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A Lagrangian Decomposition Algorithm for Optimal Emergency Voltage Control

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Abstract—Voltage control problems typically involve large networks comprising diverse components extending over considerable areas and interconnecting different grids and operators. To deal with such systems, there has been over the past few years a steadily increasing interest in model predictive/optimal control techniques as a viable solution. In such a setting, it is essential to be able to coordinate the control actions among the various grids while preserving sensitive local system data that regional operators are often not willing to disclose. In view of these discordant factors, centralized and decentralized control schemes, respectively, yield advantages and drawbacks: the former require global system data to be accessible whereas the latter may prove to be difficult to effectively implement or might achieve suboptimal performance. The present paper therefore proposes a centralized control scheme that is solved however in a distributed fashion through a Lagrangian decomposition algorithm, thus reaping the benefits of both approaches: the control problem is global, therefore intrinsically more reliable and comprehensive, but only local information is employed to achieve the overall optimum control input. The computational delay associated with the additional iterations required by the algorithm is shown to be viable for the considered application and can furthermore be inherently accounted for within the proposed optimal control scheme.

Index Terms—Optimization methods, power system dynamic stability, predictive control.

I. INTRODUCTION

THE last decade or so has witnessed a common international trend in the restructuring and privatization of electrical power systems, entailing among other things that electrical networks are nowadays placed under more physical strain to fully economically exploit the system infrastructure; the risk of outages or system collapse has thus increased, as confirmed by recent large-scale blackouts throughout the world [1], [2]. The situation is complicated by the fact that electrical power

grids are large interconnected systems spanning several regions or even countries, such that the control actions of a regional operator may adversely affect other components located hundreds of kilometers away; at the same time, however, each regional operator is typically unwilling to disclose the nature of its action and the reasons behind it, as local grid information is considered to be sensitive data pertaining to an essential asset of the region's or country's industrial infrastructure.

Appropriate countermeasures must therefore be taken to both effectively control the network and avoid that local data be made public. Clearly, these two requirements tend to contradict one another, as having access to only a limited amount of information renders the system intrinsically more difficult to control.

Traditionally voltage protection schemes have relied on rather simple criteria, such as local undervoltage protections and load shedding relays, or on static offline contingency studies [3], [4]. The recent past has however seen an increase in the interest for optimization based voltage control methods employing online measurement and computation procedures that collect available system data, analyze the predicted evolution of the system based on an analytic model of the system itself, and deduce an optimal control input [5]. This is in line with new research trends dating back to the last decade, on the basis of which it was foreseen that “the time taken by the long-term (voltage) instability to develop, while short for a human operator, would be ample for a modern computer executing efficient software to identify the problem, warn the operator and suggest or trigger corrective actions...”; such control, “... based upon online system analysis and adapting its decision to the disturbance of concern, deserves attention, although it is still beyond the state of the art” [6].

In this sense, recent research work has proposed a diversified set of solutions by employing both centralized and decentralized control approaches. Examples of the former include [7], wherein a linearized model of the network is employed and the model predictive control (MPC) problem solved by means of a heuristic tree search algorithm to determine the control inputs; a method based on the trajectory sensitivity approach [8] is used in [9]–[12] to stabilize a power network with a linear MPC scheme. A pseudo-gradient evolutionary programming technique is employed in [13] for a coordinated secondary voltage control algorithm that maintains desired global voltage profiles during emergencies.

In [14] a mixed logical dynamical (MLD) representation of the considered network and its nonlinearities is employed within a hybrid MPC algorithm. Approaches considering decentralized formulations include work done in [15]–[20] where multi-agent schemes and distributed algorithms are utilized to achieve

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overall system stability for a variety of control purposes and objectives either by assuming equivalent models for neighboring networks or by some degree of system information exchange with adjacent control areas.

Although both approaches can be made to yield good performance, it is nonetheless reasonable to assume that centralized control will typically afford more reliable overall results [21] because of its more comprehensive system modeling.

The present paper elaborates on previous results obtained in the aforementioned publications and combines advantages stemming from both lines of work. More specifically, a *centralized* quadratic programming (QP) MPC formulation is employed and the ensuing optimization problem solved online by means of a *distributed* technique, known as Lagrangian decomposition, that mirrors the different regional areas in which the network is physically partitioned: in other words, the centralized control formulation is maintained and it is only the formal solution procedure that is distributed in nature. In this sense, the chosen approach can be likened to [8]–[12] since it relies on a centralized convex optimization problem formulation which is then solved however with the proposed distributed algorithm, and it is essentially an extended and ameliorated version of work presented in [22].

Lagrangian decomposition is a classic operations research methodology [23]–[25] which has been successfully and extensively employed for a number of engineering problems and notably in the power systems field. Examples include in particular reference the unit commitment problem [26] and also network congestion control and optimal power flow techniques [27]–[30]. It should be noticed that, given the chosen QP formulation, the proposed approach does not yield particular advantages in terms of computation time, since it requires a number of iterations before optimality is reached. Rather, emphasis should be placed on the fact that grid data does not have to be made globally available, a fact which is often of crucial concern in actual industrial practice.

This paper is organized as follows. Section II gives an overview of the benchmark system and voltage control issues to be considered. In Section III, the basic principles of MPC and its application to the context of voltage control are reviewed. The proposed Lagrangian decomposition method is presented in Section IV, and numerical results for the chosen case study are shown in Section V. Finally, conclusions and further research directions are outlined in Section VI.

II. NETWORK SYSTEM AND DYNAMICS

A. Case Study

The case study under consideration is based on a simplified model of the Nordel grid describing the interconnected transmission system of Sweden, Norway, Finland, and eastern Denmark [31] and is shown in Fig. 1; an extensive description can be found in [32]. For the purpose of the proposed work, first-order dynamics [33] have been added to the loads: indeed, it is these load dynamics which are typically responsible for classical long-term voltage decay phenomena, evolving over a time frame extending from several tens of seconds to minutes.

