Contract Design For Demand Response

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Abstract—The provision of ancillary services in case of real-time imbalances is an essential part of secure power system operation. Demand side participation via direct load control is a new flexible source for reliable provision of ancillary services and supports the large scale integration of fluctuating renewable energy in-feed. However, rewarding contracts have to be designed such that the consumers have financial incentives to provide these services. This paper proposes a contract design framework based on non-linear pricing, which means that capacity reservation and the deployment of reserve energy are rewarded separately. It is (a) individually rational which means that the consumer does not make a loss in providing flexibility, and (b) incentive compatible such that the consumer is not tempted to misrepresent his costs of flexibility. A simulation study shows the impact of several design parameters of contracts such as duration, accuracy of estimated cost functions and a competitive framework.

Index Terms—Electricity Markets, Demand Response, Contract Design, Bilevel Optimization.

NOMENCLATURE

Numbers and Indices

<table>
<thead>
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<th>Symbol</th>
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<tr>
<td>$N_G$, $N_L$, $N_T$, $N_r$</td>
<td>Number of generators, consumer types, optimization horizon for capacity, number of time-periods for contracted balancing energy</td>
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<tr>
<td>$N_A$, $N_S$</td>
<td>Number of aggregators, scenarios,</td>
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<td>$N_{Q_u}$, $N_{E}$, $N_{H}$</td>
<td>Number of segments for up/down capacity cost function, discretization steps of storage, and bidding blocks</td>
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Indices $i$, $j$, $a$, $e$ Indices for generators, consumer types, aggregators, and storage segment

Indices $s$, $t$, $\tau$, $b$ Indices for scenarios, time instant for contracted capacity, time instant for contracted balancing energy, and bidding blocks

Indices $qu$ Indices for up reserve capacity cost segment

Parameters

$MC_{Cap,Up/Dn}^{i,t}$ Stated marginal costs of up/down reserve capacity of generator $i$

$MC_{En,Up/Dn}^{i,\tau}$ Stated marginal cost balancing energy production of generator $i$

$R_{Req,Up/Dn}^{t}$ Demand for up/down capacity at time $t$

$R_{En,Up/Dn}^{t}$ Demand for balancing energy at time $\tau$

$G_{Cap,Up/Dn}^{i,t}$ Maximal up/down-reserve capacity provided by generator $i$

$G_{Cap,Up/Dn,\max}^{t}$ Maximal up/down-reserve capacity provided by consumer type $j$

$\nu_{\text{disch}}^{j,t}$ Physical discharging and charging efficiency of storage of consumer type $j$ at time $t$

$E_{\min/\max}^{i,t}$ Initial/Final/Maximal charging state,

$E_{\text{reg}}^{i,t}$ $e^{th}$-segment of storage capacity of consumer type $j$ at time $\tau$ in scenario $s$,

$\omega_{s}$ Probability of scenario $s$,

$MC_{Cap,Up}^{i,t}$ $M_{\text{om}}^{i,t}$ Marginal cost of utilizing the $qu^{th}/qd^{th}$-segment of capacity cost curve of consumer type $j$ at time $t$,

$MC_{En,Up}^{i,\tau}$ Marginal cost of utilizing the $e^{th}$-segment of storage of consumer type $j$ at time $\tau$ in scenario $s$ for up/down balancing energy,

$q^{i,t}$ Cost of providing up reserve capacity of consumer type $j$ at time $t$.

$M_{A}, M_{E}$ Large constants

Variables

$G_{Cap,Up/Dn}^{i,t}$ Up/down reserve capacity provision of generator $i$ at time $t$.

$G_{En,Up/Dn}^{i,\tau}$ Up/down balancing energy provision of generator $i$ at time $\tau$ and scenario $s$.

$D_{Cap,Up/Dn}^{i,t}$ Up/down reserve capacity provision of consumer $j$ at time $t$ (based on offer from aggregator $a$ in scenario $s$).

$D_{Cap,Up/Dn}^{i,t,qu,a}$ $qu^{th}$-segment of up/down reserve capacity provision of consumer type $j$ at time $t$ (based on offer from aggregator $a$ in scenario $s$).

$D_{En,Up/Dn}^{i,t,\tau,a,s}$ Up/Down balancing energy provision of consumer type $j$ at time $\tau$ (based on offer from aggregator $a$ in scenario $s$) at time $s$.

