Abstract—The conversion of multicircuit ac transmission lines to hybrid ac/dc lines is a promising way of substantially increasing transmission capacities in areas where it is difficult to obtain new rights of way. Critical questions related to such a conversion include the impact on the electric fields and ion currents at the ground level, as well as the dc current coupling into the ac phases. In this paper, a new procedure based on the method of characteristics is proposed for solving the fully coupled ion-flow problem. The method does not make the Deutsch assumption and is fast and stable even for high assumed wind speeds. A hybrid tower is simulated with different voltages and wind speeds. The results show the influence of the onset gradient, the dc line voltage, and the wind speed on the current coupling as well as the electric field and ion current density at the ground level. It is shown that the new proposed method can serve as the tool for dimensioning real hybrid towers.

Index Terms—Corona, electric fields, finite-element methods (FEMs), HVDC transmission lines, hybrid ac/dc transmission lines, ion current, method of characteristics, wind.

NOMENCLATURE

BC Boundary conditions.
BVP Boundary value problem.
CSM Charge simulation method.
DG-FEM Discontinuous Galerkin FEM.
FEM Finite-element method.
FVM Finite volume method.
ODE Ordinary differential equation.
PDE Partial differential equation.
SUPG Streamline-upwind/Petrov–Galerkin.

I. INTRODUCTION

ALTHOUGH the idea of hybrid ac/dc transmission on the same tower was proposed several decades ago [1], it has not yet been implemented. For this reason, the simulation of the performance of this technology is still a relevant problem. The impact of ions produced by corona discharges on the electric field cannot be neglected and, thus, the fully coupled ion-flow problem needs to be solved.

The electric field and the ion current at the ground are important environmental criteria and will likely be subject to regulations. It is well established that the electric field is increased by space charges [2]. The current coupling created by the ion flow is also important. An ac ion current can be induced in the dc poles by the ac electric-field component at the poles and by ions drifting from the ac wires. But this current is usually smaller than the displacement current computed from the capacitance matrix, and the converter station can tolerate small ac components. A dc current offset produced by the ions in the ac phases is much more critical because it could drive transformers into saturation. The dc offset can be induced by the dc-field component at the ac conductors or by the ions drifting from the poles. It has also been shown that ac conductors placed below dc conductors can act as a shield and reduce the ground-level electric field at the cost of increased dc current components in the ac phases [3], [4]. Since certain experimental studies suggest that the corona onset gradient for rain is smaller for dc conductors [4], [5] and since the dc field offset at the ac conductors is very small, the drift of the ions is the dominating cause of the dc current offset.

Many different configurations are possible with regard to the placement of the ac and dc conductors on a hybrid tower (e.g., dc at the top of the tower, ac and dc on either sides, etc.). Other important parameters are the voltages level, the wind speed, and the assumed corona onset gradient (which is primarily a function of the weather condition). Therefore, a very fast and stable simulation method that is able to simulate hundreds of parameter permutations without the need for guessing suitable initial values is required. It has been shown that assuming grounded ac conductors is a good approximation for calculating the dc field. This approximation has been investigated through comparison with the fully coupled transient solution [4] and through the analysis of the drift lines [6], [7].

Over the years, different methods have been proposed for solving the ion-flow problem. The Deutsch assumption that states that the direction of the electric field is not influenced by the ions is one of the oldest methods [8], [9]. Even though the Deutsch assumption has been criticized [10], [11], it is still frequently used for solving the ion-flow problem [12], [13]. Other methods that have been used are based on the charge simulation method (CSM) [14].
Methods based on the finite-element method (FEM) have also been used [2] for the stationary problem. Recently, the transient solution for hybrid transmission lines has also been obtained with FEM [4], [15] and with FVM [16]–[18]. The FEM requires a stabilization method for the flow problem (discontinuous Galerkin (DG-FEM), SUPG, etc.). Furthermore, slow ramp up of the voltage is necessary [4]. The computation time and stability of the problem with an external velocity field (wind) are also very problematic [19].

In this paper, a new algorithm based on the method of characteristics is presented for solving the stationary ion flow of hybrid lines. The method can be compared to the flux tracing method presented by Maruvada [9], but it does not make the Deutsch assumption (fully coupled) and can handle the bipolar problem with wind. After the presentation of the ion-flow problem, the numerical method is presented and compared to the fully coupled DG-FEM and to the Deutsch method for a reduced-scale problem. Finally, the fields at the ground level and current coupling are examined for a full-scale tower for different voltages, onset gradients, and wind speeds.