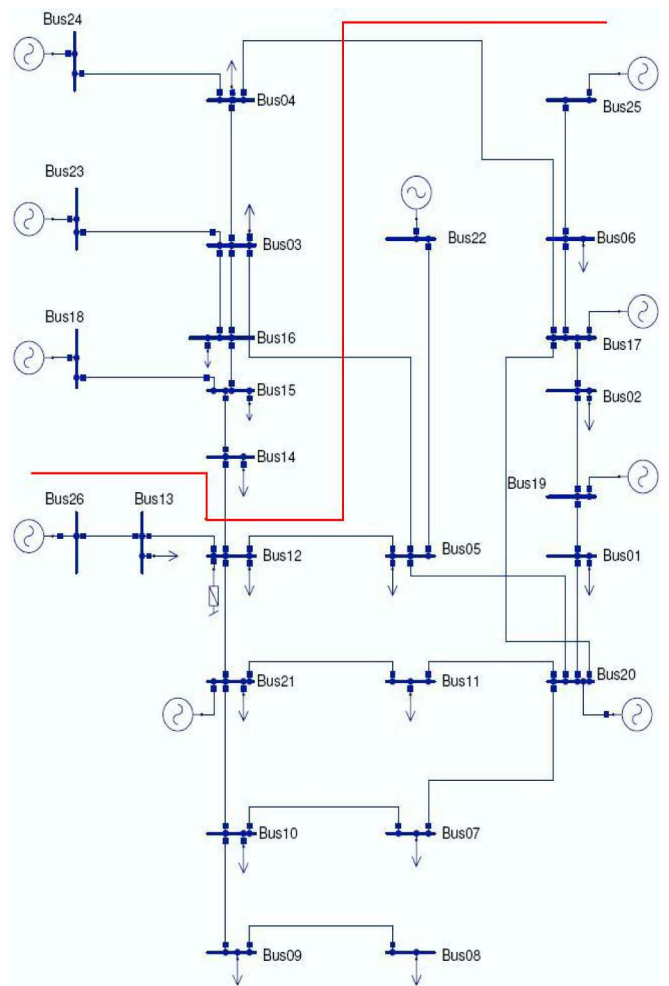


Fig. 1. Topology of the considered network.

Following a disturbance, such as the outage of a component in the grid, the load dynamics act to restore the power demand, possibly beyond the capability limits of the damaged system. As the self-restoring dynamics of the loads take place, the generators try to sustain voltages in their vicinity by means of their automatic voltage regulator (AVR) [6]. However after the capability limits have been reached, the AVR controller saturates and appropriate control actions must be taken if voltages have not yet been adequately stabilized. Local controllers are included for all generators, that is the turbine governors, the AVRs and the power system stabilizers (PSS) modulating the AVR reference signal in order to dampen frequency oscillations. The synchronous machines are represented by a fourth-order transient model describing the mechanical states plus the field and damper winding dynamics [34]. The network model overall has over 800 differential and algebraic variables. The available controls are the reference voltages of the AVRs, which lie in the interval 0.90–1.05 p.u., and the shedding factors of the loads, varying between 0 (no load shedding at all) and 1 (the whole load is shed).

B. Control Problem

Following a disruptive event, such as an outage in the network, the primary aim of the proposed emergency control

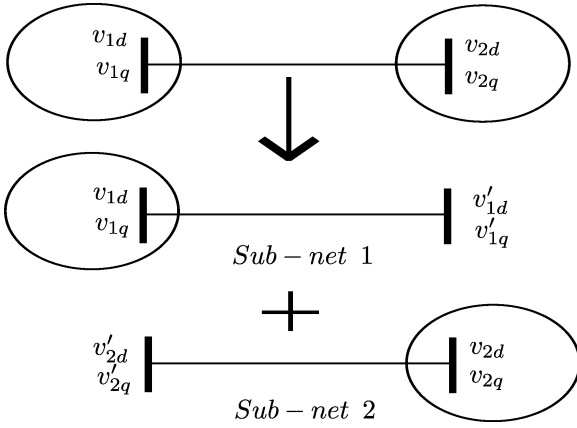


Fig. 2. Proposed decomposition scheme.

scheme is to keep all voltages at values between 0.9 p.u. and 1.15 p.u., that is sufficiently close to the nominal point of operation to ensure that the system is adequately far away from the point of voltage collapse. The secondary aim is to minimize switching of the control inputs and to effectively restore the system to a steady state point of operation with constant input settings. Lastly, load shedding must be furthermore avoided unless required to fulfill the primary objective.

III. CONTROL SCHEME

A. Model Predictive Control

MPC has been traditionally and successfully employed in a variety of industrial applications and has more recently been the object of study and investigation in power systems control research [7], [20]. This is directly related to the fact that it is a systematic control method for dealing with complex, multi-variable systems subject to constraints by directly formulating a *discrete-time control model* of the plant. The desired control aims are replicated in an appropriately selected *cost function* mirroring the order of importance of the imposed objectives. The control action is obtained by minimizing the cost function at each discrete time sampling instant over a finite horizon subject to the equations and constraints of the model, that is by solving the associated optimization problem. The first control move in the optimal sequence is then applied to the system and the procedure repeated at the successive sampling instant when the updated value of the system state is measured and taken as the new starting point for the optimal control problem. Further details about MPC can be found in [35].

B. Discrete Time Control Model

Consider the network in the upper part of Fig. 2, composed of two regional networks connected by a transmission line whose interface nodes have voltages given by the direct and quadrature axis components v_{1d} , v_{1q} , v_{2d} , and v_{2q} .

A power system network as the one under consideration can be expressed [6] in the general differential-algebraic form

$$\dot{x} = f(x, z) \quad (1a)$$

$$0 = g(x, u, z) \quad (1b)$$

$$u_{\min} \leq u \leq u_{\max}. \quad (1c)$$

In the above equations, x denotes the dynamic states of the generators with associated local controllers and of the system loads, z the algebraic variables (among which are the bus voltages), and u the system inputs (load shedding factors and AVR references). The state variables x cannot evolve discontinuously, whereas the algebraic variables z do not appear as differential terms and can thus change instantaneously by a finite amount due to changes in the input or in the network parameters; (1a) describes the system dynamics while the static part (1b) consists of the network equations which, given the inputs to the grid and the state x , can be solved to yield z . Inequalities (1c) represent the upper and lower physical bounds on the inputs.

