigenvalues and eigenvectors of the so-called Laplacian matrix to find the best possible clustering. While the eigenvalues are used to calculate the lower bound of the objective function, the eigenvectors determine a partition matrix that minimises the objective function towards this bound.

In order to apply spectral clustering for splitting a network represented by a weighted undirected graph $\mathcal{G}$, the Laplacian matrix has to be calculated at first. To do so, for each vertex $v \in \mathcal{V}$ its degree is stored in the diagonal $n \times n$ degree matrix $\mathbf{D} = [d_{vv}]$. Hereby, the vertex degree is defined as the number of edges leading to or from $v$. Given
the weighted adjacency matrix defined earlier in this Section, the Laplacian matrix can be easily calculated as:

$$\mathbf{L}(\mathcal{G}) = \mathbf{D} - \mathbf{A}. \quad \text{(2.10)}$$

With $\mathbf{L}(\mathcal{G})$, the splitting problem is solved by analysing the inherent structural characteristics of the graph using spectral graph theory methods (which is also eponymous for this ICI-approach).

This requires the calculation of the eigenvalues and eigenvectors of $L(\mathcal{G})$ and a specific definition of the objective function for the system splitting problem. For a $K$-way clustering $C = \{C_1, \ldots, C_k\}$ the corresponding objective function $J(C)$ is defined as

$$J(C) = \sum_{c=1}^{k} \frac{W_k}{|C_k|}. \quad \text{(2.11)}$$

Here, the cardinality $|C_k|$ denotes the number of vertices in the cluster $C_k$, while $W_k$ represents the cluster weight

$$W_k = \sum_{Q_k} w_{ij}. \quad \text{(2.12)}$$

While $C$ corresponds to the cut-set $Q(C)$, each cluster $C_k$ corresponds to its cluster cut-set

$$Q_k \subseteq Q(C) \quad \text{(2.13)}$$

comprising the edges that have only one end-node in $C_k$ and, therefore, connect the cluster to the rest of the graph. Herewith, $W_k$ is defined as the sum of the weights of the lines in the cluster cut-set. When the line weights represent the active power flow of the corresponding transmission line, $W_k$ with $k = 1, \ldots, K$ can be seen as a measure for the expected power flow disruption after the islanding.

In [CSZ94] it was shown that the lower bound of $J(C)$ is equal to the sum of the $K$ smallest eigenvalues of the Laplacian matrix $L(\mathcal{G})$ for any possible $K$-way clustering of $\mathcal{G}$ i.e:

$$\sum_{i=1}^{K} \lambda_i \leq \min_{c=1}^{K} \frac{W_k}{|C_k|}. \quad \text{(2.14)}$$

Moreover, eigenvectors corresponding to these eigenvalues provide the partition matrix that separates the nodes into $K$ clusters. An advanced approach for finding the $K$ smallest eigenvalue/eigenvector pairs and for the calculation of the resulting partition matrix is presented in [CSZ94].

Even though spectral clustering was one of the first ICI-approaches, in its original form presented in [LRJL05], it did not consider transient stability of the islands. Moreover, a bigger number of the power system nodes implied the increasing dimensions of the Laplacian matrix, which results in a significant growth of the time necessary for the eigenvalues / eigenvectors computation.
Multilevel Kernel K-Means Clustering

To deal with the computational time issue, an ICI-approach based on multilevel Kernel K-means was proposed in [PI09]. It omits the eigenvalues / eigenvectors calculation of a high-dimensional Laplacian matrix and is, therefore, faster than the original spectral clustering approach mentioned above. Moreover and if necessary, this method can completely omit the eigenvalues / eigenvectors calculation and instead solve an optimisation problem very similar to the standard K-means clustering according to [FCMR08, DGK07].

Multilevel kernel k-means clustering can be divided into three phases: aggregation, clustering and retrieval. Schematic overview of the procedure is depicted in Figure 2.2. Same as spectral clustering method, it presupposes that the power network is represented as an undirected weighted graph $G(V, E, W)$. The weights are, again, determined based on the absolute value of real power flow of the corresponding transmission line.

In the aggregation phase, a multilevel algorithm repeatedly coarsens the graph by merging the nodes. It operates level by level until the coarsened graph has only a small number of vertices. Hereby, a random unmerged node $x$ is selected and merged with an adjacent unmerged node connected by the edge with the highest weight, forming a so called super-node. The procedure is repeated until all the nodes have been merged. The new edge weights between the super-nodes are then equal to the sum of the edge weights between the nodes forming these. Afterwards, new level is created starting the procedure from the beginning - i.e. by selecting a random super-node and so on - until the resulting coarsened graph has reached the desired size necessary for a quick clustering [DGK05].

![Figure 2.2: Graphical overview of the multilevel clustering approach [DGK05]](image-url)

In the next phase (initial-)clustering of the coarsened graph is obtained. To do so spectral clustering method discussed above can be used. Because of the smaller size of
the graph, the clustering is fast calculating Laplacian matrix’s eigenvalues and eigenvectors. At the same time, it was shown in [DGK04] that the objective function used for the K-way spectral clustering is mathematically equivalent to the weighted kernel K-means clustering objective. Therefore, if the eigenvalues / eigenvectors calculation is unfavourable, there is an efficient alternative.

In the final phase, refinement algorithm is applied at the coarsened graph given an initial clustering obtained in the previous phase. The graph $G_{i-1}$ is retrieved extending the clustering of $G_i$ to $G_{i-1}$. Hereby, each node forming a super-node at the $i$’th level is assigned to the same cluster at the $i-1$’th level. The resulting clustering is then refined using the same kernel K-means algorithm as for the initial clustering. Afterwards, the procedure is moved to the next level $i - 2$ and is repeated until the original graph $G = G_0$ is obtained. Because of the coarsening, initial clustering ensures that the refinement algorithm can converge quickly.

In total, Multi-Level kernel K-Means turns out to be very computationally efficient. In contrast to spectral clustering it allows to perform a faster clustering of the larger graphs. According to [PI09], the advantage in computational efficiency is progressively accentuated with an increasing number of the nodes. At the same time, the ICI-approach based on the multilevel kernel K-means presented in [PI09] also provides an islanding solution minimising the power flow disruption. However, same as with spectral clustering, this ICI-approach does not explicitly consider stability of the resulting islands. Therefore, even though it can quickly provide an islanding solution, this might not always be feasible for the power system splitting.

**OBDD-Based Three Phase Clustering**

An ICI-approach that explicitly considers stability of the islands is based on the Ordinary Binary Decision Diagram (OBDD) clustering [SZL03]. The algorithm is divided into three phases which splits the computational burden between different time-layers. Herewith, many of the computations that are necessary to ensure island stability are done offline, prior to the controlled islanding. This allows to quickly provide an islanding solution even for very large networks as OBDD-based three phase clustering overcomes the combinatorial explosion of the searching space by subsequently reducing it throughout these phases.

In Phase 1, the original network is simplified by merging and removing the redundant nodes and edges. Hereby, the original network topology is nonetheless preserved as it is described in [SZL03].

In Phase 2, the simplified network is described together with the so called Separation and Synchronous Constraint (SSC) and Power Balance Constraint (PBC) as a set of boolean expressions (OBDDs). The first constraint guarantees that the generators within the island will be approximately synchronous so that the rotor angle stability of the resulting islands is promoted. The second constraint ensures that the islands are as balanced in terms of power as possible. Hereby, the frequency and voltage stability of the islands are promoted.

The obtained OBDDs are used to check if islanding solutions satisfy the SSC and
PBC as it is described in [ZSZ03]. In order to evaluate each solution, a boolean expression has to be solved. Given that after Phase 1 the network is simplified to only a couple of dozens of nodes and edged, the satisfiability check can be done very quickly. Hereby, only the solutions feasible to the SSC and PBC are left so that the solution space is significantly reduced.

In Phase 3, power-flow calculations are done for the lines that are not part of the cut-sets corresponding to the solutions left after Phase 2. Herewith, the solutions are evaluated with respect to the, so called, Rated Value and Limit Constraint (RLC). This ensures that the transmission lines and other equipment will not be overloaded after the islanding. The solution space is hereby further reduced.

In the end, only few solutions that satisfy all three constraints are left in the final searching space, as it is demonstrated in Figure 2.3. Now, the solution that fits best to the predefined objective function can be quickly selected during the actual controlled islanding.

Figure 2.3: Flowchart representing the solution space reduction during the OBDD clustering [SZL03]

The OBDD-based ICI-approach features several important advantages. Because of the modular structure of the clustering, necessary computations can be spread between different time layers. As described in [SZL03], preparatory calculations, such
as network simplification in Phase 1, are done offline. OBDDs formulation with respect to PBC is also done offline and is then, updated periodically using the given load forecast. In contrast, OBDDs describing SSC along with power-flow calculation have to be constructed online. At the same time, it was shown that even these OBDDs can be constructed by stages, as they are in some parts changeless for a given network [SZL03]. Such parts can then be quickly combined on the fly to describe the network with respect to the SSC. Once all OBDDs are constructed, the optimal solution can be found in a very short time even for the larger networks.

Because the combinatorial explosion of the solution space is overcome by its subsequent reduction due to the three constraints, and because these constraints ensure the stability of the islands as much as possible, an ICI-approach featuring the OBDD-based clustering can be applied on a real power system as it was shown in [SZL05].

**Slow Coherency Based Islanding**

To cluster a large interconnected power system, one has to overcome the combinatorial explosion of the solution space. One option is to use stochastic searching methods such as simulated annealing to find an islanding solution. Another option is to group the network elements prior to the clustering, hereby, reducing the searching space, as it is described above. Because the island stability has to be ensured, the solution space has to be systematically searched in order to reliably obtain a stable islanding solution. Most of the ICI-approaches proposed recently, support the latter idea. These approaches below have in common that the offline grouping is based on the slow coherency of the generators.

