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Active Coordination of Thermal Household Appliances for Load Management Purposes

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Abstract: In this paper, a coordination approach for thermostat-controlled household appliances is developed. Under consideration are heating and cooling devices such as refrigerators, freezers, water boilers and heat pumps, which are usually characterized by an intermittent (duty cycle) operation. Without influence from the outside, an internal hysteresis switching controller toggles the "on/off" state of a device when its temperature boundary is reached. In order to realize the approach proposed here, the devices are connected to a central control entity by two-way communication. The coordination consists of a step-wise heuristic solution of a binary optimization problem, which serves to select a number of devices in each time step that are subject to compulsory switching, i.e. toggling the "on/off" state. It allows a large group of devices to track a setpoint trajectory with its aggregated power consumption, acting like a distributed virtual energy storage, while the individual temperature bounds of the appliances are not violated. This behavior can be used for grid-control purposes, such as the provision of active power reserves. It will be shown that the group of appliances can be characterized by an approximate aggregated dynamical model consisting of a first-order differential equation together with an approximate aggregated nonlinear cost function penalizing the control actions. The developed methodology is evaluated in a numerical simulation with a small appliance cluster corresponding to a residential housing area.

Keywords: Electrical Appliances; Load Modeling; On-Off-Controllers; Load Regulation; Binary Control; Switching Algorithms.

1. INTRODUCTION AND MOTIVATION

The penetration of power systems with intermittent renewable energy sources such as wind and photovoltaic generation has been increasing rapidly during the recent years. In order to reduce greenhouse gas emissions, incentive schemes (such as feed-in tariffs) have been put into place in many countries, and renewable energy technologies tend to get cheaper over time. This suggests that the present trend will continue or even accelerate in the future, which causes the need for significant adaptation of power system operation. Although intermittent infeeds can be predicted with relatively high accuracy, which can be incorporated in the dispatch methodologies of conventional power plants, there is still a rising need for controllability of generation and/or consumption on various timescales, e.g. in order to account for the remaining infeed prediction errors.

The idea of Demand Side Management (DSM), although in principle known for decades, has been getting an increasing amount of attention during the last few years due to these developments. In its traditional form, DSM consists of the deactivation of certain loads during peak hours, e.g. through financial incentives for users (see Rasanen et al. [1995]) or remote-controlled by the electricity provider (e.g. by ripple control equipment). However, more sophisticated control tasks as e.g. the provision of ancillary services cannot be tackled with these concepts.

A number of approaches to load participation in grid control are based on a decentralized frequency-response mechanism for heating and cooling loads. The idea was already presented in Schweppe et al. [1980], and it also appears in recent publications like Short et al. [2007]. It constitutes an extension to the inherent frequency dependence of the load, which can reduce the need for primary control reserves. In these concepts, however, the impact of the control on the appliance duty cycle can be quite substantial and hard to quantify in advance. Furthermore, several studies suggest, e.g. for the German power system in Ernst et al. [1999], that the main impact of intermittent generation is not on the primary control reserves in the range of seconds but rather on larger timescales. This raises the question if the trade-off between a control contribution and the impact on the operation of a privately owned device is well-balanced in this case.

A lot of recent work in the field of non-frequency-based load management (among others Lu and Katipamula [2005], Kupzog [2006], Stadler et al. [2009]) is aimed at further increasing the controllability of heating and cooling loads. While the exploitable flexibility of the demand side is improved by the recent projects, as yet no approach is known that can impose an arbitrary curve shape, which may also be changed shortly before realization, on the aggregated active power demand of a large set of devices.

In this paper, a novel automatic load management methodology is developed which further extends the controllability of flexible loads [1]. It allows a high number of thermostat-(hysteresis-)controlled heating and cooling household appliances, equipped with two-way communication, to make an active contribution to power system control. This means that the control algorithm can freely and quickly increase or decrease the aggregated active power demand of a large group of appliances within certain limits, enabling the group to act like a virtual distributed energy storage. To avoid user comfort losses, the upper and lower temperature bounds of the appliances (switching thresholds) shall be respected at all times, and the alteration of the device duty cycle shall be kept as small as possible. Furthermore, the device must be able to function normally in the case of a communication failure.