Equations (1) can also be *equivalently* formulated as

$$\dot{x}_1 = f_1(x_1, z_1) \quad (2a)$$

$$0 = g_1(x_1, u_1, z_1, v'_{1d}, v'_{1q}) \quad (2b)$$

$$u_{1,\min} \leq u_1 \leq u_{1,\max} \quad (2c)$$

$$\dot{x}_2 = f_2(x_2, z_2) \quad (2d)$$

$$0 = g_2(x_2, u_2, z_2, v'_{2d}, v'_{2q}) \quad (2e)$$

$$u_{2,\min} \leq u_2 \leq u_{2,\max} \quad (2f)$$

$$v_{1d} = v'_{2d} \quad v_{2d} = v'_{1d} \quad (2g)$$

$$v_{1q} = v'_{2q} \quad v_{2q} = v'_{1q} \quad (2h)$$

by reconsidering the system to have been partitioned as shown in the lower part of Fig. 2. Equations (2a), (2d), (2b), (2e), (2c), and (2f) represent the system dynamics, algebraic equations, and physical input limits for subnets 1 and 2, respectively, where variables x_1 , x_2 , z_1 , z_2 , and u_1 , u_2 , functions f_1 , f_2 , g_1 , and g_2 , and values $u_{1,\min}$, $u_{2,\min}$, $u_{1,\max}$, and $u_{2,\max}$ thus pertain to either of the two subgrids. In particular, z_1 and z_2 , respectively, contain v_{1d} , v_{1q} and v_{2d} , v_{2q} .

Notice that v'_{1d} , v'_{1q} and v'_{2d} , v'_{2q} have been appended into g_1 and g_2 to represent the effect in the grid equations of the adjacent network as conveyed by the interface voltages. Since these then are constrained through (2g) and (2h), it holds that the two sets of equations for the two subnets are effectively coupled: this ensures that starting out from a given system state and physical input set, the same solution will be obtained as would have been derived from (1).

System (2) is thus effectively *mathematically identical* to the original network (1) as all that has been done is to split the network in two parts by creating a replica of the interface voltages through a formal artifice.

By linearizing (2) for a chosen operating point¹ defined by \bar{x}_1 , \bar{x}_2 , \bar{u}_1 , \bar{u}_2 , and sampling the system with period T_s through a standard zero-order hold approximation it is then possible, having set

$$w_1(k) = [u_1^T(k), v'_{1d}(k), v'_{1q}(k)]^T \quad (3a)$$

$$w_2(k) = [u_2^T(k), v'_{2d}(k), v'_{2q}(k)]^T \quad (3b)$$

to formulate the following discrete-time linear expressions:

$$x_1(k+1) = A_1 x_1(k) + B_1 w_1(k) + f_1 \quad (4a)$$

¹The current operating point at the considered sampling instant is selected to ensure a good approximation of the nonlinear system dynamics; see also Section V-B. Notice that this implies that the derived model and the associated optimal control problem change at each sampling instant.

$$y_1(k) = C_1 x_1(k) + D_1 w_1(k) + g_1 \quad (4b)$$

$$u_{1,\min} \leq u_1(k) \leq u_{1,\max} \quad (4c)$$

$$x_2(k+1) = A_2 x_2(k) + B_2 w_2(k) + f_2 \quad (4d)$$

$$y_2(k) = C_2 x_2(k) + D_2 w_2(k) + g_2 \quad (4e)$$

$$u_{2,\min} \leq u_2(k) \leq u_{2,\max} \quad (4f)$$

$$v_{1d}(k) = v'_{2d}(k) \quad v_{2d}(k) = v'_{1d}(k) \quad (4g)$$

$$v_{1q}(k) = v'_{2q}(k) \quad v_{2q}(k) = v'_{1q}(k) \quad (4h)$$

wherein k denotes the discrete time sampling instant kT_s and where the original linearizations of (2b), (2e) have been previously substituted into those of (2a), (2d) to derive (4a) and (4d). Equations (4b), (4e) furthermore represent a subset of the linearization of (2b) and (2e) in that vectors $y_1(k)$ and $y_2(k)$ are chosen to feature only the direct and quadrature axis components of the bus voltages. Among these are, respectively, v_{1d} , v_{1q} and v_{2d} , v_{2q} . The values v'_{1d} , v'_{1q} , v'_{2d} , and v'_{2q} on the other hand for the sake of convenience can be thought of as being additional fictitious inputs since their value is required to be known in order to solve (4a), (4d), (4b), and (4e). They have therefore been grouped together with u_1 and u_2 in (3). It should be noticed however that they are not effectively free variables because of (4g) and (4h).

A linearized discrete-time model of the *global* grid consisting of the subnet dynamics and algebraic (4a), (4d), (4b), and (4e) coupled via constraints (4g) and (4h) can therefore be readily obtained online at each discrete sampling instant through two separate and independent linearizations. It should be stressed again that this model is derived from (2), which is in turn equivalent to (1). This means that no approximation—other than that intrinsic in the derivation of any control model suitable for an MPC scheme—is made in the sense of assuming, for example, some form of “simplification” in the dynamics of one subsystem as seen by the other, as can be typically found in decentralized control approaches [36].

Lastly, the proposed modeling approach can intuitively be extended to any number of interconnected subnetworks featuring an arbitrary number of tie-lines with no conceptual modification of the method.

C. Optimal Control Problem Formulation

The implementation of the MPC scheme entails the formulation of a cost function J

$$J = \sum_{\ell=0}^{N-1} S(k+\ell|k) = J_1 + J_2 \quad (5)$$

$$= \sum_{\ell=0}^{N-1} S_1(k+\ell|k) + S_2(k+\ell|k)$$

penalizing the *predicted* evolution of the stage cost $S(k+\ell|k)$ from the discrete time sampling instant k over each ℓ th step of the finite horizon N ; in keeping with the objectives stated in Section II-B, $S(k+\ell|k)$ can in turn naturally be thought of as being composed of the stage costs $S_1(k+\ell|k)$ and $S_2(k+\ell|k)$ related to the two subnetworks, where for example $S_1(k+\ell|k)$ is chosen to be the expression

$$S_1(k+\ell|k) = s_1(k)^T Q_{1s} s_1(k) + \Delta u_1(k)^T Q_{1\Delta u} \Delta u_1(k) + u_1(k)^T Q_{1u} u_1(k) \quad (6)$$

and thus comprises three terms, respectively, mirroring the desired control aims.

The first term penalizes the value of the slack variables $s_1(k)$, defined by

$$\begin{cases} v_{1l} - y_1(k) \leq s_1(k) \\ v_{1u} + y_1(k) \leq s_1(k) \\ 0 \leq s_1(k) \end{cases} \quad (7)$$

where v_{1l} , v_{1u} represent constant vectors of dimensions consistent with $y_1(k)$ and $s_1(k)$ denoting the lower and upper admissible limits for the bus voltages as given in Section II-B (respectively, 0.9 and 1.15 per unit). According to the given definition of the slack variables, it can be shown [37] that at optimality, they denote the amount of violation of the condition imposed by the primary objective at the system buses. In other words, they indicate how far below or above the bus voltages are beyond the prescribed lower and upper bounds. Penalizing them thus directly captures the objective of avoiding voltage value violations.