The concept of slow coherency has been developed during transient stability studies of power systems and was originally used to perform an offline network clustering for parallel computing purposes [Cho82, YRA93]. These studies have shown that after a disturbance some generators tend to oscillate together over a longer period of time than the others. Such generators are called slowly coherent.

There are several methods to identify the groups of slowly coherent generators including both: time and frequency domain approaches. The most common approach is to identify $r$ slowest modes of the linearised model and then group the generators around those as it is extensively described in the subsequent Section 2.2. Although it uses a linearisation, the obtained slow coherency grouping appears to be valid in simulated large nonlinear system [YVW04]. Moreover, it was shown that the grouping is independent of the disturbance and can, therefore, be done offline [Cho82].

There is also another reason, why most of the recently proposed ICI-approaches use slow-coherency for the offline grouping of the generators. As it was proven in [Cho82], slow coherency implies weak interconnections between the generator groups in comparison to the connections among the generators within a group. Therefore, within the transient time-scale after an occurred disturbance, a strong flow exchange within the group is expected. At the same time, the power flow between the groups is comparatively small and the couplings between the groups appear rather weak on the short time-scale.
It appears plausible that after slowly coherent groups have been identified, in case of a disturbance, the clustering can be done across the weak couplings between the groups. By separating the system within the transient time scale, a disturbance would not be able to propagate through the entire network, because only fast dynamics can propagate through the weak connections. Therefore, if an island contains only slowly coherent generators, then the transient stability of such an island is ensured. This made slow coherency generator grouping combined with a clustering algorithm so common for the numerous ICI-approaches briefly described below.

Various ICI-approaches using slow coherency grouping to minimise the power flow disruption during the islanding have been proposed. In [WV04] minimal cut-set technique from the graph theory is used to obtain possible islanding solutions. Afterwards breadth first searching flag based on depth first searching technique is used to determine the solution that would yield minimal power flow disruption. In a recent publication [DGLF+13], the spectral clustering method described above, was used to split the predetermined slowly coherent generator groups. Because of the grouping, the dimensions of the system are kept low. Thus, the eigenvalues necessary for the clustering can be quickly calculated, while the transient stability is ensured due to the prior grouping. Therefore, the main disadvantages of the original spectral clustering method were outfoxed, as it was shown in the subsequent study [DQTKT13].

There also have been numerous propositions for an islanding approach that minimises the power mismatch within the islands. As such, [YVW04] combined slow coherency grouping with the clustering algorithm presented in [Tsa00]. It used the linearised power system model to group the generators around the \( r \) slowest modes as described in Section 2.2. Clustering algorithm will then consider topological constraints and provide a search-based list of all the possible cut sets with the power mismatch information about each islanding solution. Herewith, a solution featuring the smallest mismatch can be selected. While slow coherency grouping assures transient stability in the resulting islands, clustering algorithm pursues an islanding solution which promotes the frequency stability of the islands.

Several other ICI-approaches were proposed featuring improved clustering to reduce the computational time. In [WZS+09, WZH+10] the following three-step clustering method was presented. Given the slowly coherent generator groups, at first, power trace algorithm from [Bia96] is used to assign the loads to these groups. After each node has been assigned, initial cut-set, including all the lines connecting the nodes in different groups, is obtained. Subsequently the clustering is refined minimising generation-load mismatch in the islands. Another, presented in [LLCC09], proposes to use particle swarm optimisation to obtain the clustering considering the slowly coherent generator groups. Both ICI-approaches use the linearised power system model to identify the slowly coherent generators, as it is described in Section 2.2.

Alternatively, Krylov-subspace sethod was recently proposed to obtain slowly coherent generator groups. Combined with a spanning tree based breadth first search algorithm for clustering it was also demonstrated to provide islanding solutions minimising load generation mismatch within the islands [Naj09].

An ICI-approach that incorporated slow coherency grouping, described in Section
2.2 Slow Coherency Grouping

For a large dynamic system an intuitive reduction technique is to aggregate its elements into groups describing each group as a single entity. Thereby the system model can be formulated by a smaller number of variables.

A system is called aggregable if its elements can be grouped so that the dynamics of each group are decoupled from each other and can be described by some aggregated variables. Physical aggregation describes the process of summing up the storage quantities in a group, assuming that those are stored in an aggregated element.

Applying this idea for the purpose of grouping generators in a wide-area power system, means that each group $\alpha$ can be represented by a single aggregated generator model. Its inertia $H_\alpha$ is, then, defined as the sum of inertias of all generators in the group expressed as

$$H = \sum_{N_\alpha \subseteq \alpha} H_i$$

(2.15)

where $N_\alpha$ is the set containing the indices of the generators that belong to the group.

Grouping the elements of a dynamic system can be seen as grouping the states so that each element state $x_i$ (which can be a vector comprising several variables) is assigned to only one group. For this purpose, a notation using reference ordering of the states has to be introduced. Given that the states are to be split into $r$ different groups, we arbitrarily pick one state in each group denoting it as a reference state $x_{\text{ref}}^\alpha$.

The states can then be split into $r$ reference states $x_{\text{ref}}$

$$x_{\text{ref}} = [x_{\text{ref}}^1, x_{\text{ref}}^2, \ldots, x_{\text{ref}}^r]^T$$

(2.16)

and $(n - r)$ non-reference states $x_{\text{non-ref}}$

$$x_{\text{non-ref}} = [x_{\text{rref}}^1, x_{\text{rref}}^2, \ldots, x_{\text{rref}}^{(n-r)}]^T$$

(2.17)
so the system state vector $\mathbf{x}$ can, after corresponding permutations, be represented as

$$
\mathbf{x} = \begin{bmatrix} \mathbf{x}_{\text{ref}} \\ \mathbf{x}_{\text{nref}} \end{bmatrix}.
$$ (2.18)

In order to assign the states to the groups, we define a $(n \times r)$ matrix $\mathbf{P}$ whose columns represent the groups while each row is associated with a state so that its entries are defined as:

$$
p_{i\alpha} = \begin{cases} 
1 & \text{if the state } x_i \text{ is in the group } \alpha \\
0 & \text{if the state } x_i \text{ is not in the group } \alpha.
\end{cases}
$$ (2.19)

Using the reference ordering of the states, we can express $\mathbf{P}$ as

$$
\mathbf{P} = \begin{bmatrix} \mathbf{I}_r \\ \mathbf{L}_g \end{bmatrix},
$$ (2.20)

where $\mathbf{I}_r$ is a $(r \times r)$ identity matrix and a $(n-r) \times r$ matrix $\mathbf{L}_g$ is, a so called, grouping matrix. It is used to assign the non-reference states to the groups.

The groups can be determined depending on a predefined criterion. In order to simplify a wide-area power system dynamical model, we need a criterion focusing on the generator dynamic properties. Transient stability studies have shown that after an applied disturbance, some generators tend to oscillate together. These generators are called coherent and can be grouped defining synchronous areas, within which transient stability is assured. In fact, as it is shown in [Cho82], a power system is aggregable if the groups are formed by the coherent generators. It means that the dynamics of the groups are – on a slow time-scale – decoupled from each other so that every group can then be represented by a single generator model. Slow coherency grouping substantially simplifies the representation of the power system but, at the same time, yield a model that preserves the original dynamic behaviour.

Slow coherency grouping is an application of the two-time-scale power system modelling method. It is based on the observation of oscillations in large power systems. In particular, it has been observed, that after a disturbance, the oscillations in a power system can be classified by the system modes acting on a fast and a slow time scale.

The fast modes characterise the transients within a synchronous area. After fast intra-area dynamics have decayed, the machines in the area would swing together while the slow inter-area dynamics, characterised by the slow modes, come into effect. Groups of slowly coherent machines can be seen as a physical evidence that the connections between the synchronous areas are weaker than the connections within the areas. Therefore, in a real large power system one can always expect to find the groups of strongly interacting units coherent on a slow time scale [YVW04].

If every generator is associated with its state $x_i$, finding the slowly coherent groups essentially comes down to grouping the states based on their participation in the slow inter-area modes. Thus, to obtain $r$ slowly coherent groups, the generators represented by the corresponding states, are grouped around the $r$ slowest modes of the system.
To do so, a classical generator model can be used disregarding damping, primary frequency control, voltage regulator and automatic generation control. These, even though they can affect the simulated swing curves, do not radically change the basic network characteristics such as inter-area modes [YWW04].

The loads are assumed constant so that the grid can be represented by a node admittance matrix \( \mathbf{Y} \). Moreover, node elimination technique, as described in [Pai89], is used to reduce the network, retaining only the internal generator nodes. Reduced network is then described by the generator node admittance matrix

\[
\mathbf{Y}_G = \mathbf{G} + j\mathbf{B}.
\]  

For an \( n \)-machine system, the linearised second order model is therefore given by:

\[
\ddot{\mathbf{x}} = \mathbf{A}_G \mathbf{x}
\]  

where \( \mathbf{x} \) represent system state describing generator angle deviations \( \Delta \delta_i \)

\[
\mathbf{x} = [\Delta \delta_1, \Delta \delta_2, \cdots, \Delta \delta_n]^T.
\]

The \( (n \times n) \) system state matrix \( \mathbf{A}_G \) is obtained as

\[
\mathbf{A}_G = \mathbf{M}^{-1} \mathbf{K}
\]

where \( \mathbf{M} \) is the \( (n \times n) \) inertia matrix defined as

\[
\mathbf{M} = \text{diag} \left( \frac{2H_1}{\omega_0}, \frac{2H_2}{\omega_0}, \cdots, \frac{2H_n}{\omega_0} \right)
\]

and where the elements of the matrix \( \mathbf{K} \) are, depending on the operating point, calculated as

\[
K_{ij} = \begin{cases} 
K_{ij} = V_i V_j [B_{ij}\cos(\delta_i - \delta_j) - G_{ij}\sin(\delta_i - \delta_j)] & \text{for } i \neq j \\
K_{ii} = -\sum_{j=1,j\neq i}^{n} K_{ij} & i = j
\end{cases}
\]

Here, for \( i, j = 1 \ldots n \), \( H_i \) is the inertia constant of the \( i \)-th machine in seconds, \( \Delta \delta_i \) is the generator angle deviation in radians, \( \omega_0 \) is the system nominal frequency in radians per second, \( G_{ij} \) and \( B_{ij} \) are the real and imaginary parts of \( \mathbf{Y}_G \) with \( i, j = 1 \ldots n \).