II. BIPOLAR ION-FLOW FIELD

A. Equations

The ion-flow problem can be expressed with partial differential equations [4], [9]. The relation between the positive and negative space-charge densities \( \rho^+ \) and \( \rho^- \) and the electric potential \( \phi \) is determined by Poisson’s equation

\[ \nabla \cdot (\nabla \phi) = \Delta \phi = -\frac{\rho^+ - \rho^-}{\varepsilon} \]  

(1a)

\[ E = -\nabla \phi. \]  

(1b)

The ion current depends on the mobility of the ions in the electric field \( \mu^+ = 1.2 \times 10^{-4} \text{ m}^2/(\text{V} \cdot \text{s}) \) and \( \mu^- = 1.5 \times 10^{-4} \text{ m}^2/(\text{V} \cdot \text{s}) \), the external velocity field (for example, wind) \( v_{\text{gas}} \) and on the space-charge density. The total velocity is abbreviated as \( \beta^\pm \)

\[ j^\pm = (\pm \mu^\pm \cdot E + v_{\text{gas}}) \cdot \rho^\pm = \beta^\pm \cdot \rho^\pm. \]  

(2)

The total current is \( j = j^+ - j^- \) since the defined current is the flow current and not the electrical current. The charge conservation applies (with recombination between the positive and negative ions) leading to the ion current continuity equations

\[ \nabla \cdot j^\pm = -\frac{\partial \rho^\pm}{\partial t} - \frac{R}{e} \cdot \rho^\pm \cdot \rho \]  

(3)

where \( R = 1.8 \times 10^{-12} \text{ m}^2/\text{s} \) is the chosen recombination coefficient and \( e \) is the electron charge. Equations (1)–(3) specify the fully coupled ion-flow problem under some common assumptions as follows.  

• The diffusion is neglected, and the ion mobilities are accepted to be constant. From the Einstein relation [20], the diffusion coefficient is small with respect to the mobility: \( D/\mu = 0.0253 \text{ V} \) (at 293 K). The diffusion can be neglected if \( D/\mu \ll V \) where \( V \) is the applied voltage (1-D case). This condition is clearly respected for an overhead line. Equations (2) and (3) then represent a pure transport problem.

B. Boundary Conditions

The obtained system of equations is a third-order nonlinear PDE. Poisson’s equation is elliptic, and the transport equation is hyperbolic and, thus, a careful choice of the boundary conditions is necessary. For Poisson’s equations, the conductor voltages and the ground potential are fixed. An artificial bounding box is added for the solution with Neumann or Dirichlet constraints. This bounding box is mapped to infinity with a coordinate transformation.

For the transport problem, the boundary condition can be set for the electric field or for the charge density at the conductor’s boundaries. Attten has shown [21] that setting the BC on the electric field can lead to significant problems. The authors also observed that totally unphysical solutions can be found that almost respect the BC on the electric field, meaning that the problem is numerically unstable. For these reasons, the condition of the charge density should be used for the drift PDE. Nonreflecting boundary conditions are required; otherwise, the problem is overdetermined. For a domain \( \Omega \), the following BCs are set:

\[ \rho^\pm = \rho_{\text{bc}}^\pm \text{ on } \{ \mathbf{x} \in \partial \Omega_{\text{conductor}} : \mathbf{B}^\pm \cdot \mathbf{n} < 0 \} \]

\[ \rho^\pm = 0 \text{ on } \{ \mathbf{x} \in \partial \Omega_{\text{ext}} \mid \mathbf{B}^\pm \cdot \mathbf{n} < 0 \} \]  

(4)

where \( \mathbf{n} \) is the normal vector (point outside the domain from the boundary). The value \( \rho_{\text{bc}}^\pm \) is imposed as a BC. Since there are no discharges at the artificial bounding box (and at the ground), the charge is always zero at the boundary \( \partial \Omega_{\text{ext}} \).

However, it is very difficult to choose the charge density at the conductor boundary so that the results are predictable. For this reason, the onset gradient is chosen with an initial guess for the charge density. Afterwards, the charge density values in return are determined iteratively so that a conductor surface gradient equivalent to the corona onset gradient is obtained (cf. Section III-E). The overall boundary condition for the charge injection at the conductor is thus the onset gradient.