The second part of the cost function featuring $\Delta u_1(k) = u_1(k) - u_1(k-1)$ is utilized to penalize the variation of the control inputs between successive sampling instants. Lastly, the final term is employed to minimize the amount of load shed.

The diagonal penalty matrices Q_{1s} , $Q_{1\Delta u}$, and Q_{1u} mirror the different priority levels of the control objectives, where all diagonal elements in the penalty matrices are positive.

Identical considerations hold for $S_2(k)$; additionally, the same cost function structure can easily be extended to any arbitrary number of subgrids into which the network might be partitioned.

The control input at time-instant k is then obtained by building an online model of the system of the form (4) and minimizing the objective function (5) over the sequence of physical control inputs

$$U = [U_1^T, U_2^T]^T =$$

$[u_1(k), \dots, u_1(k+N-1), u_2(k), \dots, u_2(k+N-1)]^T$ subject to the derived model equations and constraints. This is equivalent to solving the following problem:

$$\min_{U=U_1, U_2} J = J_1 + J_2 \quad (8a)$$

$$x_1(k+1) = A_1 x_1(k) + B_1 w_1(k) + f_1 \quad (8b)$$

$$y_1(k) = C_1 x_1(k) + D_1 w_1(k) + g_1 \quad (8c)$$

$$u_{1,\min} \leq u_1(k) \leq u_{1,\max} \quad (8d)$$

$$x_2(k+1) = A_2 x_2(k) + B_2 w_2(k) + f_2 \quad (8e)$$

$$y_2(k) = C_2 x_2(k) + D_2 w_2(k) + g_2 \quad (8f)$$

$$u_{2,\min} \leq u_2(k) \leq u_{2,\max} \quad (8g)$$

$$v_{1d}(k) = v'_{2d}(k) \quad v_{2d}(k) = v'_{1d}(k) \quad (8h)$$

$$v_{1q}(k) = v'_{2q}(k) \quad v_{2q}(k) = v'_{1q}(k) \quad (8i)$$

$$k = 0 \dots N-1. \quad (8j)$$

In the above equations for the sake of notational simplicity, the argument (k) is to be understood as $(k + \ell|k)$, that is as the predicted value over the horizon. The derived optimal control input U^* yielding the optimal cost J^* is as such derived on the basis of a centralized predictive control scheme and is thus to be understood as a *globally* optimal control action. This is consistent with the fact that the full system dynamics and information are described by (8). In the following, a Lagrangian decomposition approach will be presented, through which the same U^* can be obtained by *formally* splitting the *centralized* control problem (8) into two independent parts which do not require any exchange of physical system data or information.

IV. LAGRANGIAN DECOMPOSITION APPROACH

A. Methodology

Starting out from (8), it can be observed that if (8h) and (8i) were to be relaxed, i.e., ignored, two distinct and independent networks would emerge, for which two independent (but physically inconsistent) control problems could be formulated; the two networks would now, respectively, feature v'_{1d} , v'_{1q} and v'_{2d} , v'_{2q} as *effectively new and independent inputs* representing the formerly connected external nets. The Lagrangian decomposition approach [25] employs such an artifice to solve the *full* problem (8), denoted as the *primal problem*, in a decentralized fashion by formally decomposing it into a set of independent subproblems, each one associated with a subnetwork. A Lagrangian multiplier, or *dual variable*, is introduced and associated to each relaxed equation at each time step of the horizon to account for the interaction between the subnets. Let the vector of dual variables be denoted by $\Lambda = [\lambda_a(0), \dots, \lambda_a(N-1), \lambda_b(0), \dots, \lambda_b(N-1), \lambda_c(0), \dots, \lambda_c(N-1), \lambda_d(0), \dots, \lambda_d(N-1)]^T$. The dual variables are featured in the formulation by multiplying them by the relaxed equations and adding the resulting term onto the objective function to obtain the Lagrangian cost function; specifically for the example under consideration, this would have the form

$$L(\Lambda) = J_1 + J_2 + \sum_{k=0}^{N-1} \left\{ \lambda_a(k) (v_{1d}(k) - v'_{1d}(k)) + \lambda_b(k) (v_{2d}(k) - v'_{2d}(k)) + \lambda_c(k) (v_{1q}(k) - v'_{1q}(k)) + \lambda_d(k) (v_{2q}(k) - v'_{2q}(k)) \right\} \quad (9)$$

which can then be decomposed so that the relaxed problem can be expressed as

$$\min_{U_1, V'_{1d}, V'_{1q}} J_1 + \sum_{k=0}^{N-1} \left\{ \lambda_a(k) v_{1d}(k) - \lambda_b(k) v'_{1d}(k) + \lambda_c(k) v_{1q}(k) - \lambda_d(k) v'_{1q}(k) \right\}$$

and

$$\min_{U_2, V'_{2d}, V'_{2q}} J_2 + \sum_{k=0}^{N-1} \left\{ \lambda_a(k) v_{2d}(k) - \lambda_b(k) v'_{2d}(k) + \lambda_c(k) v_{2q}(k) - \lambda_d(k) v'_{2q}(k) \right\} \quad (11a)$$

$$x_2(k+1) = A_2 x_2(k) + B_2 w_2(k) + f_2 \quad (11b)$$

$$y_2(k) = C_2 x_2(k) + D_2 w_2(k) + g_2 \quad (11c)$$

$$u_{2,\min} \leq u_2(k) \leq u_{2,\max} \quad (11d)$$

where as indicated the optimization is performed also over the free variables $V'_{1d} = [v'_{1d}(k), \dots, v'_{1d}(k+N-1)]^T$, $V'_{1q} = [v'_{1q}(k), \dots, v'_{1q}(k+N-1)]^T$, $V'_{2d} = [v'_{2d}(k), \dots, v'_{2d}(k+N-1)]^T$, $V'_{2q} = [v'_{2q}(k), \dots, v'_{2q}(k+N-1)]^T$. With this decomposed formulation then for a given value of the dual variables Λ , each subproblem can be tackled and solved independently; since this decomposition is effectively a particular instance of a relaxation, an immediate consequence [38] is that for any given value of the dual variables, the resulting cost will always be inferior or at most equal to the optimal of (8). In other words, $L^*(\Lambda) \leq J^*$, where $L^*(\Lambda)$ denotes the optimal value of the Lagrangian cost for a chosen set of Λ .