The paper is organized as follows: section 2 briefly presents a possible communication infrastructure, while section 3 introduces a modeling framework for the appliances themselves. In section 4, steady-state properties of uncoordinated appliance groups are determined that are useful for the coordination approach, which is presented in section 5. Section 6 deals with the aggregated properties of coordinated appliance clusters, followed by numerical results in section 7. Some conclusions and ideas for further research are given in section 8.

2. COMMUNICATION INFRASTRUCTURE

During the course of the LLM project, the hardware and software for a two-way communication infrastructure is being developed, providing the link from the appliances to a control center. Within the household, the system is composed of two kinds of units: one central "Load Manager Household" (LMH) device and several "Load Manager Appliance" (LMA) units which are installed in the individual appliances, as depicted in Figure 1. Note that also a number of non-thermal appliances are shown here, which do not take part in the coordination approach outlined in this paper, but will be used in a decentralized load shedding scheme investigated in parallel. The in-house link between the LMAs and the LMH can be realized with Powerline Communication (PLC) according to Konnex PL-132. For the communication between the household and the control center, two alternatives are being considered: a low-voltage network PLC to the nearest transformer station combined with a subsequent transmission over a proprietary utility communication channel, or a TCP/IP transmission over a permanent internet connection installed in the household. For further details, see Koch et al. [2009].

[1] The outlined work is part of the project "Local Load Management" (LLM) which has been conducted by a team from ETH Zurich, University of Applied Sciences North-Western Switzerland (FHNW), Atel Netz AG and Landis+Gyr since 2006. The project is financially supported by swiss*electric research*. The current project phase is called "Electricity grid security and operation taking into account distributed loads, in-feeds and storages", which commenced in 2007. Its principal goals are the development of a suitable communication infrastructure for applying a sophisticated load management scheme in private households, algorithms for coordinated appliance operation, inclusion of storages and distributed generation, decentralized under-frequency load shedding, as well as economical considerations and strategies for the regulatory or market-based introduction of Local Load Management into today's electricity systems.

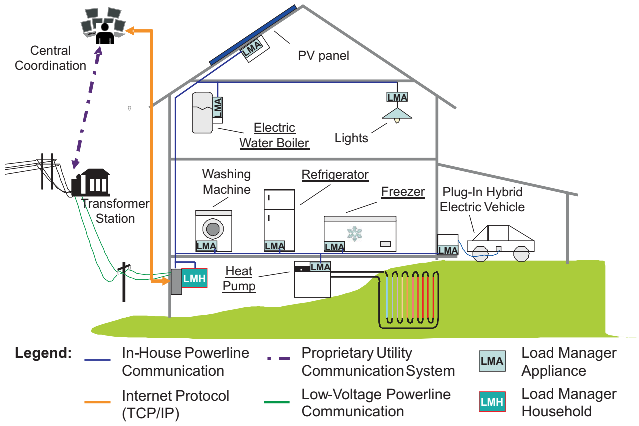

Fig. 1. Communication infrastructure in the household

3. THERMAL MODELING FRAMEWORK

In this section, a thermal modeling framework for the appliances under consideration is developed. For this purpose, a unified representation of heating and cooling devices of different nature is derived. It is assumed that all appliances are operated with a thermostat switching controller which turns the appliance "on" and "off" depending on its internal state.

3.1 Normalized expression of the appliance state

As in other known publications on thermostat-controlled heating and cooling appliances such as Bompard et al. [1996], Lu and Katipamula [2005], the dynamic state variable used here is the measured internal temperature T [°C]. In order to derive a unified description of the device state independent of the temperature level and device type, a description of the internal thermal energy relative to the ambient temperature T_{amb} [°C] can be used. This yields for cooling and heating devices:

$$E_{\text{th,cool}} = m \cdot \overline{c} \cdot (T_{\text{amb}} - T) \quad , \tag{1}$$

$$E_{\text{th,heat}} = m \cdot \overline{c} \cdot (T - T_{\text{amb}}) \quad , \tag{2}$$

where m [kg] represents the mass contained in the device and $\overline{c}$ $[\frac{\text{J}}{\text{kg K}}]$ the average heat capacity of the contents. The switching threshold temperatures of the appliance, T_{min} and T_{max} [°C], can be transformed similarly. The energy content E_{th} is now normalized to an interval of $[0, 1]$ using

$$E^{\text{rel}}_{\text{th,cool}} = \frac{T_{\text{max}} - T}{T_{\text{max}} - T_{\text{min}}} \quad , \tag{3}$$

$$E^{\text{rel}}_{\text{th,heat}} = \frac{T - T_{\text{min}}}{T_{\text{max}} - T_{\text{min}}} \quad . \tag{4}$$

The thermal energy of the ambiance is equal to zero in absolute terms (reference level) and usually negative in relative terms. The latter expression is obtained by substituting $T = T_{\text{amb}}$ in equations (3) and (4).