Using this model, system modes can be calculated and divided into a set \( \sigma_a \) containing the \( r \) slowest modes:

\[
\sigma_a = \{\lambda_1, \lambda_2, \cdots, \lambda_r\};
\]

and a set \( \sigma_a^C \) containing \( (n - r) \) remaining modes:

\[
\sigma_a^C = \{\lambda_{r+1}, \lambda_{r+2}, \cdots, \lambda_n\}.
\]
Additionally, eigenvectors \( v_i \) corresponding to the modes \( \sigma_a \) are calculated. We use

\[
V = [v_1, v_2, \ldots, v_r] = \begin{bmatrix}
w_1 \\ w_2 \\ \vdots \\ w_n
\end{bmatrix},
\]

(2.29)

to denote the \((n \times r)\) basis matrix of the \( \sigma_a \)-eigenspace that is used to find coherent states. It was shown in [Cho82] that the states \( x_i \) and \( x_j \) are coherent with respect to \( \sigma_a \) if, and only if, the \( i \)-th row \( w_i \) and \( j \)-th row \( w_j \) of \( V \) are equal. Thus, it is theoretically possible to find the coherent groups by comparing all the rows of \( V \). Hereby, we have to take into account that there might be one or more states which are not coherent to any other. Such state would constitute a coherent group containing only itself.

In general, in a real dynamic system there are more coherent groups than the number of the slow modes \( r \). It means, that coherent groups cannot always be directly used for the aggregation. Therefore, it is necessary to extend the definition of a coherent group to a group comprising near-coherent states.

For this purpose, the states \( x_i \) and \( x_j \) are defined as near-coherent with respect to \( \sigma_a \) if, and only if, there exists a \( \sigma_a \)-eigenbasis matrix \( V \) so that

\[
w_i - w_j = \epsilon_{ij}
\]

(2.30)

where \( w_i \) and \( w_j \) are, again, the rows of \( V \) and \( \epsilon_{ij} \) is a small parameter. The near-coherent states can then be grouped so that the resulting total number of groups is equal to \( r \).

If we regard the rows of \( V \) as vectors in a \( r \)-dimensional space \( \chi \), then a group of ideally coherent states would be represented by a single vector. The groups comprising the near-coherent states determined by nearly-identical rows of \( V \) are then represented by vector-clusters, each of which is contained in a cone. This representation is illustrated in Figure 2.4. There, the clusters of row vectors are represented in \( \mathbb{R}^3 \). Coherent groups 1 and 2 comprise the near-coherent states while the group 3 includes only the ideally coherent states.

In a practical case of a dynamic system with numerous states, it might not be as easy to identify the near-coherent states as it can be done in Figure 2.4. To more clearly reveal the clusters of row vectors representing the coherent groups, a coordinate transformation \( \chi \mapsto \chi' \) is applied as follows:

At first, assuming that the system states are to be split into \( r \) groups, one vector is picked from each cluster to be used as a reference. The eigenbasis \( V \) can, then, be rewritten as

\[
V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}
\]

(2.31)
2.2 Slow Coherency Grouping

Figure 2.4: Clusters of row vectors of $V$ representing some coherent groups in $\mathbb{R}^3$

where

$$V_1 = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_r \end{bmatrix} \quad (2.32)$$

is the $(r \times r)$ matrix contacting the $r$ reference vectors (that are associated with the states that we will call reference states) of the clusters, and where

$$V_2 = \begin{bmatrix} w_{r+1} \\ w_{r+2} \\ \vdots \\ w_n \end{bmatrix} \quad (2.33)$$

is the $((n - r) \times r)$ matrix containing the remaining $(n - r)$ row vectors. The row vectors in $V_1$, representing the reference states, are than taken as the coordinates of the $\chi'$. This is equivalent to

$$VV_1^{-1} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} V_1^{-1}. \quad (2.34)$$

It can be shown that, if the states within the groups are ideally coherent, then

$$V_2 V_1^{-1} = L_g \quad (2.35)$$

where $L_g$ is the grouping matrix introduced in (2.20). Analogously, if we have near-coherent states then

$$V_2 V_1^{-1} = \tilde{L}_g \quad (2.36)$$

where $\tilde{L}_g$ is a $((n - r) \times r)$ matrix that has a similar structure as $L_g$. Each row of $\tilde{L}_g$ contains only one entry which is close to 1, while all the other entries are close to 0.
In other words, if $\tilde{L}_g$ can be expressed as
\[
\tilde{L}_g = L_g + \mathcal{O}(\epsilon),
\] (2.37)
with $(i,j)$-th element of $\mathcal{O}(\epsilon)$ defined as $\epsilon_{ij}$, then the coordinate transformation (2.34) can be rewritten using (2.20) to
\[
VV_1^{-1} = \begin{bmatrix} I_r \\ \tilde{L} \\ L_g \end{bmatrix} = [ I_r \\ 0 ] + \mathcal{O}(\epsilon) = P + \mathcal{O}(\epsilon).
\] (2.38)

Therefore, reference vectors in $V_1$ become the axis of $\chi'$ while the remaining vectors are clustered around them. As a result, the clusters around the rows of the $(r \times r)$ identity matrix $I_r$ are well separated in $\chi'$. While an ideal coherent grouping yields $L_g$, a grouping of near-coherent states becomes a task of finding a $V_1$ with a set of $r$ reference states so that the resulting matrix $L$ resembles the grouping matrix $L_g$ as much as possible.

To quantify the similarity between $\tilde{L}_g$ and $L_g$ the matrix row norm
\[
\| L \| = \max_i \sum_{j=1}^n |L_{ij}|
\] (2.39)
is used. While for $L_g$ this norm is by definition $\| L_g \| = 1$, it can be shown that for every possible selection of the reference states $V_1$, $L$ will always satisfy
\[
\| \tilde{L}_g \| \geq 1.
\] (2.40)

In other words, if the near-coherent states are clustered around the unit coordinate vectors in $\chi'$ as described above, then $\| \tilde{L}_g \| \approx 1$. If, however, the states in each group are coherent so that $\tilde{L}_g$ and $L_g$ are equal, then
\[
\| \tilde{L}_g \| = \| L_g \| = 1.
\] (2.41)

Under these considerations, coherency grouping can be formulated as the following optimisation problem:

**Coherency Grouping Problem:** Find reference states minimising $\| \tilde{L}_g \|$, and grouping matrix $L_g$ minimising $\| \tilde{L}_g - L_g \|$.

To solve this optimisation problem one can apply random search trying all possible $V_1$ and evaluating $\| \tilde{L}_g \|$ and $\| \tilde{L}_g - L_g \|$. Alternatively, as it is shown in [Cho82], in order to minimise $\| L_g \|$, one can try to find $r$ reference states so that the volume of a $r$-dimensional polytope spanned by the rows of $V_1$ is maximised.

The volume of the $V_1$-polytope is given by the Gramian matrix:
\[
G(V_1) = \det(V_1 V_1^T) = (\det(V_1))^2.
\] (2.42)
2.2 Slow Coherency Grouping

\( V_1 \) approximately maximising \( G(V_1) \) can be found using Gaussian elimination which will permute the rows of \( V \) so that it gets into the upper triangular form. If then, \( p_i \) denotes the pivot of the \( i \)-th step of Gaussian elimination, then the Gramian matrix after the elimination \( G(V'_1) \) will be given by

\[
G(V'_1) = (p_1 \cdot p_2 \cdots p_r)^2.
\] (2.43)

Gaussian elimination with complete pivoting assures that the pivots \( p_i \), \( i = 1, \ldots, r \) are maximised. Because it does not necessarily assure the maximisation of the overall product \( p_1 \cdot p_2 \cdots p_r \), it is just an approximate method for maximising \( G(V_1) \). However, in comparison to the random search, it is tremendously computationally efficient, which becomes important for a wide-area power system with numerous generators.

Applying Gaussian elimination with complete pivoting to \( V \), the \( n \) rows and \( r \) columns of \( V \) are permuted so that the \( (1,1) \)-entry of it becomes the one with the largest magnitude. This entry is, then, used as a pivot \( p_1 \) for the first step of Gaussian elimination. For the next step, the procedure is repeated on the remaining \((n-1) \times (r-1)\) sub-matrix of the reduced \( V \). The elimination is terminated after \( r \) steps yielding the pivots \( p_1, p_2, \ldots, p_r \). During Gaussian elimination, every time the rows of \( V \) are permuted, the ordering of the states in \( x = [x_1, x_2, \ldots, x_n]^T \) also has to be changed because an \( i \)-th row in \( V \) is associated with the state \( x_i \). At the same time, a column permutation does not affect the ordering of the states.

After \( r \)-steps of Gaussian elimination, the first \( r \) rows of the reduced \( V \) identify the reference states. As a result, one gets \( V_1 \) and \( V_2 \) needed to calculate \( \tilde{L}_g \) using the equation (2.36). Even though Gaussian elimination might deliver only an approximate optimal solution for \( \tilde{L}_g \), it assures that two near-coherent states are not both appointed as references. In particular, due to the nature of Gaussian elimination, the rows having small entries are not used as pivots. Small entries are the result of elimination using an almost identical row which already is appointed to become reference states.

Having obtained \( \tilde{L}_g \) minimising \( \|\tilde{L}_g\| \), the grouping matrix \( L_g \) can be calculated. To do so, the biggest value (considering the sign) in each row of \( \tilde{L}_g \) is set to 1 while all the others are set to 0. The resulting \( L_g \) indicates how non-reference states are appointed to the groups around the \( r \) reference states. Hereby, the notation (2.19) - (2.20) is used for the grouping.