$D_{En,Up/Dn}^{i,\tau,a,e,s}$ Up/down balancing energy provision in storage segment $e$ of consumer type $j$ at time $\tau$ (based on offer from aggregator $a$).

$D_{En,Up/Dn}^{i,\tau,a,e,s+}$ Auxiliary variable for up balancing energy provision in storage segment $e$ of consumer type $j$ at time $\tau$.

$\phi_{Cap,Up/Dn}^{i,t,a}$ Price bid for up/down capacity of aggregator $a$ at time $t$.

$\phi_{En,Up/Dn}^{i,\tau,a,s}$ Price bid for up/down balancing energy of aggregator $a$ at time $t$.

$M_{A}, M_{E}$ Large constants

$E_{\tau,s}$ Charging level of storage of consumer $j$ in scenario $s$.

$q^{i,t,s}$, $z^{i,t,s}$ Auxiliary Variable of generator $i$/consumer type $j$ at time $\tau$ for relating limits of balancing
energy provision with limits of reserve capacity provision in scenario \( s \)

\[ C_{\text{Cap,Up/Dn}}^{j,\tau,a,s} \]

Cost of providing up/down reserve capacity of consumer type \( j \) at time \( \tau \) (based on offer from aggregator \( a \)) in scenario \( s \)

\[ u_{\text{E}}^{j,\tau,a,s} \]

Binary variable to determine segment of operation of storage

\[ u_{\text{Up/Dn},a}^{j,\tau,a,s} \]

Binary variable to ensure that only one aggregator can have a contract with consumer \( j \)

\[ N_{\text{Cap,Up/Dn}}^{j,\tau,a,s} \]

Payment (of aggregator \( a \)) to consumer type \( j \) at time \( t \) for up/down reserve capacity

\[ N_{\text{En,Up/Dn}}^{j,\tau,a,s} \]

Payment (of aggregator \( a \)) to consumer type \( j \) at time \( \tau \) for up/down balancing energy based in scenario \( s \)

\[ \bar{C}_{\text{Cap,Up/Dn}}^{t,s} \]

Prices of up/down reserve capacity at time \( t \) (in scenario \( s \))

\[ \bar{C}_{\text{En,Up/Dn}}^{t,s} \]

Prices of up/down balancing energy at time \( \tau \) in scenario \( s \)

I. INTRODUCTION

Demand response (DR) refers to the willingness of the consumer to respond to prices of electricity, or to receive incentive payments in times where grid reliability is jeopardized [1]. In this paper we focus on DR as a possibility to (a) provide an additional source of power system flexibility and (b) to increase backup capacity for ancillary services (AS) [2]. Renewable energy in-feed will most likely raise these needs [3], [4]. We refer on direct load control as the technically most viable option to achieve this goal. Consumers are directly controlled via a signal which adjusts or switches load, which gives a highly reliable and predictable tool for ancillary service provision compared to a real-time price signal for load adjustment [5].

However, in order to enable the exploitation of demand side participation via direct load control, incentive based rewarding contracts are necessary [5], [6], [7]. Different to an electricity supply contract, which arranges the relationship between a consumer and an electricity supplier [8], a demand response contract incentivizes a consumer to adapt his consumption in response to some kind of control signal. The financial reward, and hence the success of DR programs is determined by several factors:

- Demand side participation is constrained not only by capacity constraints but also by energy constraints.
- A set of demand side contracts must be designed such that they are incentive compatible. Thus, the consumer can distinguish which contract choice offers him a higher well-being.
- The flexibility of the contract framework with regards to payments and the time horizon of the contract.

The contribution of this paper is the provision of a contract design framework which allows the assessment of the previous mentioned technical and economical requirements in a market environment. *Incentive compatible* means that the customer reveals his true preferences about the costs of DR. Centralized incentive compatible contract design for load curtailment via the application of mechanism design was proposed in [9], [10]. *Non-linear* contracts consist of two separate charges, where a fixed charge deals with the remuneration of capacity costs and a usage charge with the costs of expected deployed energy [11], [12]. Finally, the role of an intermediary as a link between a retail and a wholesale market environment is assessed. In general, those may be suppliers of electric energy who also act as intermediary for DR services, third-parties, or cooperations of consumers [6], [7].

We focus on third party for-profit entities (which from now on we refer to as aggregators) and assess not only monopolistically acting intermediaries which offer contracts to consumers, but a framework with competition.