3.2 Dynamic appliance models

Now, the differential equations describing the evolution of the appliance state are introduced. Considered here are refrigerators, freezers, and water boilers.

Refrigerator and Freezer. The relative thermal energy content evolves according to

$$\frac{dE_{\text{th}}^{\text{rel}}}{dt} = -(\frac{1}{\tau} + \frac{1}{\tau_{\text{open}}} d)\,(E_{\text{th}}^{\text{rel}} - E_{\text{th, amb}}^{\text{rel}}) + \frac{k}{\tau}\,u\,, \quad (5)$$

where the thermal losses depend on the difference between the ambient energy level and the inside energy level. $u \in \{0, 1\}$ is the binary switching input variable. The time constant τ [s], the amplification factor k [-], and the initial condition of the differential equation are determined by

$$\tau = \frac{m\,\overline{c}}{A\,\overline{\alpha}} \quad , \quad (6)$$

$$k = \frac{\varepsilon_{\text{th}}\,P_{\text{el}}^{\text{rated}}}{A\,\overline{\alpha}\,(T_{\max} - T_{\min})} \quad , \quad (7)$$

$$E_{\text{th},0}^{\text{rel}} = E_{\text{th}}^{\text{rel}}(t = t_0) \quad , \quad (8)$$

with the average heat transfer coefficient $\overline{\alpha}$ $[\frac{W}{m^2\,K}]$ and the hull surface A [m^2]. ε_{th} is the coefficient of performance of the cooling aggregate (including the efficiency of the compressor), and $P_{\text{el}}^{\text{rated}}$ is the rated power consumption of the appliance [2]. Stochastic user interactions (door openings) are modeled by the disturbance input d (combined with the heuristic time constant τ_{open} during door-openings) and occasional variations of the time constant τ, which is linearly dependent on the mass.

Electric water boiler. For the water boiler, the governing differential equation looks very similar:

$$\frac{dE_{\text{th}}^{\text{rel}}}{dt} = -(\frac{1}{\tau} + \frac{\dot{m}_{\text{demand}}}{m})\,(E_{\text{th}}^{\text{rel}} - E_{\text{th, amb}}^{\text{rel}}) + \frac{k}{\tau}\,u\,, \quad (9)$$

where τ, k and $E_{\text{th},0}^{\text{rel}}$ are the same as defined in (6) - (8) and $\varepsilon_{\text{th}} = \eta_{\text{th}}$ is the thermal efficiency of the electric heating element. $\dot{m}_{\text{demand}}$ represents the mass flow of the water that is drawn from the boiler by the user, which is instantly replaced by fresh water assumed to enter the boiler at ambient temperature.

For all appliances modeled in the above way, a hysteresis switching controller acts on the input variable u which is expressed in the same normalized form:

$$u = \{ \begin{matrix} 1 & \text{if} & E_{\text{th}}^{\text{rel}} \leq 0 \\ 0 & \text{if} & E_{\text{th}}^{\text{rel}} \geq 1 \end{matrix} \quad . \quad (10)$$

3.3 Relation between thermal and electrical energy content

The relative thermal energy content, which evolves in the interval [0,1] during normal operation, represents a different span of actual thermal energy for each device between the upper and lower switching boundaries $T_{\max}$ and $T_{\min}$. This net energy is described by

$$E_{\text{th}}^{\text{net}} = m\,\overline{c}\,(T_{\max} - T_{\min}) \quad . \quad (11)$$

[2] For simplicity, the rated power is regarded as constant here, although it actually depends on the supply voltage. In the cooling appliances, active power transients during the "on" phase due to thermodynamic effects are also neglected. If desired, corresponding duty-cycle and voltage dependent factors can be included in k.