In particular, a fundamental result [39] of Lagrangian decomposition theory is that, provided the original primal problem (8) is strictly convex,² as it effectively is in view of the chosen QP framework, by fixing the dual variables to their optimal value Λ^* the solutions stemming from (10) and (11) will exactly coincide with the set of values obtained by solving the full problem (8) and thus $L^*(\Lambda^*) = J^*$. This naturally entails that (8h) and (8i), although formally relaxed, will effectively be satisfied, so that the solution is *globally optimal and physically feasible*, consistently with the overall network equations.

The question arises then as to how to determine the optimal value Λ^* of the dual variables; this is done by solving the *dual problem*, defined as

$$\Delta^* = \max_{\Lambda \in \mathbb{R}^{4N}} \Delta(\Lambda) \quad (12)$$

wherein the *dual function* $\Delta(\Lambda)$ is defined as

$$\Delta(\Lambda) = \min_{U_1, V'_{1d}, V'_{1q}, U_2, V'_{2d}, V'_{2q}} L(\Lambda) \quad (13)$$

²A presentation of the definition and properties of convex optimization problems can be found in any standard operations research textbook; for the scope of the present work, it suffices to say that positive definite quadratic optimization problems are inherently strictly convex.

i.e., it is simply the optimal Lagrangian cost value for a given Λ . The dual function is known to be concave³ and piecewise differentiable [39] so that at each Λ , the subgradient of $\Delta(\Lambda)$ can be computed. For the purpose of the present work, the subgradient may simply be thought of as an extension to the notion of the gradient of a smooth function, containing similar first-order derivative information. For the problem under consideration, the subgradient $\gamma(\Lambda)$ is given by

$$\gamma(\Lambda) = \begin{bmatrix} v_{1d}(0) - v'_{2d}(0), \dots, v_{1d}(N-1) - v'_{2d}(N-1) \\ v_{2d}(0) - v'_{1d}(0), \dots, v_{2d}(N-1) - v'_{1d}(N-1) \\ v_{1q}(0) - v'_{2q}(0), \dots, v_{1q}(N-1) - v'_{2q}(N-1) \\ v_{2q}(0) - v'_{1q}(0), \dots, v_{2q}(N-1) - v'_{1q}(N-1) \end{bmatrix}^T. \quad (14)$$

Both the dual function value and its subgradient can therefore be explicitly determined by solving (10) and (11). Note that although not explicitly formalized in (14) for notational simplicity, all variables therein effectively depend on Λ as they are the product of the solution of (10) and (11), which depend on the dual variables by construction.

One possible approach for solving the dual problem is the cutting plane (CP) method, whereby an outer hull approximation of the dual function is iteratively constructed and employed to derive an approximate value of the optimizer Λ^* . This value is then employed to further ameliorate the accuracy of the hull approximation at the successive iteration and the procedure iterates until the optimal value Λ^* is obtained.

At iteration j , multipliers $\Lambda_1, \dots, \Lambda_j$, subgradients $\gamma(\Lambda_1), \dots, \gamma(\Lambda_j)$, and dual function values $\Delta(\Lambda_1), \dots, \Delta(\Lambda_j)$ are stored in a bundle β_j . With this bundle, the CP approximation of $\Delta(\Lambda)$ is obtained as

$$\Delta_j^{CP}(\Lambda) = \min_{h=1, \dots, j} [\Delta(\Lambda_h) + \gamma(\Lambda_h)^T(\Lambda - \Lambda_h)]. \quad (15)$$

The method maximizes the analytically known function Δ_j^{CP} instead of Δ , yielding

$$\Lambda_{j+1} = \arg \max \Delta_j^{CP}(\Lambda) \quad (16)$$

and the obtained solution is used to create the updated bundle β_{j+1} and therefore refine Δ_j^{CP} to obtain Δ_{j+1}^{CP} at the next step. Problem (16) can be equivalently formulated as the following linear problem:

$$\max \mu \quad (17a)$$

$$\mu \leq \Delta(\Lambda_h) + \gamma(\Lambda_h)^T(\Lambda - \Lambda_h) \quad (17b)$$

$$\forall h = 1, \dots, j. \quad (17c)$$

A drawback of this approach is that the optimal solution of (17) may be unbounded, especially in the first few iterations; another disadvantage is that the algorithm can take large steps away from the optimum even if it has almost reached it, so that the succession of Λ_j may feature an erratic evolution. In order to

³In extremely simple terms, a concave function can be thought of as a function whose curvature is always facing upwards, like an upsidedown bowl. A trivial one-dimensional example would be the function $f(t) = -t^2$. Again, a rigorous definition can be found in any standard operations research textbook.

overcome these drawbacks, the objective function of problem (16) is penalized by a quadratic term that discourages choosing Λ_{j+1} far from $\bar{\Lambda}$, where $\bar{\Lambda}$ denotes the Λ_h yielding the highest $\Delta(\Lambda_h)$ available at iteration j . Problem (16) is thus transformed into its proximal cutting plane (PCP) formulation

$$\Lambda_{j+1} = \arg \max \Delta_j^{CP}(\Lambda) - \alpha \|\Lambda - \bar{\Lambda}\|^2 \quad (18)$$

where $\|\cdot\|$ is the Euclidean norm and α is a positive tuning parameter. Problem (18) is a quadratic problem, for which a bounded solution always exists [38], and it may be explicitly formulated as

$$\max \mu - \alpha \|\Lambda - \bar{\Lambda}\|^2 \quad (19a)$$

$$\mu \leq \Delta(\Lambda_h) + \gamma(\Lambda_h)^T(\Lambda - \Lambda_h) \quad (19b)$$

$$\forall h = 1, \dots, j. \quad (19c)$$

Formulations (19) also yield a sequence of Λ_{j+1} converging to the optimal value Λ^* . The algorithm in any case may also be terminated once a certain value $\hat{\Lambda}^*$ is obtained for which a given convergence criterion is satisfied.