As the result, given a system state matrix \( A_G \) from the \( n \)-machine reduced second order model described by (2.22)-(2.26), the slow coherency grouping algorithm can be summarised as follows:

**Step 1** Select the desired number \( r \) of slowly coherent groups.

**Step 2** Calculate the eigenvalues of \( A_G \) identifying \( r \) slowest modes \( \sigma_a \).

**Step 3** Calculate the eigenbasis matrix \( V \) corresponding to the modes \( \sigma_a \).

**Step 4** Apply Gaussian elimination at \( V \) obtaining \( r \) reference states.
Step 5 Use the ordering of the permuted vector $\mathbf{x}$ for the rows of $\mathbf{V}$ along-with the knowledge of the $r$ reference states from Step 4 to obtain matrices $\mathbf{V}_1$ and $\mathbf{V}_2$.

Step 6 Compute $\tilde{\mathbf{L}}_g$ for the set of reference states obtained in Step 4 using $\mathbf{V}_1$ and $\mathbf{V}_2$ from the Step 5.

Step 7 Construct the grouping matrix $\mathbf{L}_g$ that defines the non-reference states in each group.

2.3 K-Means Clustering

K-means clustering is a technique originally developed to identify groups (clusters) of points in a multidimensional space. Hereby, the points are clustered in such a way that the distances between the points assigned to the same cluster are small in comparison to the distances between the clusters.

Given a set $\Omega$ consisting of $n$ points in the $D$-dimensional space

$$\Omega := \{x_1, \ldots, x_n\} \quad \text{where} \quad x_i \in \mathbb{R}^D \quad \forall \quad i = 1, \ldots, n$$  \hspace{1cm} (2.44)

the K-Means clustering partitions $\Omega$ into $K$ subsets $C_k \subseteq \Omega$ so that:

$$C_k \cup C_l = \emptyset \quad \forall \quad k, l = 1, \ldots, K$$  \hspace{1cm} (2.45)

These subsets are called clusters and can be represented by their center points $\mu_k$ (centroids). The clustering objective can now be formulated as follows [B+06]:

K-Means Clustering Problem: For a given $\Omega$ find $K$ clusters $\{C_k\}$ and centroids $\{\mu_k\}$ so that the sum of squares of distances between each point and the closest $\mu_k$ is minimised.

Consequently, K-means clustering can be regarded as an iterative procedure of a two-step optimisation. Given an initial set of $\{\mu_k\}$, in the first step, each point is assigned to the cluster $C_k$ corresponding to the closest centroid. Based on the resulting partitioning of $\Omega$, in the second step, a new centroid is determined for each cluster. The procedure is repeated until convergence which is assured according to [M+67]. An example of the K-means clustering applied on a set in $\mathbb{R}^2$ is provided in Figure 2.5

There, first, the centroids marked by the blue and the red cross are initialised (a). Each point is assigned to the cluster corresponding to the closest centroid (b). Afterwards, new centroids are chosen (c). The procedure is repeated (d,e,f,g,h), while the pink line demonstrates how the cluster boundaries converge towards the final result in (h). K-means clustering has converged when the cluster boundaries remain the same for several steps (i).

A modified version of this technique presented in [KCAV10], can be also used for the power system partitioning. For this purpose, the $n$ power system nodes (buses) are regarded as the points of $\Omega$. At the same time, instead of using the euclidian distance
Figure 2.5: An example of the K-means clustering applied on a set in $\mathbb{R}^2$ [B+06]
as in the original K-means method, a custom made distance matrix \( \mathbf{D} \) determines the proximity of the buses.

In order to define \( \mathbf{D} \), the power system is represented as a weighted undirected graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \), where \( \mathcal{V} = \{v_1, \ldots, v_n\} \) is the set of \( n \) buses (vertices), while \( \mathcal{E} = \{e_{ij}\} \) is the set of power lines connecting the buses (graph-edges). Additionally \( \mathcal{W} = \{w_{ij}\} \) is the set of weights assigned to the lines and the weighted adjacency matrix of the graph is defined as an \( n \times n \) matrix \( \mathbf{A} = [a_{ij}] \) where

\[
a_{ij} = \begin{cases} w_{ij} & \text{if there is a line between buses } i \text{ and } j \text{ i.e., } e_{ij} \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}
\]  

(2.46)

The weights can be seen as line lengths and are determined by various criteria. Using \( \mathbf{A} \), the distance matrix \( \mathbf{D} = [d_{ij}] \) defining the distances between any two buses in the system, can be calculated as discussed in the next section. The weighted adjacency matrix, therefore, entirely describes the power system graph as it includes both, the topology of the network and the line lengths determining distances between the buses.

Analogously to the original K-means method, clustering is done as a two-step iterative procedure. Given the desired number of clusters \( K \) and an initial set of buses \( \{\mu_1, \ldots, \mu_K\} \) used as centroids, in the first step, every other bus is assigned to one of the clusters corresponding to the closest \( \mu_k \) according to the distance matrix. This assignment is described by the \( n \times K \) partitioning matrix \( \mathbf{P} = [p_{nk}] \) where

\[
p_{nk} = \begin{cases} 1 & \text{if } k = \arg \min_i d_{ni} \\ 0 & \text{otherwise} \end{cases}
\]  

(2.47)

In the second step, new centroids for each cluster are chosen as the buses with the minimum distance to all the other buses in the cluster. This can be mathematically described as:

\[
\mu_k = \arg \min_j \sum_{i=1}^{M} d_{ij}
\]  

(2.48)

Here, \( d_{ij} \) is the distance between buses \( v_i \) and \( v_j \) with \( v_i, v_j \in C_k \), while \( M \) is the total number of the buses in the cluster. After new centroids have been determined, the rest of the buses is reassigned to the closest centroid returning to the first step. The procedure is repeated until convergence [KCVA10].
3 Methods

Having chosen an ICI-approach and having provided the necessary theoretical background in the last Chapter, in this Chapter, we explain the methods we use to fulfil the research objective. First, we will extend the ICI-approach so that it can account for the existing VSC-HVDC links. The resulting underlying ICI-algorithm is described in Section 3.1. In the subsequent Section 3.2, we will develop the VSC-HVDC control scheme that is used to help stabilising the islands. At last, in Section 3.3 we will describe how we have simulated the controlled islanding using the extended ICI-algorithm and the developed control scheme. To show how controlled VSC-HVDC links can support the controlled islanding, we define various cases for the simulation, before we presents the simulation-results in the next Chapter.

3.1 Controlled Islanding Algorithm

In Chapter 2 we explained why controlled islanding can be seen as a clustering problem. To assure that the resulting islands satisfy various constraints mentioned in Section 2.1, we can adjust the distances between the buses. Hereby, we modify the approach presented in [KCVA10] so that it will also take into account for the VSC-HVDC links. The result is the ICI-algorithm that we use for the clustering determining the islands bounds.

Given a weighted adjacency matrix $A$ characterising the network, a distance between any buses $x$ and $y$ can be defined. To do so, each path

$$p = \{v_0, v_1, \ldots, v_k\} \ \forall \ v_i \ \text{with} \ i = 1, \ldots, n \quad (3.1)$$

from a bus $v_0 = x$ to the bus $v_k = y$ is assigned a weight $w(p)$ that is defined as the sum of the weights of the lines constituting $p$:

$$w(p) = \sum_{i=1}^{k} w(v_{i-1}, v_i) \quad (3.2)$$

The distance between $x$ and $y$ is, then, defined as the weight of the shortest path between them:

$$d_{xy} = \begin{cases} 
\min w(p) : x \leadsto y & \text{if there is a path between } x \text{ and } y \\
\infty & \text{otherwise}
\end{cases} \quad (3.3)$$

25
Summarising the distances in the $n \times n$ distance matrix $D = [d_{xy}]$, allows us to use K-means clustering for partitioning of the power system.

To determine the line weights, we use various criteria. These are chosen so that the K-means clustering is enforced to satisfy the controlled islanding constraints. As we pointed out in Section 2.1, trying to perfectly satisfy all of the constraints is rather unpractical. Therefore, we choose criteria that enforce clustering, yielding islands which are as stable and as balanced (in terms of load-generation) as possible. Additionally, we ensure that the islands are connected via the HVDC-links so that the energy can be exchanged compensating the eventual load-generation imbalance. The following criteria are used in this study in order to determine the line weights:

**Criterion 1** Slowly Coherent Groups  
**Criterion 2** Rotor Angle Difference  
**Criterion 3** HVDC-Links  
**Criterion 4** Active Power Imbalance

With the weights initialised as $w_{ij} = 1$ for $\forall i, j$, we apply the above criteria one after another throughout the ICI-algorithm. Weighted adjacency matrix modified to promote slow coherency within the islands (Criterion 1) is then used for the further modification applying Criterion 2, and so on.

As a result, starting with a network of equidistant nodes ($w_{ij} = 1$), the line weights are modified so that, for example, the distances between coherent generators are reduced promoting that their buses will be assigned to the same cluster. Such weighted adjacency matrix modification is schematically demonstrated in Figure 3.1. There, the HVDC-links are represented in green while the VSCs are represented as the green dots. The red-lines depict the obvious clustering.

In accordance to [KCVA10], ICI-algorithm can be divided into the on- and the offline parts to improve computational efficiency. So, the slowly coherent groups can be determined in advance. The corresponding weight modification can, therefore, be done offline. rotor angle difference and active power imbalance based modifications, on the other hand, require information about the operational state of the system (rotor angles, active power generation and consumption) and, therefore, have to be done online. The HVDC-modification has to consider previous modification must, therefore, also be done online. The resulting K-means clustering based ICI-algorithm is similar to the one described in [KCVA10] and is presented as a flow-chart in Figure 3.2. Its steps are now described in detail.