As the relation between thermal and electrical input power is given by

$$P_{\text{th}}^{\text{rated}} = \varepsilon_{\text{th}}\,P_{\text{el}}^{\text{rated}} \quad , \quad (12)$$

the same can be stated for the thermal and electrical energy contents. Taking into account equations (7) and (11), it is easy to show that the net electrical energy span between the two switching boundaries is

$$E_{\text{el}}^{\text{net}} = \frac{\tau}{k}\,P_{\text{el}}^{\text{rated}} \quad . \quad (13)$$

Furthermore, it can be shown using the results from equations (1) – (4) that the electrical and thermal relative energy contents are the same:

$$E_{\text{el}}^{\text{rel}} = E_{\text{th}}^{\text{rel}} \quad . \quad (14)$$

4. STEADY-STATE PROPERTIES IN UNCOORDINATED OPERATION

In order to characterize a group of several hundred appliances in steady-state uncoordinated operation, stochastic considerations can be made which will be useful when developing the coordination approach. The goal here is to derive the steady-state mean value and standard deviation of the overall power consumption of the appliance cluster depending on certain appliance properties.

The power consumption behavior of a switch-controlled appliance, which takes part in a large cluster, can be expressed as a Bernoulli-distributed stochastic variable $X \in \{0, 1\}$ which is scaled by the rated power consumption $P_{\text{el}}^{\text{rated}}$ [W] of the device:

$$P_{\text{el}} = P_{\text{el}}^{\text{rated}} \cdot X \quad . \quad (15)$$

Note that this representation does not include any information about the duty cycle length, which is not of interest here. The probability of the device being switched on ($p[X = 1]$) is equal to a value p_{on} which can be derived by taking into account the daily energy consumption $W_{\text{el}}^{\text{daily}}$ [kWh/d] of the device:

$$p_{\text{on}} = (\frac{1000}{24\text{ h}}\,W_{\text{el}}^{\text{daily}})/(P_{\text{el}}^{\text{rated}}) \quad . \quad (16)$$

Based on straight-forward stochastics, it is possible to derive the expected value $\mathrm{E}[X]$ and the variance $\mathrm{Var}[X]$ for a weighted sum of Bernoulli distributions representing n thermal appliances. As the appliances are operated independently of each other, the covariances between them can be assumed equal to zero. Thus, a diverse set of n uncoordinated appliances, each of which is here defined by the tuple $(P_{\text{el},i}^{\text{rated}}, p_{\text{on},i})$ with $i = 1 \ldots n$, has the following aggregated stochastic characteristics:

$$P_{\text{el,mean,unc}}^{\text{total}} = \mathrm{E}\left[\sum_{i=1}^{n} P_{\text{el},i}^{\text{rated}}\,X_i\right] = \sum_{i=1}^{n} P_{\text{el},i}^{\text{rated}}\,p_{\text{on},i}\,, \quad (17)$$

$$\mathrm{Var}\left[P_{\text{el, unc}}^{\text{total}}\right] = \mathrm{Var}\left[\sum_{i=1}^{n} P_{\text{el},i}^{\text{rated}}\,X_i\right] \quad (18)$$

$$= \sum_{i=1}^{n} (P_{\text{el},i}^{\text{rated}})^2\,p_{\text{on},i}\,(1 - p_{\text{on},i}) \quad . \quad (19)$$

For obtaining the average electrical energy content of the cluster, the mean value of each relative thermal energy evolution can be calculated by integrating the time solution of equations (5) and (9), the stochastic user interaction terms set to zero. Supposing again that the duty cycles of all the appliances are uncorrelated, the mean value of the average total electrical energy can be expressed as

$$E_{\mathrm{el,mean,unc}}^{\mathrm{total}} = \sum_{i=1}^{n} E_{\mathrm{el},i}^{\mathrm{net}}\, E_{\mathrm{el,mean},i}^{\mathrm{rel}} \approx 0.5\, E_{\mathrm{el,max}}^{\mathrm{total}} \quad .(20)$$

5. COORDINATED OPERATION

5.1 The control rationale

The proposed strategy for coordinating the appliance cluster consists of a local and a global computation module which exchange information through a defined interface. This interface consists of a momentary price offer from the appliance to the control center, which would be paid if the appliance was to toggle its current "on/off" state in that very instant, instead of later by its own switching controller. The algorithm running in the control center can then "accept" the offer at a specific instant, sending a switching impulse to the device and paying the switching cost to the customer. Thus, two different computation tasks have to be solved: a calculation of a momentary switching price curve on the device level, and a time-step-wise global selection of devices to be switched compulsorily, taking into account their price offers. A suitable way of calculating the switching offers within the household is regarded in detail in Koch et al. [2009] as well as the distribution of the calculation tasks on the Load Manager Appliance and Load Manager Household. Further details about this are beyond the scope of this paper.