In many Lagrangian decomposition schemes, a commonly used criterion is to determine the $\hat{\Lambda}^*$ that yields a sufficiently small *duality gap* Γ defined as

$$\Gamma = 100 \frac{J_j - \Delta(\Lambda_j)}{J_j} \quad (20)$$

where J_j is the value of the primal cost J available at the j th iteration. Notice however that this implies that a feasible set of values [that is, satisfying (8h) and (8i)] has to be extracted at each iteration from the solution of (10) and (11) in order to establish the value J_j [which according to (8) requires feasibility]. This is possible with an elementary additional procedure, albeit at the cost of disclosing a slightly higher amount of system information. For the sake of simplicity, however, the value of the subgradients can also be taken as a convergence criterion [30], since when full convergence is achieved, both optimality and feasibility are guaranteed, that is $\gamma(\Lambda^*) = 0$ (see Sections V-B and C). Other choices are also possible: for example, even simply verifying that the dual variables have not varied by more than a given tolerance over the last few preceding iterates is a viable criterion for practical convergence evaluation of the dual algorithm.

Algorithm Summary

The crucial aspect of the described procedure is that its implementation does not require that the subnetworks exchange any data concerning internal network values. At iteration j , each subarea simply receives the multipliers with which to perform its local optimization (10) and (11), and the *only* information it sends out to the external control center running the dual PCP algorithm are its obtained local optimal cost together with the calculated interface bus voltages so that $\Delta(\Lambda_j)$, $\gamma(\Lambda_j)$ can be evaluated and Λ_{j+1} thus determined. For the sake of clarity, this is further detailed in the following algorithmic summary:

- 1) At sampling instant k , derive and pose global optimal control problem (8), requiring two independent linearizations for subareas 1 and 2.
- 2) Relax (8h) and (8i) and obtain Lagrangian cost (9).

- 3) Decompose resulting problem, i.e., each subarea independently formulates (10) and (11).
- 4) Fix initial value Λ_1 for iteration $j = 1$ and *separately* solve (10) and (11) to obtain $\Delta(\Lambda_1)$ and $\gamma(\Lambda_1)$.
- 5) The acquired information is sent out from the subareas to the external control center to build and solve problem (19) and thus obtain Λ_{j+1} .
- 6) Return to Step 4) and repeat for new iteration $j + 1$. Iterate until convergence criterion is satisfied, i.e., until value $\hat{\Lambda}^*$ is obtained.
- 7) Apply set of control inputs U_1, U_2 obtained at the last iteration of Step 4) to the physical system.

Notice that due to the chosen (inherently approximate) termination criterion, the obtained solution set coming from (10) and (11) will be neither fully optimal nor strictly feasible as regards (8h) and (8i), as there will still be a small residual error in relation with the tolerance employed. This is not a crucial problem, however, since as featured in the last step of the above summary, it is only the real control inputs which need be applied to the physical system whereas the associated values $V'_{1d}, V'_{1q}, V'_{2d}$, and V'_{2q} are discarded. The evolution of the physical system (1) is then by definition consistently and uniquely defined starting from a given vector of physical inputs, as (8h) and (8i) merely reflect the formally introduced variable duplication *in the control model*.

Lastly, the proposed approach can be extended to any number of interconnected subnetworks featuring an arbitrary number of tie-lines with no conceptual modification to its structure.

B. Implementation Issues

The presented algorithm features the advantage of yielding the global optimum to a centralized MPC control scheme for a safety-critical application without requiring that sensitive local system be made externally available. This however comes at a cost, namely that the solution procedure involves the iterative dual algorithm presented above. This in turn will typically feature a number of iterations that will increase with the number of dual variables, i.e., with the number of coupling network constraints being relaxed. This must be taken into account when evaluating the overall viability of the method for a real-time application. Indeed, tackling the full problem (8) directly would give the same optimal values in usually less time, due to the chosen problem formulation.⁴

Although this is a limitation of the method, there is still the possibility of envisaging such an approach specifically in the case of the considered context, as explained in the following.

First of all, as highlighted in Section II typical voltage decays in power grids usually feature dynamics exhibiting relatively "slow" evolutions occurring over a time span from tens of seconds to several minutes [7], [11]. This implies that the sampling time T_s can be selected accordingly and which means that a relatively "long" amount of time is available for computation, as further illustrated in Section V.

⁴This holds for the chosen (convex) QP problem framework and arguably even more so for a linear problem formulation. For such classic optimization programs, there exists a large body of advanced algorithms and software tools [40] which can handle problems featuring hundreds or even thousands of variables efficiently.

Secondly, as mentioned above, the iterations required for convergence normally increase with the number of dual variables, which means that the situation will worsen for long prediction horizons. However, in relation with the considered system dynamics, it can be intuitively argued that even a prediction horizon of $N = 2$ is enough to capture the system evolution with satisfactory accuracy. This is due to the fact that traditional long-term voltage decays typically manifest monotonically (decreasing) *quasi-linear* temporal evolutions [5], [13], [41], [42]. By then considering the effect of the selected control input at the very beginning (i.e., at the sampling instant, since the system voltages are algebraic variables which can change instantaneously) and after one time step in the horizon, it is possible to derive for each bus, starting from only these two values, a reasonable approximation of the "linearly" decreasing voltages. To a certain extent, prolonging the horizon would simply add a predicted sequence of values along the same linear evolution and therefore not add any intrinsically novel knowledge. Similar data could be obtained for example by simply increasing the discrete time step over which (4) are derived.

Finally, it should also be mentioned that since the proposed method is specifically devised for adjacent *independent* networks, it can also sometimes be the case that the number of interconnecting tie-lines is in any case relatively small as the grids' interface is comparatively weakly meshed. This means that even for more complex or detailed grids than the one used in the present work, the number of interconnecting lines will not necessarily explode. Furthermore, as pointed out in [7], there exist standard network reduction techniques through which it is possible to restrict one's attention to major load centers and generation units without having to model all grid elements, thereby reducing the number of measurement and control points and consequently allaying the computational effort.

C. Computation Delay

All of the foregoing remarks made in Section IV-C are in any case in line with more general considerations that are usually made in conjunction with MPC schemes, the use of which always entails a certain computation delay to obtain the optimal sequence of control inputs. Specifically, this means that starting from sampling instant k , the state is measured and the optimal control problem solved, but whatever the solution method, the optimal input will only be available after a certain interval t_{delay} , so that if t_{delay} is relatively large or even worse if $t_{\text{delay}} \geq T_s$, this will disrupt controller operation.