### 3.1.1 Slow Coherency Grouping

The determination of the $K_{coh}$ slowly coherent generator groups is done according to the theoretical description provided in Section 2.2.
3.1 Controlled Islanding Algorithm

**Figure 3.1:** Illustration of the weighted adjacency matrix modification adopted from [KCVA10]

**Figure 3.2:** K-means clustering based ICI-algorithm
3.1.2 Criterion 1 - Slowly Coherent Groups

Assuming that slowly coherent generators have been previously identified, in this step, we modify the path weights between them so that after the distance matrix is calculated, coherent generators appear closer to each other. As the result, the probability of those, being assigned to the same cluster, will increase promoting the transient stability in the islands.

For this purpose, we have to find the shortest path between a pair of coherent generators, and then, modify the weights of all the lines constituting that path. Additionally, we have to make sure that the shortest path between each pair has been modified this way.

Finding the shortest path between two nodes of an undirected weighted graph is a task that can be solved using Dijkstra’s algorithm [CLR+01]. For MATLAB, its implementation is available within the GAIMC-toolbox [Gle09], which we used for our study. For the graph represented by the weighted adjacency matrix $\mathbf{A}$, Dijkstra algorithm calculates the shortest path

$$p^* = \{v_0, \ldots, v_k\}$$

as a set of nodes lying on the way between the specified first node $v_0$ and the last node $v_k$. The lines which have to be traversed following the shortest path from $v_0$ to $v_k$, are now the ones whose weights are reduced by the slow coherency modification factor

$$C_{\text{coh}} < 1.$$  \hspace{1cm} (3.5)

The new line weights are then given by:

$$w_{v_0v_1}^{\text{new}} = C_{\text{coh}} w_{v_0v_1}$$
$$w_{v_1v_2}^{\text{new}} = C_{\text{coh}} w_{v_1v_2}$$
$$\vdots$$
$$w_{v_{k-1}v_k}^{\text{new}} = C_{\text{coh}} w_{v_{k-1}v_k}.$$

Having a slowly coherent group of $m$ generators, $l$ paths, with

$$l = \frac{m^2 - m}{2},$$

have to be modified. Defining a set $S$ containing the shortest path between each pair of the generators in the groups as

$$S = \{p_1^*, \ldots, p_l^*\},$$

we now need to ensure that we modify every line weight of every path $p_i^* \in S$, with $i = 1, \ldots, l$.

To do so, we use the iterative procedure demonstrated in Figure 3.3.
have illustrated an example of a slowly coherent group consisting of four generators. We have also denoted all (shortest) paths between the generators with the dashed lines. We now select the first generator from the group (let it be generator 1) and modify the paths between the generator 1 and the rest (generators 2,3,4), denoting these paths with the solid lines. After that, we exclude the generator 1 and repeat the procedure by selecting another generator from the diminished group (generator 2). The procedure is repeated as long as there is more than one generator left in the group. Applying it group by group, we not only ensure that every (shortest) path has been modified, but also that it has been modified only once.

![Diagram](image_url)

**Figure 3.3:** Iteration procedure to ensure that the shortest path between each pair of generators from a same group is modified

Summarising, the Criteria 1 application can divided into two parts:

1. finding the shortest paths between slowly coherent generators
2. modifying the line weights

This step is represented as a flow chart in Figure 3.4

### 3.1.3 Criterion 2 - Rotor Angle Difference

In this step, we modify the weighted adjacency matrix $A$ based on the rotor angle difference between the generators. Using the real-time information on the rotor angles, we determine the generator groups with similar angles. Next, we modify $A$, promoting that the machines with the similar angles will be assigned to the same cluster.

Given the real-time knowledge of the system operational state, we can group the machines by their rotor angles, obtaining what we will call *rotor angle groups*. To determine these, we apply the original K-means clustering method from [B+06]. Hereby,
Figure 3.4: Criterion 1 - slow coherency - based modification of the weighted adjacency matrix
we regard the rotor angles as points in the 1-dimensional Euclidian space. We also make sure that the number of rotor angle groups $K_{\text{ang}}$ is equal to the number of slowly coherent groups:

$$K_{\text{ang}} = K_{\text{coh}}$$

Having obtained the rotor angle groups, we make sure that the generators having similar angles and, therefore, belonging to the same group, are brought closer together. At the same time, we also increase the distances between the groups, making sure that the machines with significantly different rotor angles are departed from each other.

To increase the distances between the groups, for every pair of the machines belonging to different rotor angle groups we need to:

1. find all paths connecting the generators
2. for each path, set the first and the last line weight to $C_{\text{diffang}} \gg 1$

Finding all paths between two vertices in a graph is not a trivial task. As mentioned in [KCVA10], this is an $NP$-problem that cannot be solved within a polynomial time. However, given the fact that we are looking only for the first and the last line of each path, the search can have polynomial complexity ($P$-problem).

For this task, we apply the procedure that we call different angle algorithm and present as a flow chart in Figure 3.5. It is an iterative procedure that goes through every pair of generators with different rotor angles. Starting with a copy $A'$ of the weighted adjacency matrix $A$

$$A' = A,$$  \hspace{1cm} (3.9)

for each pair, we find the shortest path, obtaining its first and last lines. We call these lines $\alpha_i$ and $\omega_i$ and store them in a set $W$:

$$W = \{\alpha_1, \omega_1, \ldots, \alpha_i, \omega_i, \ldots\}$$  \hspace{1cm} (3.10)

At the same time, we remove those from the graph, setting the corresponding weights in $A'$ to zero:

$$w'(\alpha_i) = w'(\omega_i) = 0$$  \hspace{1cm} (3.11)

Next, the shortest path is calculated again, for the graph without the removed lines. For each pair this procedure is repeated as long as Dijkstra's Algorithm is able to find a path between the generators. Note, that with each iteration, $A'$ will become more and more sparse as we remove lines from the graph. The complexity of the path search will therefore decrease with each iteration yielding that different angle algorithm will be accomplished in polynomial time ($P$-problem). At the end of the different angle algorithm, we modify the line weights of the lines stored in $W$ by setting the corresponding elements of $A$ to:

$$w(\alpha) = w(\omega) = C_{\text{diffang}}$$  \hspace{1cm} (3.12)

for $\forall \alpha, \omega \in W$
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weighted adjacency matrix A, generator pairs with different rotor angles, C_{diffang}

take a new generator pair

Is there any unprocessed pairs left? Yes

Figure 3.5: Different angle algorithm
On the other hand, in order to bring the generators within a rotor angle group closer together, for every pair of the machines belonging to the same group we:

1. find the shortest path between the generators

2. modify the line weights belonging to the shortest path by the factor $C_{\text{simang}} < 1$

Hereby, we apply the algorithm which we call *similar angle algorithm* and present as a flow chart in Figure 3.6. It is almost identical to the Criterion 1 - algorithm applied for the slowly coherent groups which was extensively described in the corresponding subsection.

![Figure 3.6: Similar angle algorithm](image)

Criterion 2 application can be summarised as a procedure consisting of three parts as it is demonstrated in Figure 3.7. Given the real-time information on the rotor angles, we first obtain the rotor angle groups. Then, we apply the different angle algorithm on the weighted adjacency matrix to increase the distances between the groups. The modified weighted adjacency matrix is then used to bring the generators within the groups closer together. As a result, we promote that the generators having similar rotor angles will be assigned to the same island in the subsequent $K$-means clustering.
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**Figure 3.7:** Criterion 2 - rotor angle difference - based modification of the weighted adjacency matrix

### 3.1.4 Criterion 3 - HVDC-Links

In this step, we promote the clustering across the HVDC-links so that after the controlled islanding, as many islands as possible are interconnected via HVDC-lines. For this purpose, we separate the HVDC-link terminals (VSCs) by increasing the distance between them.

Following the idea illustrated in Figure 3.1, we rely on geometrical considerations presented in Figure 3.8. In order to promote that the VSCs are assigned to the different clusters we need to ensure a large distance between them according to the distance matrix. As it is shown in Figure 3.8, we achieve this by increasing the weight of the lines, connecting the VSCs to the rest of the network. To do so, we apply an algorithm similar to the different angle algorithm described above. Following two-step procedure is applied for each VSC-HVDC link where in the first step we:

1. find the shortest path \( p_1 = \{a^1_1, \ldots, a^1_n\} \) between the VSCs
2. store the first and the last line \( a^1_1 \) and \( a^1_n \) in a line register \( W \)
3. cut these lines
4. repeat until no path can be found between the VSCs
and in the second step, we modify the line weights belonging to the found paths \{p_1, \ldots, p_m\}. Hereby, \(p_m\) represents the longest path, and the line weights are modified as follows:

\[
\begin{align*}
  w'(a_1^m) &= C_{\text{hvdc}} + w(a_1^m) \\
  w'(a_n^m) &= C_{\text{hvdc}} + w(a_n^m) \\
  w'(a_1^{m-1}) &= (1 + \eta_{\text{hvdc}})C_{\text{hvdc}} + w(a_1^{m-1}) \\
  w'(a_n^{m-1}) &= (1 + \eta_{\text{hvdc}})C_{\text{hvdc}} + w(a_n^{m-1}) \\
  &
\end{align*}
\]

Here, \(w'(a_i^j)\) and \(w(a_i^j)\) with \(i = 1, \ldots, m\) and \(j = 1, \ldots, n\), represent the new and the old line weight respectively. The modification factors \(C_{\text{hvdc}}\) and \(\eta_{\text{hvdc}}\) are specified by the user and can be adjusted based on the ICI-algorithm requirements. Increasing these parameters increases the importance of the Criteria 3 in modifying the weighted adjacency matrix.

As a result, the shortest path between the VSCs might become the longest and vice versa - depending on how \(C_{\text{hvdc}}\) and \(\eta_{\text{hvdc}}\) were chosen. In any way, the distance between the VSCs is increased so that the clustering across the HVDC lines is promoted. In Figure 3.9, the weight modification is demonstrated for an example, where 3 paths have been found between the VSCs of an HVDC link.