To calculate the switching price of a single appliance, the expression

$$c_{\mathrm{sw}}(t) = \lambda_{\mathrm{sw}}(t)\, P_{\mathrm{el}}^{\mathrm{rated}} = \lambda_{\mathrm{p}}\, \frac{\Delta t_{\mathrm{dwell}}(t)}{\Delta t_{\mathrm{dwell}}^{\mathrm{full}}(t)}\, P_{\mathrm{el}}^{\mathrm{rated}} \qquad (21)$$

is proposed, where $c_{\mathrm{sw}}(t)$ [cent] is the current switching price offer, λ_{p} [cent/W] is a fixed price for a change in power consumption of the cluster (which can be specific for a certain appliance type), $\Delta t_{\mathrm{dwell}}(t)$ [h] is the estimated dwell time that the appliance will still remain in its current "on/off" state before reaching a switching boundary and $\Delta t_{\mathrm{dwell}}^{\mathrm{full}}(t)$ [h] is the full dwell time in the current state (including the already elapsed time).

In the discussions about "smart" homes, "smart" electricity grids and "smart" metering, concerns about user privacy and data protection have emerged which need to be addressed accordingly. Under the requirement that an individual household participating in the Local Load Management scheme should not be transparent to the operating company in terms of size, number, and state of installed appliances, an individual appliance addressing should be avoided. This can be achieved as follows: The individual switching impulses sent to a number of appliances in every optimization time step can be replaced by the transmission of an accepted switching ("clearing") price, to which the appliances react autonomously. Thus, any appliance offering a lower price than the accepted one is switched by a local decision. This avoids the need for the individualized collection of data. Furthermore, communication requirements are much lower in a "broadcast" scheme compared to an individual addressing.

5.2 Selection of the accepted switching prices

The global optimization algorithm is running in discrete time steps t_k with the fixed time step size $h_{\mathrm{opt}} = t_{k+1} - t_k$. Note that the appliances can in principle switch their "on/off" state asynchronously within the time span h_{opt}, which may lead to significant deviations from the power setpoint and cannot be influenced until the next optimization time step t_{k+1}. This is avoided in the following way: all appliances that are going to switch in the upcoming time interval ($\Delta t_{\mathrm{dwell}}(t_k) < h_{\mathrm{opt}}$) are switched compulsorily and "slightly prematurely" in the current optimization step t_k.

The algorithm being executed in each time step t_k thus has to aggregate all the relevant information and calculate the accepted relative switching prices (clearing prices) λ_{on}^* and λ_{off}^* [cent/W]. The price determination resembles a "pay-as-bid" auction where all the appliances are paid according to their submitted bids when the offer is accepted, not according to the clearing price.

The calculation of the clearing prices is somewhat similar to a "priority list" methodology for unit commitment as outlined in Senjyu et al. [2002]. After assessing the need for a power reduction or increase in the current time step, the relative switching price offers λ_{sw} are grouped into "on" and "off" appliances and sorted in ascending order. In this way, a "merit order" over the cumulated sum of the sorted rated powers is constructed for both "on" and "off" appliances. This allows the determination of clearing prices that lead to a certain power change.

The algorithm is described below in detail:

(1) Assemble the information: build the cluster vectors of "on/off" states $\mathbf{u}$, dwell times $\mathbf{\Delta t}_{\mathrm{dwell}}$ and relative switching prices λ_{sw}. Split λ_{sw} according to $\mathbf{u}$ to distinguish whether the device *could be* switched on or off:

$$\lambda_{\mathrm{on}} = \lambda_{\mathrm{sw}}(\mathbf{u}=0)\,, \quad \lambda_{\mathrm{off}} = \lambda_{\mathrm{sw}}(\mathbf{u}=1) \quad . \quad (22)$$

(2) Predict the switchings that will take place in the upcoming time step: $\mathbf{\Delta t}_{\mathrm{dwell}} < h_{\mathrm{opt}}$ (element-wise).