Therefore the model considers three forms of strategic interaction:

- Between the consumer and the aggregator through incentive compatible contracts.
- Between the aggregators on a retail level which compete for consumer subscription.
- Between the aggregators and the generators on a wholesale level auction for ancillary service provision.

In order to assess our approach in terms of efficiency we compare it with a benchmark approach which assumes full truthful preference revelation of consumers and participation in a central dispatch problem without an aggregator as an intermediate.

Ref. [7] provides a comprehensive survey with regards to issues and open research in DR contract design and assesses the appropriateness of certain contract types. In ref. [5] the authors analyze the role of contracts compared with real-time price signals in case of DR for ancillary services and find that DR contracts are the more viable option to achieve reliable responses in case that system services are requested. In ref. [6], a novel contract design framework based on a decentralized cooperative decision processes has been assessed.

In Section II we present the modeling framework for incentive compatible DR with endogenous valuation of procured DR. Section III comprises a simulation study where we investigate different aspects of contract design for DR. Finally, in Section IV we conclude and give a future outlook.

II. INCENTIVE COMPATIBLE DR CONTRACT DESIGN

The design of an efficient contract requires information prior to and after signing the contract. However, in many real-life situations there exists asymmetric information. In particular, the consumer may hide some information that he does not want to share with the aggregator. An efficient contract can come about if the aggregator provides enough incentives such that the consumer agrees on the contract when he profits from it (*individual rationality*) and the consumer reveals his true preferences about his costs of comfort loss (*incentive compatibility*) [13][9]. We establish a bilevel optimization problem in order to endogenously value procured AS capacity and energy procured by the aggregator. We define the upper level problem as the profit maximization problem of one or several aggregators. We assume a finite number of consumer
types. The aggregator has estimates about the cost functions of the consumer types [10]. The lower-level problem includes the price bids from the aggregators and the generators in order to cope with certain AS capacity and balancing energy requirements. The objective of the upper level problem is defined as:

$$\max \sum_{j=1}^{N_A} \sum_{a=1}^{N_L} \left( c_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Up}}^{j,t,a} + c_{\text{Cap,Down}}^{j,t,a} D_{\text{Cap,Down}}^{j,t,a} - \kappa_{\text{Cap,Up}}^{j,t,a} - \kappa_{\text{Cap,Down}}^{j,t,a} \right) + \sum_{s=1}^{N_g} \sum_{a=1}^{N_L} \left( \sum_{t=1}^{N_T} \left( \sum_{r=1}^{N_r} \left( r_s^{j,t,a} D_{\text{En,Up}}^{j,t,a} + r_s^{j,t,a} D_{\text{En,Down}}^{j,t,a} - \kappa_{\text{En,Up}}^{j,t,a} - \kappa_{\text{En,Down}}^{j,t,a} \right) \right) \right)$$

subject to (2)-(19). Equation (2) ensures the individual rationality of the consumer with cost functions:

$$\forall j \in \mathbb{N}_s \sum_{j=1}^{N_A} \sum_{a=1}^{N_L} \left( c_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Up}}^{j,t,a} + c_{\text{Cap,Down}}^{j,t,a} D_{\text{Cap,Down}}^{j,t,a} - \kappa_{\text{Cap,Up}}^{j,t,a} - \kappa_{\text{Cap,Down}}^{j,t,a} \right) + \sum_{s=1}^{N_g} \sum_{a=1}^{N_L} \left( \sum_{t=1}^{N_T} \left( \sum_{r=1}^{N_r} \left( r_s^{j,t,a} D_{\text{En,Up}}^{j,t,a} + r_s^{j,t,a} D_{\text{En,Down}}^{j,t,a} - \kappa_{\text{En,Up}}^{j,t,a} - \kappa_{\text{En,Down}}^{j,t,a} \right) \right) \right) \geq 0, \forall s$$

(2)

For brevity we state in equ. (3)-(10) and (20)-(19) only the costs for up reserve capacity and energy. Constraint (3) ensures the individual rationality of the consumer with cost functions:

$$\forall j \in \mathbb{N}_s \sum_{j=1}^{N_A} \sum_{a=1}^{N_L} \left( c_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Up}}^{j,t,a} + c_{\text{Cap,Down}}^{j,t,a} D_{\text{Cap,Down}}^{j,t,a} - \kappa_{\text{Cap,Up}}^{j,t,a} - \kappa_{\text{Cap,Down}}^{j,t,a} \right) + \sum_{s=1}^{N_g} \sum_{a=1}^{N_L} \left( \sum_{t=1}^{N_T} \left( \sum_{r=1}^{N_r} \left( r_s^{j,t,a} D_{\text{En,Up}}^{j,t,a} + r_s^{j,t,a} D_{\text{En,Down}}^{j,t,a} - \kappa_{\text{En,Up}}^{j,t,a} - \kappa_{\text{En,Down}}^{j,t,a} \right) \right) \right) \geq 0, \forall s$$