3.1.5 Compute Distance Matrix

To compute the distance matrix \(D\), we have to calculate the shortest path weight \(w(p^*)\) for each pair of buses using the previously modified weighted adjacency matrix. To do so, we use the Dijkstra’s algorithm to calculate \(\min w(p)\) for every pair of buses. As the result, the distance between any buses \(x\) and \(y\) is given as the corresponding element of the distance matrix \(D = [d_{xy}]\).

3.1.6 Criterion 4 - Active Power Imbalance

To reduce the active power imbalance within the islands, in this step, we decrease the distances between every generator and the neighbouring loads so that it can fully supply. By bringing the buses, at which the power is fed in, closer to the ones, at which its consumed, we promote that such balanced entity will be assigned to the same island and, therefore, the active power imbalance within the islands will be reduced as much as possible.

For this step of the ICI-algorithm, two inputs are required. First, we need the real-time information on the active power production/consumption at each bus. Second,
Figure 3.8: Line weight modifications done to increase the distance between the VSC terminals of the HVDC-link

Figure 3.9: Line weight modifications shown for an example where three paths were found between the VSCs
the distance matrix $\mathbf{D}$ computed in the previous step is necessary to determine the neighbouring loads which can be fully supplied with power so that we can bring them closer to the corresponding generator by a further modification of $\mathbf{D}$.

The algorithm applied for each generator $G$ can be described according to [KCVA10] as follows:

1. Given a set $L = \{l_i\}$; with $\forall i = 1, \ldots, m$ of power system load buses, we sort these buses by their distance to the selected generator. Hereby, we define an array $\mathbf{L}_B$ as:

$$
\mathbf{L}_B = \begin{bmatrix}
    l_1 & P_1 \\
    \vdots & \vdots \\
    l_m & P_m
\end{bmatrix}
$$

(3.13)

where $l_1$ is the closest load bus and $P_1, \ldots, P_m$ are the active power consumptions of the corresponding buses.

2. We compute the active power cumulative sum defining an array $\mathbf{L}^{\text{sum}}_B$ as:

$$
\mathbf{L}^{\text{sum}}_B = \begin{bmatrix}
    l_1 & P_1 \\
    l_2 & P_1 + P_2 \\
    \vdots & \vdots \\
    l_m & P_1 + \cdots + P_m
\end{bmatrix}
$$

(3.14)

3. We subtract the power production $P_G$ of the selected generator defining an array $\mathbf{L}^{\text{netsum}}_B$ as:

$$
\mathbf{L}^{\text{netsum}}_B = \begin{bmatrix}
    l_1 & P_1 - P_{G_j} \\
    l_2 & P_1 + P_2 - P_{G_j} \\
    \vdots & \vdots \\
    l_m & P_1 + \cdots + P_m - P_{G_j}
\end{bmatrix}
$$

(3.15)

4. We obtain the buses that can be supplied by the selected generator. These are the ones which have negative entries in the second column of $\mathbf{L}^{\text{netsum}}_B$. We store those buses in a set $S$.

5. At last, we reduce the distances for $\forall b_i \in S$ by a factor $C_{\text{imb}} < 1$ so that the new distances are given by:

$$
a^{\text{new}}_{G_j l_i} = C_{\text{imb}} \cdot d_{G_j l_i}
$$

(3.16)

The flow-chart diagram of the Criterion 4 application, summarising the above procedure, is presented in Figure 3.10.

### 3.1.7 $K$-Means Clustering

In this step, the $K$-means clustering method described in Section 2.3 is applied using the distance matrix to determine $K$ clusters of the power system buses. The number
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Distance matrix $D$, bus active power $P$

1. Select a new generator $G$.
2. Sort the load buses by distance from $G$.
3. Compute active power cumulative sum $\sum L^B$.
4. Subtract the power production of $G$.
5. Obtain the buses fully supplied by $G$.
6. Modify the distances between $G$ and the buses in $S$.

Is there any unprocessed generator left?

Yes

Figure 3.10: Criterion 4 - active power imbalance - based modification of the distance matrix $D$
of clusters is hereby set equal to the number of slowly coherent and rotor angle groups:

\[ K = K_{\text{coh}} = K_{\text{ang}}. \] (3.17)

The \( K \)-means clustering algorithm used in this step is presented as a flow-chart in Figure 3.11.

As we have also mentioned in Section 2.3, centroids \( \mu_k \) with \( k = 1, \ldots, K \) have to be initialised before the clustering. To do so, we will take the \( K \) buses of the generators with the largest power production in each rotor angle group, as it was originally done in [KCVA10].

Subsequently, we start with the iterative procedure, described in Section 2.3. After the initialisation, we assign each bus to the cluster, corresponding to the closest \( \mu_k \) according to the distance matrix. Next, we find new centroids for the resulting clusters. If \{\( \mu_k \)\} remained the same for a predefined number of iterations \( T \), we assume that \( \mu_k \) and \( C_k \) \( \forall k = 1, \ldots, K \) have converged. The \( K \)-means clustering algorithm can now be terminated, yielding us the clusters of power system buses \{\( C_k \)\}, which define the island boundaries.

**Figure 3.11: \( K \)-means clustering algorithm**
3.1.8 Islanding

Based on the K-means clustering results, the corresponding cut-set has to be identified. The lines from this cut-set are then tripped to accomplish the islanding. In a practical application, a specialised algorithm will be necessary to find the cut-set based on the clustering. However, for this study, we disregarded this step. Given a clustering, we can easily identify the cut-set by hand, and then use it for the subsequent controlled islanding simulation.

3.2 VSC-HVDC Control

In the last Section, we have introduced an extended ICI-algorithm based on the K-means clustering. Our extension allows to split the system so that the islands are interconnected by the existing VSC-HVDC links. In this Section, we introduce a simple control strategy for active and reactive power of those links. Herewith, VSC-HVDC can be used to stabilise the islands in terms of frequency and voltage. We will use our control strategy for the simulations described in the next Section.

3.2.1 Model

For our study, we use a simplified second-order model of a VSC-HVDC link proposed in [IIA14]. It consists of the physical line model and the internal VSC control scheme. With these controllers, the active power of the link behaves as a second-order response, and the reactive power behaves as a first-order response.

According to [IIA14], for the physical line model (Figure 3.12), it is assumed that all dynamic elements, such as capacitors or inductances, are replaced by their steady state equivalents (reactances). Moreover, we assume that a converter is modelled as a lossless two-level Pulse Width Modulation (PWM) converter so that

$$P_{AC} = P_{DC}. \tag{3.18}$$

The line itself is modelled as a bipolar HVDC cable with the resistance $R_{dc}$. The HVDC link including the converters is connected to the AC system by a transformer, represented by the reactance $X_t$. A phase reactor, represented by $Z_r = R_r + jX_r$ at each side of the link, is also included. The VSC-HVDC link is controlled by three external control inputs: the rectifier is controlled by active and reactive power signals $P_{1,\text{ref}}, Q_{1,\text{ref}}$ respectively, and the inverter is controlled by the reactive power signal $Q_{2,\text{ref}}$. The DC voltage $U_{\text{DC1}2}$ is assumed to be constant and its control input is disregarded.

All AC components are described using a rotating $dq0$-reference frame. The $q$-axis is aligned with the AC system voltage $u_{s,i}$ so that $u_{d,i} = 0$ and

$$u_{s,i} = j \cdot u_{q,i}^s \tag{3.19}$$
with $i = 1$ representing the VSC acting as a rectifier and $i = 2$ the VSC acting as an inverter. By aligning the dq0-reference frame to the system voltage $u_{s,i}$, the apparent power of the link
\[
S_i = u_{s,i}i = P_i + jQ_i
\] (3.20)
can be rewritten as
\[
S_i = j \cdot u_{s,i}^q(i_1^d + j \cdot i_1^q) = j \cdot u_{s,i}^q i_1^d - u_{s,i}^q i_1^q.
\] (3.21)
Thus, the active power transmission through the link
\[
P_i = u_{s,i}^q i_1^q
\] (3.22)
and the reactive power produced by a VSC
\[
Q_i = u_{s,i}^q i_1^d
\] (3.23)
can be independently controlled by the corresponding currents $i^q$ and $i^d$. This is a key feature of a VSC-HVDC link that allows to use it not only for active power transmission, but also for the voltage control.

To model the internal control scheme of a VSC-HVDC link, we use the simplification proposed in [IIA14]. Given an external active power signal $P_{\text{ref}}$, the link including its internal controllers, can be approximated as a second-order response
\[
H_P(s) = \frac{P_1}{P_{\text{ref}}} = \frac{K_0}{s^2 + 2\zeta\omega_n s + \omega^2}.
\] (3.24)
Hereby, damping ratio $\zeta$, undamped natural frequency $\omega_n$ and the scaling factor $K_0$ can be derived using measurable quantities, such as the peak time $t_p$ and the overshoot $M_p$, as:
\[
\zeta = -\ln(M_p) \frac{1}{\sqrt{\ln(M_p)^2 + \pi^2}}
\] (3.25)
\[ \omega_n = \frac{\pi}{t_p \sqrt{1 - \zeta^2}} \]  
(3.26) 

\[ K_0 = \omega_n^2 \]  
(3.27) 

These quantities \( t_p \) and \( M_p \) can be seen as the model parameters and are obtained from the step-response measurement of the modelled VSC-HVDC link. Moreover, the VSC response for an external reactive power input \( Q_{1,\text{ref}} \) can be approximated by a first-order response

\[ H_{Q_i}(s) = \frac{Q_i}{Q_{i,\text{ref}}} = \frac{1}{\tau_{Q_i} s + 1}. \]  
(3.28) 

Here, \( \tau_{Q_i} \) is the only model parameter and represents the step-response settling time of the modelled link.