III. ITERATIVE METHOD OF CHARACTERISTICS

A. Method Presentation

The proposed method, called the iterative method of characteristics (IMoC), follows the same philosophy as the flux tracing method presented by Maruvada [9], [12], [22]. The key advantage of the proposed procedure is that the Deutsch assumption is not necessary. The direction of the electric field is influenced by the ions. An iterative method has been proposed in [23] by Davis and Hoburg. This method combines FEM for solving the
Poisson equation and the method of characteristics for solving the flow problem but has several limitations:

- not adapted for setting the boundary conditions required for the corona discharges;
- restricted to the unipolar ion-flow field; for a bipolar line, the method can only be used to compute the space-charge density at the ground level; the method is an approximation since the discharges between the poles are neglected and the field lines are not exact;
- the current coupling cannot be extracted (unipolar flow field).

The IMoC method can handle the fully coupled bipolar ion-flow problem for overhead lines. In addition, an external velocity field can be added for handling the impact of wind. The only limitation of the IMoC implementation is the fact that it only works for stationary problems. However, there are no fundamental barriers for a transient implementation in the future.

The basic idea of IMoC is to iterate between the Poisson and the ion-flow equations. The simplified flow diagram is summarized in Fig. 1 as follows.

1) At the beginning, the positive and negative space-charge densities are set to zero. Then, Laplace’s equation is solved with FEM (or with any other method).
2) A certain number of characteristic lines (drift lines or flux lines) are extracted so that the complete domain is covered. This can be seen like a meshing with lines (the mesh is done in the direction of the transport).
3) The charge density at the boundaries of the conductors [see (4)] is chosen with respect to the chosen corona model.
4) The ion-flow problem is solved on the obtained characteristic lines, leading to a solution of the charge distribution with respect to the considered electric field. But at this point, the back coupling of the charge distribution on the electric field (Laplace field) is not considered, such that the solution is not exact.
5) The space-charge densities (known on the characteristic lines) are interpolated on the FEM mesh. The Poisson’s equation (and no longer the Laplace) is solved with the same BC as the Laplace’s equation. After the FEM solution process, the new characteristic lines can be extracted.
6) The procedure is repeated until convergence is reached (based on comparing the deviation between two iterations). If the solution remains stable between two iterations, it means that the solution of the ion flow obtained with the method of characteristics does not only respect the continuity equations but also Poisson’s equation.

B. Method of Characteristics

The electric field, velocity field, and the BC for the charge density on the line are known, and the charge density along the line needs to be solved. For solving the transport problem, the flux lines are classified into three different categories as follows.

1) Unipolar lines without wind. Such a line starts at the boundary of a conductor. The end of the lines can be at the ground plane or at the artificial bounding box. Since the ions are drifting along the characteristics, only one polarity is present on these lines: the polarity of the discharges that can occur at the conductor.
2) Bipolar lines without wind. The lines start and end at two conductors of different polarities. Therefore, positive ions can be created by one conductor and negative ions by the other.
3) Lines with wind. The case with wind should be treated separately. This comes from the fact that the drift lines are not the same for both polarities due to different ions’ mobilities and drift directions [see (2)]. Even for a bipolar region, the problem should be solved separately for the positive and negative ions (with recombination). Then, all of the characteristics are solved for one polarity line even if the region contains both polarities.

C. Case With Wind

Here, the case of the lines with wind is presented (the other types of lines are special cases of this category). The divergence of total velocity can be written as (from (2) and (3))

$$\nabla \cdot \{\vec{\beta}^\pm \cdot \rho^\pm\} = -\frac{R}{\epsilon} \cdot \rho^+ \cdot \rho^- \cdot \rho^-.$$

(Splitting the divergence operator yields

$$\rho^\pm \cdot \left(\nabla \cdot \vec{\beta}^\pm\right) \pm \left(\nabla \rho^\pm\right) \cdot \vec{\beta}^\pm = -\frac{R}{\epsilon} \cdot \rho^+ \cdot \rho^- \cdot \rho^-.$$

If the wind is curl free, the divergence of the velocity field can be written with (1)

$$\nabla \cdot \vec{\beta}^\pm = \nabla \cdot \left(\pm \mu^\pm \cdot E + v_{gs}\right) = \pm \mu^\pm \cdot \frac{\rho^+ - \rho^-}{\epsilon} \cdot \frac{R}{\epsilon} \cdot \rho^+ \cdot \rho^- \cdot \rho^-.$$