To avoid this problem, MPC schemes can be implemented with a one step delay, that is it is assumed (and verified through an analysis of the computational complexity of the resulting optimization problem instance) that the optimal input can be obtained within the time interval T_s and it is then applied at the *successive* instant $k + 1$, so that it will be for sure available at the beginning of that period. To account for the introduced time shift, the optimal control problem is then fed not with the state measured at instant k but with the state *predicted* on the basis of the measured state itself and of the applied control input at instant k (which will have been computed at instant $k - 1$). A similar approach can be employed also in the present case; however,

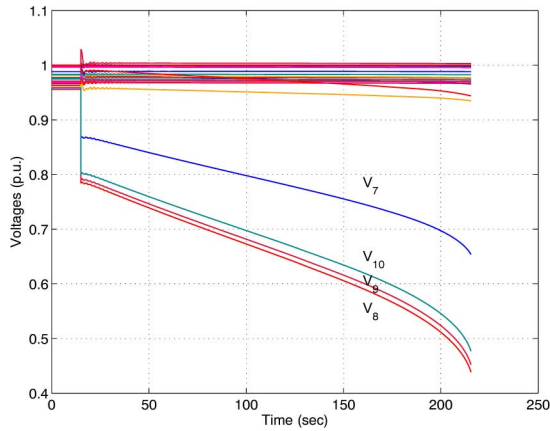


Fig. 3. Proposed simulation scenario.

this would lead to the outage occurring in the network being accounted for with one step delay. Although additional provisions could be made to avoid this (e.g., as soon as the fault is detected the model could be re-derived and the optimization procedure re-initiated independently of the chosen sampling time periodicity), for the sake of simplicity, a delay equal to 3 s is hypothesized for the worst-case scenario in terms of the required computation interval. After this time interval, it is thus assumed that the desired optimal control action is available and can be applied to the system *within the current* sampling interval, which has duration $T_s = 30$ s (see Section V-B). This assumption can then be substantiated on the basis of the results reported in Section V-C. In any case, the introduced time shift is again accounted for by considering the starting state as predicted at instant $kT_s + 3$ on the basis of the measured state at instant kT_s and of the current control input: this implies that the associated delay is consistently and intrinsically included in the overall procedure.

V. SIMULATION RESULTS

A. Sample Case Study

As a sample case study, the outage at $t = 15$ s of the line connecting nodes 21–10 of the network shown in Fig. 1 is considered, thereby creating a radial connection from nodes 20 to 8. Fig. 3 depicts the subsequent evolution of the network voltages if no control action is taken. As can be seen, following the outage, fast oscillations initially arise in the network voltages due to the generators' dynamics, but are ultimately dampened out by the local controllers. Subsequently the loads' slower self-restoring dynamics come into play and progressively evolve, causing a gradual decay in the voltages of buses 7 through 10. After about 220 s, the configuration of the grid is physically unsustainable, ultimately leading to collapse. Adequate control action must therefore be taken to counteract the possibility of system blackout and restore the grid to an acceptable operating state.

B. Controller Derivation and Closed-Loop Performance

For the MPC scheme, a sampling time T_s of 30 s has been chosen, that is the optimal control problem is formulated and

solved with this periodicity. To build the control model, the current state and input settings are chosen for \bar{x}_1 , \bar{x}_2 , \bar{u}_1 , and \bar{u}_2 . For the presented results, the full state is assumed to be measurable, i.e., also the internal states of the generators' local controllers are considered to be accessible; a more realistic choice would have been to use a simplified model of the system generators. In any case, there exist in the literature analyses [5], [11], [43] concerning the use of such simplified models for predictive voltage control, on the basis of which it can be argued that no crucial loss of model accuracy occurs because of this approximation. Furthermore, since what one would obtain would simply be an alternative set of linearized equations and expressions all of the considerations and remarks made in the present work hold also in such cases without any loss of generality. All AVR references are available as controls, whereas it is assumed that load shedding is only possible at buses 2–4, 6, 7, 9, and 13–16.

The prediction step length of the horizon is set to 30 s and the length of the horizon is chosen to be $N = 2$, in accordance with the considerations made in Section IV-C. A move blocking strategy is employed, that is u_1 and u_2 are assumed to be constant over the horizon to simplify the optimization problem by reducing the numbers of degrees of freedom, i.e., the number of free optimization variables or inputs over the horizon [35]. Concerning the weights to be used in the cost function, the entries in the diagonal penalty matrices Q_s , $Q_{\Delta u}$, and Q_u are, respectively, set to be 100, 10, and 5.

The grid is decomposed into two subnetworks by "cutting" the transmission lines connecting nodes 12–14, 16–5, and 4–6 as indicated by the red line in Fig. 1, thus overall entailing the introduction of 24 dual variables throughout the prediction horizon (four for each line at each step of the horizon). For the PCP algorithm, the choice $\alpha = 1$ is made⁵ and the initial values Λ_1 are all set to zero at each sampling instant.

As termination criterion, a relative mismatch of 0.5% of the interface bus voltages as provided by the subgradient is taken, that is for example from the first component of $\gamma(\Lambda_j)$ [see (14)] the condition

$$\left\| \frac{v_{1d}(0) - v'_{2d}(0)}{v_{1d}(0)} \right\| \leq 0.005 \quad (21)$$

has to be met, and likewise for all other components.

Fig. 4 depicts the closed-loop simulation results stemming from the developed controller. As can be seen following, the outage immediate control action is taken on the AVR references which however is not sufficient due to the severity of the fault. Extensive load shedding must then be applied at the available nodes in order to alleviate the strain under which the network would otherwise be placed and restore system voltages to acceptable levels. Notice that although the fault occurs within the lower portion of the network, the load profile and grid settings have been chosen such that control actions must be taken in both subnets in order to safeguard system operation according to the chosen globally optimal criterion: again, since the proposed centralized control methodology inherently captures this, "locally"

⁵Notice that since the optimization problem changes at each sampling instant in relation to the online linearization of the system (2), it is not realistic to try to tune this parameter, so that the nominal value $\alpha = 1$ is selected.