Using the equations (3.22) - (3.28), the simplified internal control scheme can be described as it is demonstrated in Figure 3.13. Together with the physical line model discussed above, we obtain our VSC-HVDC link model that is controlled by the three external inputs \( P_{\text{ref}}, Q_{1,\text{ref}} \) and \( Q_{2,\text{ref}} \). We use these inputs to control the link in such a way that it will stabilise the islands in terms of frequency and voltage as we describe below.

![Figure 3.13: Simplified internal control scheme adopted from [IIA14].](image_url)

### 3.2.2 External Control

In order to help stabilising the islands using the VSC-HVDC links between them, we introduce an external control scheme with the inputs \( P_{\text{ref}}, Q_{1,\text{ref}} \) and \( Q_{2,\text{ref}} \), as demonstrated in Figure 3.14. For this purpose, we implement three independent PI-controllers. While the active power transmission through the link will be controlled in such a way that the active power mismatch within the islands is reduced as much
as possible (P-control), the reactive power control of the VSCs will ensure that the voltage at the buses where VSCs are located, is kept at its nominal value of 1 p.u. (Q-control).

\[ \Delta \omega = \omega_1 - \omega_2, \quad (3.29) \]

which depends on the mismatch in the islands connected by the HVDC-link. Herewith, if, for example, an active power mismatch in island 1 has lead to \( \omega_1 > \omega_2 \), then the power will be transmitted to island 2. This will enforce that the frequencies in both islands become equal in a steady state. In the end, the total mismatch is equally spread between both islands, supporting frequency stability in them.

For this purpose, we apply a controller demonstrated in Figure 3.15. The controller output

\[ P_{\text{ref}} = P_{\text{set}} + dP \quad (3.30) \]

comprises a set-power signal \( P_{\text{set}} \) and the additional power signal \( dP \) provided by the PI-controller with the input \( \Delta \omega \). At the same time, \( dP \) is calculated by the PI-controller that can be tuned using the proportionality terms \( K_P \) and \( G_P \). Moreover, \( P_{\text{ref}} \) is limited by the nominal active power of the VSCs (1 p.u.). Note that, \( P_{\text{set}} \) is defined externally and might be used by another, higher level control scheme for an additional power flow control in the grid. PI-controller can be fine-tuned in order to
meet the requirements of the grid.

![Figure 3.15: P-controller](image)

The Q-control, responsible for maintaining the voltage $U_{s,i} = |u_{s,i}|$ of the AC terminal busses of the HVDC link at its set-value $U_{s,i,set}$, is implemented as two independent $Q_i$-controllers depicted in Figure 3.16. This controller, first, calculates the difference $\Delta U_{s,i}$ between the set-voltage $U_{s,i,set}$ and measured voltage $U_{s,i}$. It is, then, used as an input by the PI-controller which can be tuned using the proportionality terms $K_Q$ and $G_Q$. Additionally, the controller output $Q_{i,ref}$ is limited by half of the VSC-rating (0.5 p.u.) which usually corresponds to the maximal reactive power a VSC can produce. Similar to the P-Control, the set-voltage $U_{s,i,set}$ is defined externally and can be used for a higher level voltage control. The PI-controller can be fine-tuned in order to meet the requirements of the grid.

![Figure 3.16: Q_i-controller](image)

Based on the presented control scheme, three different control types for a VSC-HVDC link become available:

**P-control**: only active power is controlled

**Q-control**: only voltage is controlled

**PQ-control**: active power and voltage are controlled
Next, we have to simulate the controlled islanding with VSC-HVDC links to if at least one of these control types can help stabilising the frequency and the voltage after the islanding. To do so, we conduct the simulation described in the next Section.

3.3 Simulation

To simulate the controlled islanding we proceed as follows. First, we apply our ICI-algorithm for the clustering, obtaining the island boundaries for a given test system. Then, we perform dynamic simulations where we simulate the islanding and observe subsequent dynamics in the islands. For our study, we take two test systems and define various cases to extensively investigate how controlled VSC-HVDC links can support the islands after the islanding. The cases are also described in this Section.

For our investigation, we have chosen two different test systems. The first one, represented in Figure 3.17, is called Kundur Two-Area system and is extensively described in Example 12.6 of [KBL94]. This simple power system includes four generators with the transformers and two load buses - each with an attached active and reactive loads. Moreover, we have added four internal generator buses considering the internal impedance. The resulting system has 15 buses in total denoted in Figure 3.17. The two easily identifiable areas are connected by a 220 km long doubled tie line. In the normal operational state, this line transmits approximately 400 MW of power from the left area to the right one where it is consumed. All the parameters of this system are listed in [KBL94].

The second system is the IEEE-39 system, also known as New England power system presented in Figure 3.18. Its parameters are described in [Sad06]. This test-system represents a practical grid with a more complex topology and dynamics in comparison to the first system. It is a ten machine system with, originally, 39 buses. To those, we have, again, added the ten internal generator buses so that there has became in total 49 buses, denoted in Figure 3.18. Nonetheless, we will still call it as the 39 bus system.

To determine the island boundaries, we implement the ICI-algorithm described in Section 3.1 using MATLAB-software [MAT14]. To do so, we input both test systems using the MATPOWER format [ZMSG14]. For the algorithm, we use the parameters denoted further below.

To perform dynamic simulations, we use the corresponding module of NEPLAN-software [NEP14]. Both systems are extended by an UFLS scheme with the frequency relays placed at every load bus. The details on the used UFLS scheme are provided further below. We disregard the over-frequency generator trip relay, but keep in mind that some generators have to be switched off once the frequency exceeds a certain bound.

In the original publications [KBL94] and [Sad06], neither of the test-systems included any VSC-HVDC links. Therefore, we had to include those into our simulation. For the ICI-algorithm, such links were defined in a simple list that was considered throughout the clustering. For the dynamic simulations, we use the model and the
VSC-HVDC control scheme described in Section 3.2. Their parameters are provided further below.

By placing VSC-HVDC links at different locations and varying the number of islands provided by the ICI-algorithm, we define various cases. For each case, the island boundaries are, first, calculated in MATLAB, and then, the dynamic simulation is done in NEPLAN. The cases are described at the end of this Section.

![Kundur Two-Area system](image)

**Figure 3.17:** Kundur Two-Area system

### Controlled Islanding

For the ICI-algorithm, we use the parameters provided in Table 3.1. These include modification factors for the four criteria, and the $K$-means clustering termination condition $T$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{coh}$</td>
<td>0.8</td>
</tr>
<tr>
<td>$C_{diffang}$</td>
<td>50</td>
</tr>
<tr>
<td>$C_{simang}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$C_{hvdc}$</td>
<td>100</td>
</tr>
<tr>
<td>$\eta_{hvdc}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$T$</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table 3.1:** ICI-algorithm parameters

### Load Shedding

We use a basic load shedding scheme described on page 626 in [KBL94]. It consists of three UFLS steps, listed in Table 3.2. Once the frequency falls down to one of the
Figure 3.18: IEEE-39 system
thresholds $\omega_{th}$, a part of the load $-\%P_{load}$ is disconnected, after a circuit breaker delay of 50 ms. The relay delay is assumed to be 0.2 s.

<table>
<thead>
<tr>
<th>$\omega_{th}$ (p.u.)</th>
<th>$-%P_{load}$</th>
<th>$f_{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>0.9866</td>
<td>10%</td>
</tr>
<tr>
<td>Step 2</td>
<td>0.9800</td>
<td>15%</td>
</tr>
<tr>
<td>Step 3</td>
<td>0.9666</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 3.2: Applied UFLS scheme [KBL94]

VSC-HVDC Control

For the dynamic simulations, the model and the control scheme described in Section 3.2, has to be implemented in NEPLAN. To do so, we use the user-defined 2-port block that allows to include any systems described in SYMDEF format. For the VSC-HVDC model the parameters are taken from the ABB M6 HVDC-Light link and are listed in Table 3.3 [ABB10]. For the control scheme, we take the controller parameters denoted in Table 3.4. All PI-controller parameters are not fine-tuned, which can be a subject for further investigations.

| base power | 570 MVA |
| base voltage | 195 kV |
| $X_t$ | 0,113 p.u. |
| $X_r$ | 0,1648 p.u. |
| $R_t$ | 0 Ω |
| $R_{DC}$ | 0,0079 Ω |
| DC voltage | 150 kV |
| $\omega_n$ | 38,95 |
| $\zeta$ | 0,591 |
| $K_0$ | 0,7797 |
| $T_{Q1}$ | 0,02 |
| $T_{Q2}$ | 0,02 |

Table 3.3: Parameters of the ABB M6 HVDC Light link [ABB10]

Cases

To investigate how controlled VSC-HVDC links can support the islanding, we defined various cases summarised in Table 3.5. There, $K$ represents the number of created islands, while $\Delta P_{load} < 0$ represents the excessive load in the corresponding island after the separation.

In Case 1, we use Kundur Two Area system to illustrate how the VSC-HVDC link affects the dynamics after the islanding. We first simulate the islanding without the
HVDC link (Case 1.1). Then, we insert the link between the buses 11 and 13 and apply different control types (P, Q- and PQ-control) to see which one can stabilise the island more effectively. Afterwards, we make similar simulations on the IEEE-39 system that is also separated into two islands to draw final conclusions about the necessary control type. In the end, we simulate a three-island separation of the IEEE-39 system, inserting two VSC-HVDC links to see if these could contract each other hindering stabilisation of the islands.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>System</td>
<td>K2A</td>
<td>IEEE-39</td>
</tr>
<tr>
<td>VSC-HVDC links</td>
<td>none</td>
<td>1 - 11</td>
</tr>
<tr>
<td>K</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\Delta P_{\text{load1}}$</td>
<td>+400</td>
<td>+400</td>
</tr>
<tr>
<td>$\Delta P_{\text{load2}}$</td>
<td>-350</td>
<td>-350</td>
</tr>
<tr>
<td>$\Delta P_{\text{load3}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: Simulated cases
4 Results

To show how the controlled VSC-HVDC links can help stabilising the islands, we have simulated the controlled islanding for various Cases according to the simulation description in Section 3.3. In this Chapter, we present the simulation results. For each case, we, also, provide a short description. Moreover, we explain why the results fulfil the research objectives drawing some intermediate conclusions.