Substituting (7) in (6) gives

\[
(\nabla \rho^\pm) \cdot \vec{\beta}^\pm = -\rho^\pm \cdot \left(\pm \mu^\pm \cdot \frac{\rho^+ - \rho^-}{\epsilon} \cdot \frac{R}{\epsilon} \cdot \rho^+ \cdot \rho^- \cdot \rho^-\right).
\]
If the equation is solved along the flux lines, then the scalar product with the velocity field can be replaced and the following ODE is derived:

\[
\frac{d\rho^\pm}{ds} = +\rho^\pm \cdot \frac{\mu^\pm \cdot (\rho^+ - \rho^-)}{\varepsilon \cdot \beta^\pm} - \frac{R}{\varepsilon \cdot \beta^\pm} \cdot \rho^+ \cdot \rho^-.
\] (9)

The obtained ODE has the form

\[
\frac{dy(t)}{dt} = c(t) \cdot y(t)^2 + d(t) \cdot y(t)
\] (10a)

\[
y(t) = y_0.
\] (10b)

This ODE can be expressed explicitly and a vectorial solver can be used

\[
C_1(t) = \int_0^t d(s) \, ds
\] (11a)

\[
C_2(t) = \int_0^t - \exp(C_1(s) \cdot c(s)) \, ds
\] (11b)

\[
y(t) = \frac{y_0 \cdot \exp[C_1(t)]}{1 + y_0 \cdot C_2(t)}.
\] (11c)

After a coefficient matching the following parameters can be obtained for the ion-flow problem

\[
c^\pm = -\frac{\mu^\pm}{\varepsilon \cdot \beta^\pm}
\] (12a)

\[
d^\pm = \frac{\mu^+ \cdot \rho^+}{\varepsilon \cdot \beta^\pm} - \frac{R}{\varepsilon \cdot \beta^\pm} \cdot \rho^+.
\] (12b)

The problem is that the negative charge density has an impact on the ODE for the positive charge density. Since corona discharges can occur at both ends of a characteristic line, a boundary value problem (BVP) needs to be solved. In [22], it has been shown that such nonlinear BVPs are complex to solve without very good initial values. Therefore, an iterative method is proposed and illustrated with a flowchart in Fig. 2. The flowchart corresponds to the particular block that solves the method of characteristics in Fig. 1.

The idea is to solve the positive and negative ion flow separately and to iterate between the polarities. As the initial values, the charge concentrations of the previous iteration are taken (the values used for Poisson’s equation). The positive lines (along \(\beta^+\)) are solved with the corresponding ODE from the BC to the other end of the lines. The same procedure is applied to the negative lines (along \(\beta^-\)). After that, the positive charge density is known on the positive lines and the negative density along the lines of \(\beta^-\). The values are interpolated on the lines with the other polarity, and the iteration is repeated until convergence.

D. Case Without Wind

Without wind, the computation is easier since the drift lines are the same for the positive and negative ions (along the electric field). For a unipolar line, the ODE with the corresponding BC is an initial value problem, so that (9) can be directly evaluated. For the bipolar lines, again a BVP needs to be solved. The same iterative procedure is applied as before. But the iterations can be done separately for all of the lines (no interpolation is required). The computational cost for the case without an external velocity field is lower compared to that with an external field.

E. Choice of the Charge at the Conductor

The boundary condition from (4) is determined by the choice of the corona discharge model. A commonly used model [9], [17], [22] based on the onset gradient is employed. The charge density can be computed from a first- or a second-order method (secant) on \(\Omega_{\text{conductor}}\).

\[
\rho_i = \rho_{i-1}. \left[1 + \frac{E_{i-1} - E_{\text{on}}}{E_{i-1} + E_{\text{on}}} \right].
\] (13a)

\[
\rho_i = \rho_{i-1} + \frac{E_{\text{on}} - E_{i-1}}{E_{i-1} - E_{i-2}} \cdot (\rho_{i-1} - \rho_i).\] (13b)

The method is applied between the iteration of the IMoC. The first-order method is very stable and is used for the first iterations and afterwards the secant method is used. Local averaging is used to tolerate small violations of the onset gradient as was done in [4]. This is done to avoid the ill-conditioned problem with the BC on the electric field and for avoiding an unphysical distribution of the charge at the boundary (discharges at only one point around the conductor). The wires are taken as perfect cylinders (not stranded) because the local surface condition is not known and because the onset gradient is taken from measurements. The effect of the stranded conductors is thus incorporated in the chosen onset value.