(3) Using that, calculate the minimum accepted relative switching prices that have to be paid in order to prevent asynchronous switching:

$$\lambda_{\mathrm{on,min}}^* = \max \lambda_{\mathrm{on}}(\mathbf{\Delta t}_{\mathrm{dwell}} < h_{\mathrm{opt}}) \quad , \quad (23)$$

$$\lambda_{\mathrm{off,min}}^* = \max \lambda_{\mathrm{off}}(\mathbf{\Delta t}_{\mathrm{dwell}} < h_{\mathrm{opt}}) \quad , \quad (24)$$

and the corresponding consumption change $P_{\mathrm{el,sync}}^{\mathrm{total}}$.

(4) Compare the current consumption with the power setpoint $P_{\mathrm{el,set}}^{\mathrm{total}}$, taking into account the switchings caused by the prevention of asynchronous behavior in order to determine the required power to be switched additionally:

$$P_{\mathrm{el,req}}^{\mathrm{total}} = P_{\mathrm{el,set}}^{\mathrm{total}} - P_{\mathrm{el,current}}^{\mathrm{total}} - P_{\mathrm{el,sync}}^{\mathrm{total}} \quad . \tag{25}$$

(5) Construct the merit orders for positive and negative compulsory switchings by sorting the price offers and building the cumulative power change curves that can be achieved (”avd”) by a certain clearing price: $\lambda_{\mathrm{on}}^{*}(P_{\mathrm{el,on,avd}}^{\mathrm{total}})$ and $\lambda_{\mathrm{off}}^{*}(P_{\mathrm{el,off,avd}}^{\mathrm{total}})$.

(6) Approximate the required power change by determining accepted switching prices for ”on” and ”off” switching actions (while observing the minimum values computed in step (3)) with

$$P_{\mathrm{el,avd}}^{\mathrm{total}} = P_{\mathrm{el,on,avd}}^{\mathrm{total}}(\lambda_{\mathrm{on}}^{*}) - P_{\mathrm{el,off,avd}}^{\mathrm{total}}(\lambda_{\mathrm{off}}^{*}) \tag{26}$$

by ”walking up the merit order” in the required direction such that $P_{\mathrm{el,avd}}^{\mathrm{total}} \approx P_{\mathrm{el,req}}^{\mathrm{total}}$.

(7) Broadcast the two clearing prices to all devices which then toggle their state if their own bid is lower than the relevant clearing price and log the paid sums $c_{\mathrm{SW}}(t_k) = \lambda_{\mathrm{SW}}\, P_{\mathrm{el}}^{\mathrm{rated}}$ locally in the household.

6. APPROXIMATELY EQUIVALENT DYNAMICAL MODEL AND COST FUNCTION

In this section, an approximation for the coordinated appliance cluster as a whole will be derived which enables it to be operated as a single entity showing quasi-continuous behavior, represented by a first-order dynamical system and an associated cost function.

6.1 Approximate first-order cluster representation

First, the dynamical behavior of the ”energy storage content” of the appliance cluster depending on the consumed electrical power will be analyzed, which is essential to assess the ability of the cluster to perform a certain control action without reaching the storage limits. For this purpose, a differential equation of the form

$$\dot{E}_{\mathrm{el}}^{\mathrm{total}} = f(E_{\mathrm{el}}^{\mathrm{total}}, P_{\mathrm{el}}^{\mathrm{total}}) \tag{27}$$

is sought. Note that the stochastic user interactions with the thermal appliances are neglected for the moment in order to derive a clear understanding of the deterministic dynamic behavior. It is easy to see that the current total electrical energy content of the distributed energy storage can be described by the weighted sum

$$E_{\mathrm{el}}^{\mathrm{total}} = \sum_{i=1}^{n} E_{\mathrm{el},i} = \sum_{i=1}^{n} E_{\mathrm{el},i}^{\mathrm{rel}}\, E_{\mathrm{el},i}^{\mathrm{net}} \quad . \tag{28}$$