4.1 Case 1

For Case 1, according to the Table 3.5, we have simulated the islanding of the Kundur Two-Area system without and with a VSC-HVDC link (Cases 1.1 and 1.2 respectively). As we can see from the islanding results (Figures 4.1 and 4.2), the VSC-HVDC link connects the islands allowing active power transmission between them. At the same time, VSCs can participate in the voltage control.

The dynamic simulation shows (Figure 4.3) that in Case 1.1, controlled islanding results in a significant frequency deviation in both islands. While in the generation-rich island 1 (upper plot) the frequency steeply rises due to a positive active power mismatch, in the load-rich island 2 (lower plot) the falling frequency stipulates the first load shedding step at about 4 s after the islanding.

However, if there is an VSC-HVDC link connecting the islands (Case 1.2) and, at the same time, the active power exchange is controlled by either P- or PQ-control, then the excessive power is transmitted from the island 1 to the island 2 (Figure 4.4). Hence, the mismatch is quickly reduced in both islands and the frequency is stabilised without any load shedding. From the Figures 4.3 and 4.4, we also see that PQ-control requires less active power to be transmitted and is able to stabilise the frequency more effectively.

Moreover, similar results are obtained for the voltage (Figure 4.5). While in Case 1.1 the voltage deviates by more than 10% from its nominal value, PQ-controlled VSCs effectively regulate the voltage at their buses. They bring it back to its nominal value almost immediately after the islanding by independently controlling the reactive power generation (Figure 4.6). Though voltage stability is rather a local phenomena, VSCs herewith assist the automatic voltage regulators of the generators so that the voltage stability in the island is further promoted.

Based on these results, we can conclude, that the active power control is necessary to stabilise the voltage. Even though P-control does not directly regulate the voltage at the VSC buses, it still reduces its maximal deviation after the islanding. On the other hand, we see that Q-control, alone, can not effectively stabilise the voltage. After
the islanding, the controller saturates very quickly as the maximal reactive power generation of 0.5 p.u. is achieved (Figure 4.6).

Obviously, neither can it assist in stabilising the frequency, and a load shedding is still necessary. According to the Figure 4.3, Q-control even increases the maximum frequency deviation in both islands.

Summarising the results of Case 1, we can conclude that active power control and, hence, either P- or PQ-control type is required to help stabilising the islands, hereby, preventing the load shedding. At this point, PQ-control seems to be more effective than any of the others. It not only requires less active power to be transmitted, but also can quickly bring the voltages back to its nominal value at the VSC buses. To strengthen this conclusion, we simulate the controlled islanding with a VSC-HVDC link of the larger, IEEE 39-system, presenting the results in the next Section.

4.2 Case 2

For Case 2, we simulate the controlled islanding of IEEE-39 system. The test system is split into two islands having different boundaries without (Case 2.1) and with (Case 2.2) a VSC-HVDC link, as it is depicted in Figures 4.7 and 4.8 respectively. Same as in Case 1.2, the link connects the islands allowing active power transmission and voltage control by the VSCs.

Even though the island boundaries are different in Cases 2.1 and 2.2, the non-coherent generator 1 is isolated from the larger part of the system in both cases. However, the island 2 enlarged in Case 2.2 results in an increased power mismatch denoted in Table 3.5.

Nonetheless, the dynamic simulation shows (Figure 4.9) that the increased mismatch can easily be mitigated by the active power transmission through the VSC-HVDC link (Figure 4.10). Same as in Case 1, P- and PQ-control allows to quickly stabilise the frequency in both islands. In contrast to the Q-control and the islanding without the VSC-HVDC link (Case 2.1), no load shedding is required, while the active power can be transmitted through the link.

To further compare controlled islanding results of the dynamic simulation, we represent the voltages at the buses 26 and 11 in Figure 4.11. We have chosen these voltages instead of the ones at the VSC terminals because they are assigned to island 1 and 2 respectively in, both, Case 2.1 and 2.2, allowing us a better comparison.

From the Figure 4.11, we see that the islanding leads to a significant voltage deviation in both islands (bus 26 in island 1 and bus 11 in island 2). At the same time, we observe that, both, P- and PQ-controlled VSC-HVDC links very much participate in voltage control of the islands. While P-control reduces the voltage drop solely by reducing the active power mismatch, PQ-control, being slightly more effective, additionally regulates the reactive power generation of the VSCs (Figure 4.12). In contrast, Q-control, alone, does not provide any improvement, as the controller is quickly saturated.

Summarising the results of Case 2, we can say that our previous conclusion about
Figure 4.1: Case 1.1: Controlled islanding

Figure 4.2: Case 1.2: Controlled islanding
the necessity of active power transmission and about the effectiveness of the different control types for stabilising the islands still holds for a bigger test-system. At last, we have to see if different VSC-HVDC links present in the system can counteract each other, impeding the stabilisation after the islanding. To do so, we go over to Case 3.

### 4.3 Case 3

In Case 3, we simulate the controlled islanding of IEEE-39 system again. However, this time, ICI algorithm has to split the system into three islands instead of two. Moreover, we place two VSC-HVDC links to see how they affect the controlled islanding, and if they can counteract each other in stabilising the islands.

Our ICI-algorithm splits the system without any VSC-HVDC links (Case 3.1) as it is shown in Figure 4.13. After placing two PQ-controlled VSC-HVDC links according to Table 3.5 (Case 3.2), the algorithm considers them, providing new island boundaries shown in Figure 4.14.

In both cases, we obtain two small generation-rich islands which isolate the included generators from the larger part of the system. Dynamic simulation shows (Figure 4.15), that without the VSC-HVDC links the frequency will steeply rise in these islands (2 and 3) so the generators will have to be tripped in order to stabilise the frequency.

In the same Figure 4.15, we see that the PQ-controlled VSC-HVDC links can quickly stabilise the frequency so that there is no need to either trip the generators, or shed a part of the load.

To show that a similar result holds for the voltage stability, we represent the voltages after the islanding at the buses 26, 41 and 45 in Figure 4.17. These buses belong to the same islands in, both, Cases 3.1 and 3.2 allowing us a better comparison. Again, we see that the VSC-HVDC links assist in stabilising the voltage in all three islands. Especially in island 1 that has two VSCs belonging to the different links, we can observe a significant improvement of voltage stability.

Therefore, we can conclude from Case 3 that even if several VSC-HVDC links are connecting an island to the others, the locally controlled VSCs will not counteract each other in neither stabilising the island in terms of frequency or voltage.
Figure 4.3: Case 1: COI frequency in island 1 (upper plot) and island 2 (lower plot)
Figure 4.4: Case 1: Active power of the VSC-HVDC link
Figure 4.5: Case 1: Voltage at bus 11 within the island 1 (upper plot) and at bus 13 within the island 2 (lower plot)
Figure 4.6: Case 1: Reactive power generated by the VSCs at bus 11(upper plot) and at bus 13(lower plot)
Figure 4.7: Case 2.1: Controlled islanding
Figure 4.8: Case 2.2: Controlled islanding
Figure 4.9: Case 2: COI frequency in island 1 (upper plot) and island 2 (lower plot)
Figure 4.10: Case 2: Active power of the VSC-HVDC link
Figure 4.11: Case 1: Voltage at bus 26 within the island 1 (upper plot) and at bus 11 within the island 2 (lower plot)
Figure 4.12: Case 2: Reactive power generated by the VSCs at bus 39 (upper plot) and at bus 44 (lower plot)
Figure 4.13: Case 3.1: Controlled islanding
Figure 4.14: Case 3.2: Controlled islanding
Figure 4.15: Case 3: COI frequencies in Case 3.1 (upper plot) and Case 3.2 (lower plot)
Figure 4.16: Case 3: Active power of the VSC-HVDC links
Figure 4.17: Case 3: Voltage at buses 26, 41 and 45 within the islands 1, 2 and 3 respectively. Case 3.1 is demonstrated in the upper plot. Case 3.2 is demonstrated in the lower plot.
5 Conclusion

In this study, we have shown that VSC-HVDC links can support the controlled islanding. The islanding will often result in a frequency and voltage instability within the islands. We have shown, that these negative consequences can be mitigated by the VSC-HVDC links while those can help stabilising the islands, both, in terms of frequency and voltage.

To show that this is the case, we have extended an existing ICI-approach so that it can consider the VSC-HVDC links present in the power system. Hereby, the island boundaries can be determined in such a way that the resulting islands are interconnected by the VSC-HVDC links. These links can then be used to help stabilising the islands.

This is, however, only possible with an appropriate control. For our study, we have introduced a control strategy of the VSC-HVDC links that allows to locally regulate the active and reactive power of the links in such a way, that they can effectively assist in the stabilisation of the islands they are connecting. Our control strategy also allowed to vary between different control types to see which one is the most effective.

In various simulations, we have demonstrated that either P- or PQ-control type can help stabilising the islands. Hereby, PQ turned out to be more effective as it can also quickly regulate the voltage at the VSC buses. Moreover, we have also shown that even simple PI controllers for, both, active and reactive power can prevent an eventual load shedding and stabilise the frequency and voltage very quickly after controlled islanding. Herewith, the negative consequences of the islanding are minimised for the consumers.

It is important to note, that in our study we did not simulate the disturbance that has lead to a controlled islanding. Therefore, an idea for a further investigation is to test our extended ICI-algorithm along-with a fine-tuned control strategy on larger systems to see which results can be obtained for a grid comparable in its size to a practical power system, given a realistic disturbance. Another idea, is to apply a centralised control strategy for a more effective stabilisation if numerous VSC-HVDC links are present in the system. As with this study, herewith, we will provide an additional argument for the controlled islanding as a novel emergency protective measure for the power systems of the future.
Bibliography