F. Advantages of the Method

The first advantage of the IMoC is that the Deutsch assumption is waived and the fully coupled problem is solved. The algorithm is very stable even for the case of high ion densities without an initial guess. This is not the case for the Deutsch method [22] or with FEM [4] where a slow ramping of the model is required. For the full FEM method, the presence of wind poses many stability problems [19]. The proposed method remains very stable with high wind speeds. Furthermore, the method is mesh free (for the flow problem). The problem of
The radius of the conductors is 13.8 mm for both models.

need a fine mesh in regions where high space charge gradients occur is solved automatically by the presence of many field lines in this domain. Since the ODE integration can be vectorized, the solution process for the flux lines can be parallelized, and the computational cost of the method is rather low.

The method is iterative; thus, it is possible to carry out parameter variation studies very quickly, the last iteration of the previous configuration is taken as the first iteration of the next one. The last important feature of the method is that the problem is solved in the drift direction, giving a direct and valuable insight into the physical interpretation of the results.

IV. VALIDATION CASE

The proposed procedure has been validated with different existing methods (Deutsch approximation and fully coupled FEM) for reduced scale configurations. The Deutsch method has been used as presented in [9]. The FEM solution has been obtained from a DG-FEM and SUPG formulations. A bipolar reduced-scale model with bundled conductors has been simulated. The geometry is shown in Fig. 3(a). Fences have been added for providing a confinement to the ion-flow problem to show that the IMoC method can handle arbitrary geometries. The voltage of the conductors is ±150 kV and the onset gradient is 10 kV/cm. The onset gradient was chosen with respect to the saturated corona model [5]. But the choice of the onset gradient is not critical since the goal is to compare the results with the Deutsch assumption, DG-FEM, and IMoC.

The same corona model is used for all of the methods so that the same charge density is applied at the boundary of the conductors [see (13)]. This choice has been made in order to compare the numerical method for solving the PDE and not the corona model.

The electric field, ion density, and current at the ground level are shown in Fig. 4. One can see that the electric field with the space charges is about three times bigger than the Laplace electric field. The bundle conductor (positive polarity) does not have a clear impact on the electric field for the fully coupled solutions but is visible for the Deutsch method. This comes directly from the fact that this method uses the Laplace field lines without any dependence between the different lines. The glitch is observed at the point where the field lines starting for the ground are going to the second wire consisting of the bundle. The position where the electric field is zero (change between the two polarities) is also shifted between the fully coupled solutions and the Deutsch/Laplace fields. The field lines of the fully coupled problem are shifted by the space-charge density such that the field lines are mostly bipolar and are not going toward the ground.

For the ion density, the agreement between the Deutsch approximation and the fully coupled solution is very good. Once again, the effect of the bundle is larger for the Deutsch method and there is a shift of the polarity inversion. For the fully coupled solutions, a very small inflexion due to the bundle can be seen. For the current density, the effects seen for the electric field and the space-charge density are multiplied.

No significant difference can be seen between the IMoC and the DG-FEM solutions, demonstrating the validity of the proposed method. The Deutsch approximation is mostly correct except for the representation of the bundle impact and the transition location between the positive and negative polarity. The current at the boundary of the conductor deviates significantly (maximum deviation of about 50%) from the fully coupled solution and the Deutsch approximation, implying that the Deutsch method is inaccurate for computing the current coupling in the wires. For other reduced-scale geometries (unipolar), a deviation of 50% has also been observed for the maximal ion current density at ground level with the Deutsch approximation.

V. FULL-SCALE HYBRID MODELS

A. Model Parameters

The standard double-circuit towers of the Swiss transmission grid are used as a basis for the full-scale model. One ac system has been replaced with dc. The resulting geometry can be seen in Fig. 3(b). The ac and dc system are placed at either sides of the tower. The placement of the positive, negative, and neutral conductor has been chosen arbitrarily. The presented configuration should, in theory, lead to lower ground level ion current densities due to lower positive ion mobility and a higher positive onset gradient. From other perspectives, such as audible noise, radio interference, and lightning flashover, it would, however, be more favorable to place the positive pole on top. Unless otherwise mentioned, the nominal dc voltage has been chosen as...
±400 kV and the onset gradient as 10 kV/cm. This value corresponds to a low value for the rainy condition [5].