Consequently, the overall electrical storage capacity $E_{\mathrm{el,max}}^{\mathrm{total}}$ is obtained by setting $E_{\mathrm{el},i}^{\mathrm{rel}} = 1$ for $i = 1, \ldots, n$. The evolution of the storage content over time can be seen when all the participating appliances, represented by a set of parameterized equations (5) and (9), are combined with equation (28). This leads to

$$\dot{E}_{\mathrm{el}}^{\mathrm{total}} = \sum_{i=1}^{n} \left(\frac{-1}{\tau_i} (E_{\mathrm{el},i} - E_{\mathrm{el,amb},i}) + P_{\mathrm{el},i}^{\mathrm{rated}}\, u_i \right) . \tag{29}$$

Note that, because of the presence of a weighted sum in (29), it is not easily possible to derive a closed-form differential equation which allows a direct solution in time. However, the following reasoning can be applied: It is supposed that the differential equation below shall represent the dynamic behavior of the appliance cluster:

$$\dot{E}_{\mathrm{el}}^{\mathrm{total}} = -\frac{1}{\overline{\tau}} (E_{\mathrm{el}}^{\mathrm{total}} - E_{\mathrm{el,amb}}^{\mathrm{total}}) + P_{\mathrm{el}}^{\mathrm{total}} \quad , \tag{30}$$

for which the parameters have to be determined. The weighted sum from (29) has been substituted by ordinary sums forming $E_{\mathrm{el}}^{\mathrm{total}}$ and $E_{\mathrm{el,amb}}^{\mathrm{total}}$ and a mean approximate cluster time constant $\overline{\tau}$. This time constant can be obtained by considering the steady-state solution of (30), which yields

$$\overline{\tau} = (E_{\mathrm{el,ss}}^{\mathrm{total}} - E_{\mathrm{el,amb}}^{\mathrm{total}})/(P_{\mathrm{el,ss}}^{\mathrm{total}}) \quad , \tag{31}$$

where the stochastically derived values from equations (17) and (20) can be inserted as the steady-state quantities. The initial condition to (30) is given by substituting $E_{\mathrm{el},i}^{\mathrm{rel}} = E_{\mathrm{el},0,i}^{\mathrm{rel}}$ for $i = 1 \ldots n$ in (28). This completes the aggregated first-order representation of the cluster behavior.

6.2 Approximation of switching costs

To conclude the theoretical part, an approximate cost function for the appliance cluster is presented which describes the overall switching cost depending on the energy level $E_{\mathrm{el}}^{\mathrm{total,rel}} = E_{\mathrm{el}}^{\mathrm{total}}/E_{\mathrm{el,max}}^{\mathrm{total}}$ of the cluster. The approximation is based on the following statements:

(1) A certain amount of switching cost $\dot{c}_{\mathrm{tight}}$ is caused by the tightness of the control even when the cluster is kept at its expected value in steady state operation.

(2) The switching cost for individual cluster consumption power changes is negligible compared to the steady-state cost incurred by keeping the cluster at a certain energy level.

(3) The uncoordinated average energy content value of the cluster is approximately $E_{\mathrm{el}}^{\mathrm{total,rel}} = 0.5$.

(4) The coordination effect is similar to moving the lower switching threshold for $E_{\mathrm{el}}^{\mathrm{rel}}$ upwards when $E_{\mathrm{el}}^{\mathrm{total,rel}} > 0.5$ and the upper switching threshold downwards for $E_{\mathrm{el}}^{\mathrm{total,rel}} < 0.5$ such that the average value of all $E_{\mathrm{el},i}^{\mathrm{rel}}$ is equal to $E_{\mathrm{el}}^{\mathrm{total,rel}}$.

(5) In this case, the overall duty cycle time of all appliances i, $\Delta t_{\mathrm{cycle},i}^{\mathrm{full}}$ [h], decreases approximately linearly with a decline of $2\,|E_{\mathrm{el}}^{\mathrm{total,rel}} - 0.5|$ compared to uncoordinated operation and the necessary switchings increase thus by $1/\Delta t_{\mathrm{cycle},i}^{\mathrm{full}}$.