(3)

We assume a quadratic marginal cost function for the costs of DR provision of the form:

$$MC_{\text{Cap,Up}}^{j,t,a} = a_{\text{Cap,Up}}^{j,t,a} + b_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Up}}^{j,t,a} + c_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Up}}^{j,t,a}^2$$

(4)

where $a_{\text{Cap,Up}}^{j,t,a} > 0, b_{\text{Cap,Up}}^{j,t,a} > 0, c_{\text{Cap,Up}}^{j,t,a} > 0$. In real power systems it is difficult to get access to data and derive consumer cost functions. Ref. [10] highlights one possibility for the calibration of consumer cost functions. Equation (4) is piecewise-linear approximated with $qu$ steps for up-reserve storage. Equations (7)-(13) attribute to every $e'$th charging level costs of up/down balancing energy provision:

$$\tilde{C}_{\text{En,Up}}^{j,t,e,s} = \sum_{e=1}^{N_E} M_{\text{En,Up}}^{j,t,e,s} (D_{\text{En,Up}}^{j,t,e,s} - D_{\text{En,Up}}^{j,t,e,s})$$

(7)

$$\sum_{e=1}^{N_E} (D_{\text{En,Up}}^{j,t,e,s} - D_{\text{En,Up}}^{j,t,e,s}) = \tilde{D}_{\text{En,Up}}^{j,t,e,s}, \forall j, t, e$$

(8)

$$0 \leq \tilde{D}_{\text{En,Up}}^{j,t,e,s} - D_{\text{En,Up}}^{j,t,e,s} \leq u_{E}^{j,t,e,s} M_{E}, \forall j, t, e$$

(9)

$$0 \leq \tilde{D}_{\text{En,Up}}^{j,t,e,s} \leq (1 - u_{E}^{j,t,e,s}) M_{E}, \forall j, t, e$$

(10)

$$\sum_{e=1}^{N_E} u_{E}^{j,t,e,s} = 1, \forall j, t, s$$

(11)

Equations (7)-(11) handle the problem of having a multiplication between a continuous and a binary decision variable when choosing segment-wise cost curves for energy provision [15]. Storage operation is modeled via equ. (14)-(17):

$$E_{j,t+1,s} = E_{j,t,s} + \sum_{a=1}^{N_A} (\mu_{\text{ch}}^{j,t,e,s} D_{\text{En,Down}}^{j,t,e,s} + \mu_{\text{disch}}^{j,t,e,s} D_{\text{En,Up}}^{j,t,e,s}), \forall j, s$$

(14)

$$E_{j,t+1,s} = E_{\text{inst}}, \forall j, s$$

(15)

$$E_{j,t+1,s} = E_{\text{final}}, \forall j, s$$

(16)

$$0 \leq E_{j,t+1,s} \leq E_{\text{max}}, \forall j, t, s$$

(17)

We normalize the costs of storage operation with respect to a certain charging level, and in case of deviations from this charging level, the provision of balancing energy comes with different costs dependent upon the direction (see also Fig. 1). For example, in case of a nearly empty storage, it is more costly to provide upward balancing energy than downward balancing energy. Equ. (14) states that the sum of accepted contracts from different aggregators influences the storage content of the consumer. Constraints (18)-(19) ensure that consumer type $j$ can only be contracted by one aggregator $a$:

$$0 \leq D_{\text{Cap,Up}}^{j,t,a} \leq u_{\text{Up}}^{j,t,a} M_{A}, \forall j, t, a$$

(18)

$$\sum_{a=1}^{N_A} u_{\text{Up}}^{j,t,a} = 1, \forall j, t$$

(19)