B. Variation of Voltage and of the Onset Gradient

The voltage level of the dc poles was varied between the corona onset value (about ±155 kV) up to ±530 kV. Simulations were carried out for three different onset gradients (6, 10, and 14 kV/cm). The lowest onset gradient corresponds to the 50% corona saturation for rainy or foggy conditions [5] and can be considered as a worst case. The highest value could be considered as a conservative value for fair weather [24], [25].

In Fig. 5, the maximal value of the electric field and ion current density at the ground level is shown for different voltage levels. As a comparison, the Laplace field is also shown. One can see that the increase of the electric field due to space charges is critical (factor 4). At ±400 kV, the electric field is about 40 kV/m. As a first approximation, it can be stated that the increase of the electric field is linearly dependent on the voltage level and that the impact of the onset gradient is not very large, particularly at high voltages.

The ion current at the ground level does not scale linearly with the voltage. If a limit of 100 nA/m² is taken [25], the maximum voltage level is about ±400 kV.

The onset gradient is important for determining the maximum possible voltage. The results at the ground level show that it will be difficult to choose a dc voltage greater than ±400 kV for the conversion of the existing ac tower to the hybrid tower.

C. Variation of the Wind Speed

The wind speed is an important parameter for the ion-flow field because it is far away from the conductors \( v_{gas} > \mu^{\pm} \cdot E \), implying that the drift due to the wind is dominating [25]. With FEM, high wind speeds can lead to stability problems [19] while the IMoC method remains stable with wind. A complete analysis of a range of wind speeds has been done for the field at the ground level and for the current coupling in the phases. The wind speed was varied between ±4 m/s (x component). The resulting electric field and ion current density profiles are shown in Fig. 8.

The dashed black curves represent the case without wind. The maximum of the curves with zero wind speed coincides with the position of the positive pole. The curves are rather symmetric but near \( x = -26.5 \text{ m} \), and the electric field and the ion current are zero. This point corresponds to the transition between the two polarities (as in Fig. 4 for the reduced-scale model). At the left side, far away from the tower, the field lines at the ground level are going to the negative pole placed at the top of the tower.
But this effect has no practical meaning since the fields are very small in this region.

The maximum value of the electric field does not vary significantly with wind speed. It decreases slightly for nonzero wind. With wind, the ions are pushed away from the axis of the pole. Consequently, the field enhancement due to the ions is shifted with respect to the maximum of the Laplace field, which is located below the pole. The electric field is enhanced in the direction of the wind. The size of the corridor where the field is larger than 5 kV/m is \((-13.8, 31.4)\) m without wind. With the considered wind, the size is \((-38.3, 53.7)\) m. The asymmetry of the corridor can be explained by the position of the poles at the right side of the tower and by the shielding effect of the phases at the left side.

The ion current at the ground level is strongly influenced by the wind. The ions are pushed in the direction of the wind. Domains exist at the ground level where the current is zero. This phenomenon can only occur with wind; otherwise, a field line starting at the ground level should end at a wire. In comparison to the electric field, the displacement of the maximum is larger for the ion current density and the value of the maximum is also increased by the wind. This can be seen in (9) where increased velocity (due to the increased electric field and to the wind itself) implies a larger space-charge density and, thus, a greater ion current. The peak of the current is slightly larger for wind in the positive direction because the phases do not shield the ground. Another important effect of the wind is that the width of the corridor where high ion currents occur is dramatically increased.

If a negative \(y\) component is added to the wind, the electric field and the current at the ground level are significantly higher. This acts as an increased ion speed toward the ground and, thus, larger current. Due to the backcoupling of the problem, the electric field is also increased.

With a laminar flow, a \(y\) component of the wind is, strictly speaking, not possible. But this case is still interesting for a coarse examination of the sensitivity of the solution for different wind conditions. Vertical wind flow is possible in corridors with extreme topography (as in Switzerland) or if buildings are located near the overhead line.

The current coupling in the ac phases is also strongly influenced by wind as shown in Fig. 9. The upper plot shows the coupling in the phases for different wind speeds. The current coupling is increased by 170% in the phase T. Near the maximum speed of \(-4\) m/s, the beginning of current saturation can be seen. For wind in the positive direction, the coupling is reduced to zero.