Considering these points, the mean cost per time caused by the entire appliance cluster at a certain energy level can be approximated as

$$\frac{dc}{dt} = \dot{c}_{\mathrm{tight}} + \sum_{i=1}^{n} \frac{P_{\mathrm{el},i}^{\mathrm{rated}}}{\Delta t_{\mathrm{cycle},i}^{\mathrm{full}}} \cdot \frac{2\,|E_{\mathrm{el}}^{\mathrm{total,rel}} - 0.5|}{1 - 2\,|E_{\mathrm{el}}^{\mathrm{total,rel}} - 0.5|} \lambda_{\mathrm{p}} \,. \tag{32}$$

For a better approximation, the left and right branches of this curve can be scaled by different weight factors w_{lower} and w_{upper} determined e.g. by least-squares approximation techniques. Equation (32) can be integrated piece-wise for piece-wise constant values of $E_{\text{el}}^{\text{total,rel}}$ in order to yield an approximation for the accumulated switching cost incurred by the evolution of the cluster energy content.

7. NUMERICAL RESULTS

Finally, the proposed control strategy is tested with an appliance cluster of 300 refrigerators, 200 freezers and 100 water boilers (total installed power of 385.4 kW, total electrical storage capacity between the temperature bounds of 27.77 kWh), simulated over six hours. A variable simulation step size and zero-crossing detection is used, the optimization step size is equal to $h_{\text{opt}} = 0.01$ h. The group is created with statistically distributed parameter sets and it is very diverse in terms of time constants (minimum 1.8 h, maximum 43.8 h), rated power (minimum 100 W, maximum 4 kW) and duty cycle times (minimum 4.8 min, max 3.2 h). In Figure 2, a power consumption setpoint trajectory is imposed on the cluster. The setpoint is equal to the expected cluster power consumption value (46.84 kW) in the beginning, which is then decreased by 40 % for half an hour from $t = 1$ h and increased in the same way from $t = 2$ h. From $t = 4$ h, a sinusoid is imposed to show that arbitrary curve shapes can be tracked. The plots show (top to bottom) the individual relative energy contents of selected representative appliances within the cluster (demonstrating the effect of the "earlier-than-normal" switchings on individual appliances), the aggregated electrical energy storage content between the upper and lower bounds, the aggregated cluster power consumption and its setpoint, the accumulated switching cost and the instantaneous switching costs over time (with λ_{P} arbitrarily set to 2 cent/kW). It can be seen that the control algorithm keeps the overall power consumption close to the setpoint, and that the approximations developed in section 6 are valid.

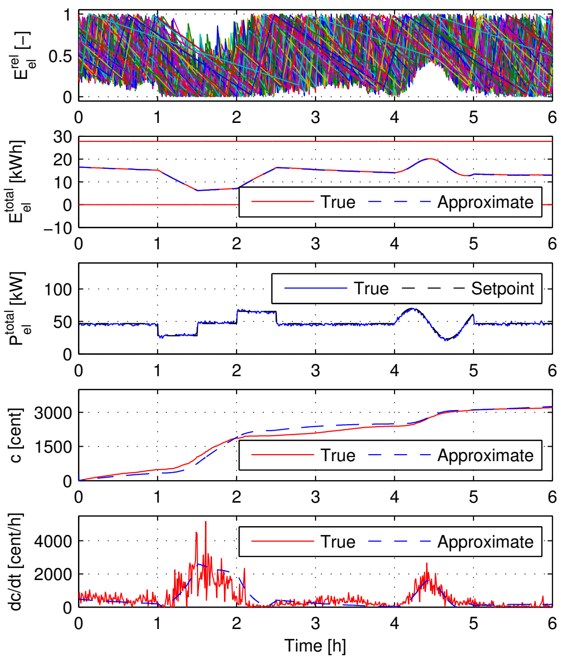

Fig. 2. Coordinated cluster of 600 appliances

8. CONCLUSION AND OUTLOOK

This paper demonstrated the ability of large groups of thermal household appliances, coordinated by a novel control methodology, to act like a distributed energy storage. Further research will address the voltage-dependency of the loads, the active power transients in compressor-operated cooling appliances, effects of stochastic user interactions and the evaluation of different switching cost function designs. Furthermore, the possibility to transform the aggregated cost function into easier (e.g. quadratic) forms will be explored. This may facilitate the incorporation of coordinated appliance clusters in economic dispatch problems, e.g. for the utilization by ancillary service providers or in decentralized energy management systems.

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