Constraint (20) ensures incentive compatibility of the contract proposals of the aggregators for up-reserve contracts. The individual payment that consumer $j$ receives from aggregator $a$ must be such that he doesn’t misrepresent his preferences to be like the ones from $j'$, where $j'$ refers to all other consumers. Further aggregator $a$ has to ensure that his contract offer is...
preferred over all other contract offers from $a'$, where $a'$ refers to all other aggregators:

$$
\sum_{i=1}^{N_A} \left( \kappa_{\text{Cap,Up}}^{j,t,a} - \delta_{\text{Cap,Up}}^{j,t,a} \right) + \sum_{\tau=1}^{N_T} \left( \kappa_{\text{En,Up}}^{j,t,a,s} - \delta_{\text{En,Up}}^{j,t,a,s} \right) \geq \sum_{i=1}^{N_A} \left( \kappa_{\text{Cap,Dn}}^{j,t,a} - \delta_{\text{Cap,Dn}}^{j,t,a} \right) + \sum_{\tau=1}^{N_T} \left( \kappa_{\text{En,Dn}}^{j,t,a,s} - \delta_{\text{En,Dn}}^{j,t,a,s} \right), \forall j, t, a, s.
$$

(20)

Equation (20) is similar for down reserve contracts. Variable $\vartheta_1$ is a vector of decision variables which comprises of

$$
\vartheta_1 = \left\{ D_{\text{Cap,Up}}^{j,t,a}, D_{\text{Cap,Dn}}^{j,t,a}, D_{\text{En,Up}}^{j,t,a,s}, D_{\text{En,Dn}}^{j,t,a,s}, \right\}, \forall j, t, a, s.
$$

The decision variables $D_{\text{Cap,Up}}^{j,t,a}$, $D_{\text{Cap,Dn}}^{j,t,a}$, $D_{\text{En,Up}}^{j,t,a,s}$, and $D_{\text{En,Dn}}^{j,t,a,s}$ lie in the feasible set of the lower level problem which is defined as a dispatch problem of the system operator with the aim to minimize procurement costs of reserve capacity and balancing energy:

$$
\min \, f(\vartheta_2) = \sum_{i=1}^{N_A} \sum_{t=1}^{N_T} \left( MC_{\text{Cap,Up}}^{i,t} G_{\text{Cap,Up}}^{i,t} + MC_{\text{Cap,Dn}}^{i,t} G_{\text{Cap,Dn}}^{i,t} \right) + \sum_{a=1}^{N_A} \sum_{j=1}^{N_T} \left( \alpha_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Up}}^{j,t,a} + \beta_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Dn}}^{j,t,a} \right) + \sum_{a=1}^{N_A} \sum_{\tau=1}^{N_T} \omega_{a}^{\tau} \left( \sum_{i=1}^{N_A} \left( \kappa_{\text{En,Up}}^{i,t,a,s} - \delta_{\text{En,Up}}^{i,t,a,s} \right) G_{\text{En,Up}}^{i,t,a,s} + \sum_{a=1}^{N_A} \left( \kappa_{\text{En,Dn}}^{j,t,a,s} - \delta_{\text{En,Dn}}^{j,t,a,s} \right) G_{\text{En,Dn}}^{j,t,a,s} \right)
$$

$$
= \max(0, R_{\text{Rec,Up}}^{\vartheta_2}, 0, \forall t, \vartheta_2)
$$

(23)

$$
\min \, f(\vartheta_2) = \sum_{i=1}^{N_A} \sum_{t=1}^{N_T} \left( MC_{\text{Cap,Up}}^{i,t} G_{\text{Cap,Up}}^{i,t} + MC_{\text{Cap,Dn}}^{i,t} G_{\text{Cap,Dn}}^{i,t} \right) + \sum_{a=1}^{N_A} \sum_{j=1}^{N_T} \left( \alpha_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Up}}^{j,t,a} + \beta_{\text{Cap,Up}}^{j,t,a} D_{\text{Cap,Dn}}^{j,t,a} \right) + \sum_{a=1}^{N_A} \sum_{\tau=1}^{N_T} \omega_{a}^{\tau} \left( \sum_{i=1}^{N_A} \left( \kappa_{\text{En,Up}}^{i,t,a,s} - \delta_{\text{En,Up}}^{i,t,a,s} \right) G_{\text{En,Up}}^{i,t,a,s} + \sum_{a=1}^{N_A} \left( \kappa_{\text{En,Dn}}^{j,t,a,s} - \delta_{\text{En,Dn}}^{j,t,a,s} \right) G_{\text{En,Dn}}^{j,t,a,s} \right)
$$

$$
= \min(0, R_{\text{Rec,Up}}^{\vartheta_2}, 0, \forall t, \vartheta_2)
$$

(24)

$$
\vartheta_2 = \left\{ G_{\text{Cap,Up}}^{i,t,a}, D_{\text{Cap,Dn}}^{j,t,a}, G_{\text{En,Up}}^{i,t,a,s}, D_{\text{En,Dn}}^{j,t,a,s}, \right\}, \forall i, j, t, a, s.
$$