A \(y\) component has been added to the wind speed, and the coupling in the phase S has been examined (second plot in Fig. 9). Such wind is possible locally between the conductors and can totally change the dc offset. At the conductor surface, the drift due to the electric field is dominating compared to the wind. For example, an electric field pointing in the exterior direction implies that only negative ions can reach this phase independently of the wind speed. The coupling is reduced with positive \(y\) wind speed because the ions from the positive pole are attracted by the ground and the ions from the negative pole are pushed away from phase S. With a negative \(y\), the coupling is increased because the negative ions are drifting in the direction of phase S. It can be concluded that the coupling is very sensitive to local wind variation, particularly for the phases placed far away from the poles because the electric field is smaller and because the longest drift lines are more sensitive to deviations (phase S in the chosen configuration).

The phenomena explained before can be seen in Fig. 10 where the positive current \(j^+\) is shown with a wind speed of \(-2\) m/s in the \(x\) direction. The discharges are mostly located in the direction of the negative pole, the neutral conductor, and to the ground. Only a small part of the current flows to the ground and
to the phases. The zone where the current is zero at the ground can also be observed as a consequence of the wind.

It is also clear that the positive ions cannot reach phase S, creating a zone without positive ions. There, the impact of the wind is particularly important. Beyond phase R, the current is reduced, showing the shielding effect of the phase with respect to the dc corona. If the wind is flowing with a negative y component, a discontinuous current profile can be observed at the ground level because the ions should travel through the phases. But the impact of the phase shielding at the ground level is very local and does not change the general shape of the current profile.

VI. CONCLUSION

A. Numerical Method

- The presented IMoC method has been shown to be valid for the bipolar ion-flow problem with and without wind. The solution has been successfully compared with the well-known DG-FEM method. As stated in [4], the hybrid ion-flow problem can be reduced to the dc problem as an acceptable approximation.
- The numerical method is stable for all considered parameter variations, including arbitrary wind speeds, and does not require an initial guess. The algorithm enables direct physical interpretation of the results since the problem is solved along the characteristic lines. The IMoC is a fast method, allowing the computation of different configurations and parameter sweeps. On a standard desktop computer (Intel Core i7), the computation time is less than 10 min with very fine discretization.

B. Hybrid Overhead Line

- A standard double-circuit overhead line can be converted to a hybrid line with a reasonable dc current coupling in the ac phases (about 400 $\mu$A/km). The corona losses at the dc poles appear to be in the typical range for HVDC lines [5].
- At a dc voltage of 400 kV, the maximum ground-level electric field is about 36–43 kV/m and the ion current density is about 89–131 nA/m², depending on the assumed onset gradient. The electric-field scales are almost linear with the chosen voltage with a much steeper slope than the Laplace field.
- The coupling is strongly dependent on the wind speed and direction. At the ground level, the effect of the wind enlarges considerably the size of the required corridor. It can be concluded that the external velocity field cannot be neglected for the computation of ac/dc lines.

ACKNOWLEDGMENT

The authors would like to thank Swissgrid (the swiss TSO) for providing the data of a typical tower geometry.

REFERENCES

Thomas Guillod was born in Neuchâtel District, Switzerland, in 1989. He received the M.Sc. degree in electrical engineering and information technology from ETH Zurich, Zurich, Switzerland, in 2013, with a focus on power electronics, numerical analysis, and field theory.

In 2013, he joined the Power Electronic Systems Laboratory, ETH Zurich, as a Ph.D. student. His main research interests are numerical methods for PDE and ODE and properties of insulating materials.

Christian M. Franck (M’04–SM’11) received the Ph.D. degree in physics from the University of Greifswald, Greifswald, Germany, in 2003.

He was with the Swiss corporate research center of ABB from 2003 to 2009 as a Scientist and Group Leader for gas circuit breakers and high-voltage systems. Currently, he is Assistant Professor for High Voltage Technology at the Swiss Federal Institute of Technology (ETH), Zurich, Switzerland.

Martin Pfeiffer was born in Cotonou, Benin, in 1986. He received the M.Eng. degree in engineering and business finance from University College London, U.K., in 2008 and the M.Sc. degree in energy science and technology from ETH Zurich, Zurich, Switzerland, in 2012.

In 2012, he joined the High Voltage Laboratory at ETH Zurich as a Ph.D. student. His main research topics are corona effects on HVDC overhead lines and their impact on potential hybrid ac/dc transmission systems.