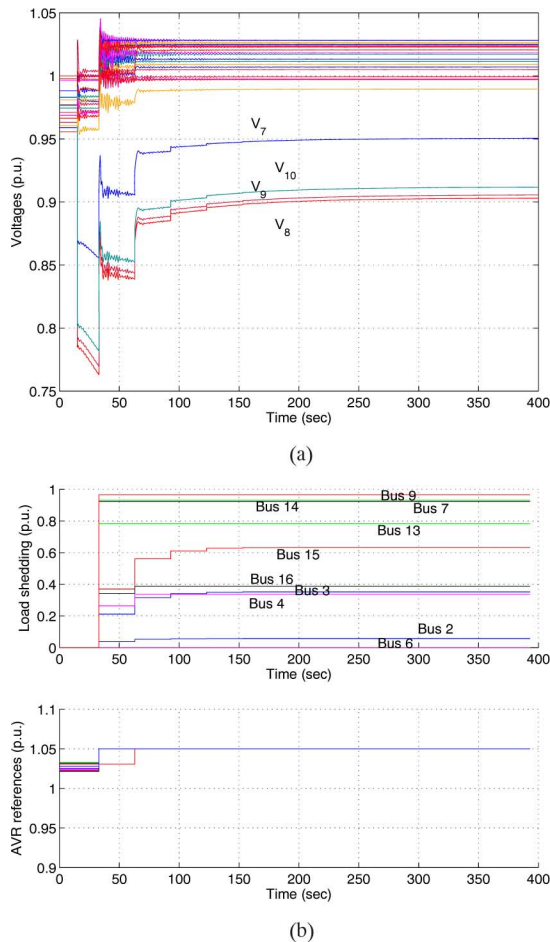


Fig. 4. Simulation results.

optimal decisions whereby for example load shedding is only used in the portion of the network directly in the proximity of the outage are avoided.

By using the aforementioned termination criterion (21) an average of 922.2 iterations were required for convergence over the considered sampling instants, with the slowest and fastest series requiring 964 and 912 iterations respectively.

C. Analysis of Dual Algorithm Performance

The foregoing closed-loop evolution results simply reflect the findings already obtained in [7], [11], and [13] in terms of the adequacy of centralized predictive control schemes for emergency voltage control. The proposed work rather presents an approach for solving the same problem through the decomposition algorithm described in Section IV-A, whose performance must therefore be assessed.

The analysis was carried out by considering 100 random operating points in the state space, i.e., 100 arbitrarily chosen instances of problem formulation (8). For these points, the main issue consists then in verifying the speed of convergence of the optimization procedure in relation to the chosen termination criterion (21): an average of 945.5 iterations was required for the considered operating points, with a maximum of 978 and a minimum of 801 iterations.

One salient aspect is perhaps that a relatively “high” number of iterations are required to satisfy the selected convergence criterion. This is obviously an issue as regards the time required for the optimization and information exchange at each step. Such limitations however should be put into perspective since a number of considerations can be made, as detailed in the following, where a worst-case scenario of 1000 iterations is assumed.

Firstly, concerning the computation times, it should be highlighted that with the given computation platform, consisting of a standard Pentium 4™ 3-GHz PC running the commercial solver CPLEX™ in Matlab™ environment, the reported average calculation times amounted to slightly less than a millisecond for (10), (11), and (18) for the 100 problem instances considered in the analysis. In view of the computational power that would then be realistically available for the power system regulation of a region or even of an entire country, it is only reasonable to assume that these calculation times could further decrease considerably. Although it is also possible to argue that even larger or more complex grids would have to be envisaged, it is likewise true that QPs scale efficiently [40] to problems with many times more variables, so that the overall balance would appear to still be in favor of the proposed approach; additionally with only elementary provisions, a similar linear programming (LP) problem could also be derived, for which formulation the degree of solution efficiency in relation to problem size is arguably even more pronounced. Since (10) and (11) are solved in parallel but sequentially with respect to (18) an overall time of 2 s is allotted for the required optimization, assuming 1 millisecond for each optimization iteration.

Secondly concerning the speed of communication between the local subareas performing the optimization of (10), (11), and the control center solving (18), it should be highlighted that the transmission velocity primarily consists of two terms, respectively, related to the bandwidth of the communication channel and to the latency time. The bandwidth relates to how much data can be transported per unit time, whereas the latency denotes how well the communication channel performs with respect to the theoretical physical limit of light travelling through the fiber of the transmission cables. In particular, high-speed long-distance Ethernet connections [44] exist which enable transmission protocols over several tens of kilometers operating with bandwidths in the range of 1 Gigabit and above and with latency times down to the hundreds of microseconds; indeed, an overall delay time of 1 millisecond is mentioned for a distance of up to 200 kilometers in [45]. In view of the fact that the data transmitted (i.e., the dual variables, dual function values, and subgradients) amounts to a few kilobits at most (assuming for example a 24-bit representation for the transmitted variables) with a few elementary calculations, it can reasonably be argued that the time required for data exchange both ways (each iteration implies communication to and from the subareas and control center) would amount to at most a fraction of a second for the assumed 1000 iterations, so that by considering also the optimization times, an overall delay of 3 s is obtained.

Additionally, the employed PCP algorithm is perhaps not the fastest way in absolute terms to solve the dual problem. A certain improvement in performance could be obtained by consid-

ering possible refinements [46] or other algorithms [47], [48] or even by just tuning the parameter α in the above simulations. This value was however intentionally set to 1 for the sake of simplicity and generality in order to avoid the dependence of the obtained results on the specifically chosen settings. It should be noticed furthermore that on the basis of the performed simulations, the adopted algorithm appears to be substantially predictable in its rate of convergence. This is a relevant factor for a real-time critical application such as the one considered in that it contributes to its general reliability. In this sense and in view also of the above remarks, it would appear plausible to claim that the overall set-up provides a framework that adequately captures the desired features.

Lastly, even if a longer delay were necessary to execute the overall algorithm, there is still a certain portion of the 30 s within the period T_s available ([11] employs a sampling interval of 50 s for a similar problem formulation) and this will in any case be inherently accounted for by the compensation described in Section IV-D.

VI. CONCLUSIONS

Model predictive techniques have emerged over the past few years as a possible way of addressing control problems in power systems research. A Lagrangian decomposition algorithm for long-term voltage stability problems has been illustrated through which the resulting overall optimization problem can be fully solved in a completely distributed manner. This allows regional operators to maintain sensitive internal system data undisclosed and thus combine the comprehensive validity of a global system model with the necessity of satisfying the political constraints imposed by power systems operation practice.

The proposed methodology is overall admittedly still at an early phase of development and no definitive conclusions at the operational level can be drawn at this stage. The presented work and results however show that with the appropriate assumptions, the described method is reasonably within reach of currently available computation and communication hardware.

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