Variable $\vartheta_2$ is a vector of decision variables which comprises of

Objective (21) is subject to constraints (22)-(32). Equation (22)-(24) state the balance for reserve capacity and up/down balancing energy respectively:

$$
R_{\text{Cap,Up}}^{\vartheta_2} = \sum_{i=1}^{N_A} G_{\text{Cap,Up}}^{i,t,a} - \sum_{a=1}^{N_A} \sum_{j=1}^{N_T} D_{\text{Cap,Up}}^{j,t,a} = 0, \forall t
$$

(22)

$$
R_{\text{Res,Up}}^{\vartheta_2} = \sum_{i=1}^{N_A} G_{\text{Res,Up}}^{i,t,a} - \sum_{a=1}^{N_A} \sum_{j=1}^{N_T} D_{\text{Res,Up}}^{j,t,a} = 0, \forall t
$$

(23)

$$
R_{\text{Res,Dn}}^{\vartheta_2} = \sum_{i=1}^{N_A} G_{\text{Res,Dn}}^{i,t,a} - \sum_{a=1}^{N_A} \sum_{j=1}^{N_T} D_{\text{Res,Dn}}^{j,t,a} = 0, \forall t
$$

(24)

$$
G_{\text{Cap,Up}}^{i,t,a} = \max(0, x_{j,t,a}^{i,t,a}), \forall i, j, t, a
$$

(25)

$$
G_{\text{En,Up}}^{i,t,a,s} = \max(0, -x_{j,t,a}^{i,t,a}), \forall i, j, t, a
$$

(26)

Equation (25)-(30) ensure that either up or down balancing energy is supplied by generator $i$ or consumer type $t$ in time instant $\tau$ respectively. Equation (31)-(32) ensures that capacity limits of the generator are fulfilled:

$$
0 \leq G_{\text{Cap,Up}}^{i,t,a} \leq G_{\text{Cap,Up},\text{max}}^{i,t,a}, \forall i, t
$$

(31)

$$
0 \leq D_{\text{Cap,Dn}}^{j,t,a} \leq D_{\text{Cap,Dn},\text{max}}^{j,t,a}, \forall j, t, a
$$

(32)

All decision variables are greater or equal zero. We assume that the number of price/volume blocks of aggregator $a$ is equal to the number of different payments to the consumer types. However, in real operation the aggregator has most likely a different bidding in the wholesale market compared to its retail level activity. This is a point of future research. The stated problem formulation (1)-(32) can be solved by expressing the lower level problem (21)-(32) as KKT - conditions and including it in the upper level optimization problem (1)-(19) [16], [17].

### III. SIMULATION FRAMEWORK

We assume a generic test system in a day ahead market with 2 different consumer types and one generator which has no storage or ramping constraints. System data are given in the appendix. AS capacity and energy is contracted on a hourly basis. The time horizon is 24 hours. In this simulation no grid constraints are considered, but the model can be extended in this direction.

We compare the presented approach with a benchmark, which is a central dispatch problem. It does not contain an aggregator as intermediate, and comprises all technical and financial constraints except incentive compatibility. The
objective is the minimization of procurement costs of AS capacity and balancing energy.

We show results with regards to the duration of payment periods, the sensitivity with respect to a change of costs of providing DR on consumer side, and the resolution of the piecewise-linear approximation of the cost function. Further, we assess the effect of having two aggregators in competition. Data in VI are normalized by (4.97,10.5) for the load types. The costs of providing up reserve capacity per load type in Table VI have to be multiplied with the normalized elements of Table VI respectively. Cost of providing dn reserves per load type have to be multiplied with the inverse of the normalized elements respectively. The maximal physical capabilities for DR are 10\% of the demand in Table VI for Type I and 20\% for Type II. The requirements for reserve capacity are 10 times the requirements according to the ENTSO-E formular [18]. The demand per time instant $\tau$ is drawn from a normal distribution $R_{\text{Req,En}}^{\tau,s} \sim \mathcal{N}(0, \frac{1}{15}) * R_{\text{Req,Cap,Up/Dn}}^{\tau,t}, \forall (t,s)$. The reason is that the deployment of balancing energy is only weakly connected to a specific time instant of a day. Out of computational reasons we assume 2 scenarios with equal probability $\varphi = 0.5$. We repeated all simulations 10 times for higher robustness. Future research comprises the scenario generation based on an autocorrelated process and proper scenario reduction. We assume initially 12 piece-wise linear steps for the cost function for reserve capacity and 13 steps for the cost functions for reserve energy. The approach was implemented in MATLAB using the interface YALMIP and IBM ILOG CPLEX as solver [19], [20],[21].

A. Change in duration of contracting period

Fig. 2 shows the payments of the aggregator to the consumer if the payment periods are varied from a hourly basis to the entire optimization horizon. We find crosscurrent tendencies for energy and capacity in terms of DR exploitation compared with the benchmark approach. Fig. 3 shows the cost efficiency per procured quantity unit from the consumers. We find that whereas the overall DR exploitation decreases with the contract duration, the reduction does not go linearly but stepwise. The impact of this analysis depends upon if the aggregator is aiming for a high exploitation of DR potential or on cost efficiency in terms of buying capacity and energy as cheap as possible.

B. Change in cost function resolution

The resolution of the cost function is correlated to a) the accuracy of available information (i.e. metering data), and b) the number of contracts that a consumer gets provided. Both effects can be assessed via changing the resolution of the cost curves for capacity and energy. We only changed the resolution for capacity and Fig. 4 shows the effect of a reduction of the piecewise-linear steps of the cost function. We find that on the one hand the number of steps increases slightly the exploitation potential for capacity to a certain extent. On the other hand, the cost efficiency is reduced with a decrease in the number of steps. This suggests to keep the accuracy of the cost function estimates as high as possible.

C. Several Aggregators

We assess the effect of several aggregators in the model. Table I shows that the exploitation of DR increases for energy in case of 2 aggregators. Simulations show that the consumers profit in terms of total payments. We assumed equal aggregators with the same estimates about probability distribution of load types, no other costs of operation, and wholesale bidding according to payments to the consumers. Therefore there exists no rule on how the exploitation and the payments are shared among the aggregators which is point of further research.
IV. CONCLUSION AND FUTURE WORK

In this paper we presented a contract framework for the provision demand response for ancillary services based on a bilevel optimization problem. We modeled an aggregator as an intermediate between wholesale energy market operation and retail markets. The aggregator has to provide incentive compatible contracts which ensure the sufficient rewarding of reserve capacity and balancing energy due to energy storage limitation on the consumer side. We assessed several design parameters of the contract which show that a high time resolution for contracts may not be necessary, the costs of demand response determine the validity of intermediate units such as aggregators, and the availability of accurate information increases cost efficiency in DR exploitation. The existence of several aggregators shows that a reduction in the overall payments for DR is possible. In all approaches we assumed for simplicity reasons no grid and no explicit modelling of market operation for traded energy. Future research will be expanded in these directions.

REFERENCES


A. System Data

Tables II-VI contain relevant system data used for the simulations.

### TABLE II: Economic and Technical Data of Generators

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<th>Capacity</th>
<th>MW</th>
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<tr>
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<td>$\text{En,Down}$</td>
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<td>$\text{Cap,Down}$</td>
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### TABLE III: Technical Data of Load

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<tbody>
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<td>$\text{P}_{\text{load}}$</td>
</tr>
<tr>
<td>$\text{Q}_{\text{load}}$</td>
<td>$\text{Q}_{\text{load}}$</td>
</tr>
<tr>
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<td>$\text{P}_{\text{Up/Down}}$</td>
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<tr>
<td>$\text{Q}_{\text{Up/Down}}$</td>
<td>$\text{Q}_{\text{Up/Down}}$</td>
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### TABLE IV: Cost factors of up/down reserve capacity for different load types providing DR ($\text{MW}_\text{Up/Down}$)

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<tr>
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</thead>
<tbody>
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<td>$\text{C}_{\text{Up/Down}}$</td>
</tr>
<tr>
<td>$\text{C}_{\text{Up/Down}}$</td>
<td>$\text{C}_{\text{Up/Down}}$</td>
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### TABLE V: Costs of down-reserve energy per storage segment for different load types providing DR ($\text{MW}_\text{Up/Down}$)

<table>
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<td>$\text{C}_{\text{Up/Down}}$</td>
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### TABLE VI: Time series of load dependent upon consumer type (MW)

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