"Incorporation of N-1 Security into Optimal Power Flow for FACTS Control"

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***Abstract*— As the electric power consumption increases continuously, the stress on the power grid grows and therefore, failures of system components become more probable. When the system is not in an N-1 secure state, an outage of a single component may trigger cascading failures in the worst case resulting in a blackout. In this paper, a new current injection method is developed such that the apparent power flows on lines in case of a line failure are very accurately determined by a linear equation system. Based on this, a method using FACTS devices to keep the system in an N-1 secure state is presented. The set values of the FACTS devices are determined by an Optimal Power Flow (OPF) control where the current injection method is used to include N-1 considerations focused on line failures. Thus, a control strategy is outlined which reduces the risk of cascading failures.**

I. Introduction

Electric energy is an essential need of today's society. The experience has shown that an extensive failure of the electric power system may paralyze entire countries. The reasons for such blackouts are manifold but often they occur as a consequence of cascading events. As the stress in a grid is increased when a component fails subsequent failures become more probable. Examples are various blackouts which occurred in the last few years [1], e.g. in Sweden or Italy. Therefore, it is important to anticipate failures and keep the system in a secure state. In [2], security is defined as

> *the ability of the power system to withstand sudden disturbances such as electric short circuits or unanticipated loss of system components.*

In this paper, the focus lies on the part concerning losses of components and particulary on line failures. The intention is to keep the system in an N-1 secure state. Here, this means that any component may fail and all other components are still below their limit.

There are various possibilities to control the system state and therefore to enforce the system to be in a secure state. One of them is to use FACTS devices [3], [4]. As these devices provide the opportunity to influence the system without generation rescheduling, they enable an electricity market which is less constrained by security aspects.

The set values of FACTS devices are in this paper determined by Optimal Power Flow calculations [5], [6], including the objective to keep the system in an N-1 secure state as defined above. But this means that line loadings in case of a line failure have to be determined. In general, this is done by load flow calculations for the grid topology in the faulted situation. Taking into account several possible line faults in such a way, the number of variables in the optimization problem grows rapidly. Therefore, a method has to be found where the line loadings resulting from a line fault can be determined without carrying out an entire load flow calculation. A possibility is to apply the current injection method. In this paper, this method is extended, while the form presented in [7] yields results which are not accurate enough in the here discussed application.

In the first section, an overview over the current injection method and the realized extension are given. In simulations, it is shown how accurate this extended method is compared with the exact load flow calculations. Sect. III elaborates on the modelling of FACTS devices and how these models are included in the current injection method. The control of these FACTS devices is based on Optimal Power Flow and is derived in Sect. IV. It is explained how the developed current injection method is included into Optimal Power Flow calculations. Simulations demonstrate the effectiveness of this control. Finally, the conclusion in Sect. V completes the paper.

II. Current Injection Method

In case of a line failure, bus voltages and currents change due to the modification in the grid topology. In order to find these altered voltages and currents, generally, the entire load flow calculations have to be redone. To avoid this, the current injection method [7], [8] can be used.

A. Basic Method

The idea is to compensate for the line failure by introducing virtual injection currents at the buses adjacent to the faulted line. If the injection currents are determined such that their values are equal to the currents resulting on the line which is actually faulted, then the effect is the same as if the line is not in operation.

In Fig. 1, the concept of the current injection method is visualized. On the left hand side, the situation before the fault is given and in the middle, the actual situation when the line is faulted. On the right hand side, it is shown how the faulted situation is approximated by keeping the line in operation but introducing injection currents $\underline{I}_{Si}$ and $\underline{I}_{Sj}$. As these injection currents influence the currents on line ij, they cannot just be set to the values of the line currents before the fault, i.e. $\underline{I}_{ij0}$ and $\underline{I}_{ji0}$. Therefore, the influences of the injection currents on the line currents have to be found, in order to find the values for $\underline{I}_{Si}$ and $\underline{I}_{Sj}$.

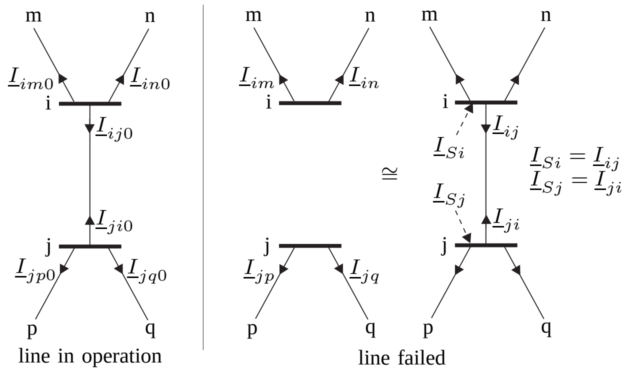

Fig. 1. Injection Current Method

The situation with all lines in operation is described by

$$\underline{U}_0 = \underline{Z}_0 \cdot \underline{I}_0, \tag{1}$$

where $\underline{U}_0$ and $\underline{I}_0$ are the bus voltages and currents, respectively and $\underline{Z}_0$ is the bus impedance matrix. When a line fails, the corresponding equation is

$$\underline{U}_B = \underline{Z}_B \cdot \underline{I}_B. \tag{2}$$

Here, $\underline{U}_B$ and $\underline{I}_B$ are the bus voltages and currents in the faulted situation and $\underline{Z}_B$ is the modified bus impedance matrix.

Now, the vector of injection currents $\underline{I}_S$ is introduced whose elements are the injection currents at the buses of the grid. All elements in $\underline{I}_S$ except the ones corresponding to the buses adjacent to the failed line are set to zero. With this vector the voltages in the faulted situation are calculated by

$$\underline{U}_B = \underline{Z}_0 \cdot (\underline{I}_B + \underline{I}_S). \tag{3}$$

The changes in voltages are then derived from (1) and (3) as

$$\Delta\underline{U} = \underline{U}_0 - \underline{U}_B = \underline{Z}_0 \cdot (\underbrace{\underline{I}_0 - \underline{I}_B}_{\approx 0} - \underline{I}_S) \tag{4}$$

where it is assumed that the changes in bus currents are negligible. The changes in line currents consequently result in

$$\Delta\underline{I}_{line} = \underline{Y}_p \cdot \Delta\underline{U} = \underbrace{-\underline{Y}_p \cdot \underline{Z}_0}_{\underline{D}} \cdot \underline{I}_S = \underline{D} \cdot \underline{I}_S \tag{5}$$

where $\underline{Y}_p$ is the line admittance matrix of the situation without fault and $\underline{D}$ is the matrix of distribution factors defining the influence of the injection currents on line currents.

From (5), the equations for the calculation of the injection currents are set up, which say that the injection currents at the buses adjacent to the faulted line have to be equal to the currents flowing on this line, thus,

$$\underline{I}_{S_i} = \underline{I}_{ij0} - \Delta\underline{I}_{ij} = \underline{I}_{ij0} - \underline{D}_{i,i} \cdot \underline{I}_{S_i} - \underline{D}_{i,j} \cdot \underline{I}_{S_j} \tag{6}$$

$$\underline{I}_{S_j} = \underline{I}_{ji0} - \Delta\underline{I}_{ji} = \underline{I}_{ji0} - \underline{D}_{j,i} \cdot \underline{I}_{S_i} - \underline{D}_{j,j} \cdot \underline{I}_{S_j} \tag{7}$$

The notation $\underline{D}_{i,j}$ indicates the element in row i and column j of matrix $\underline{D}$ and $\underline{I}_{S_i}$ the i-th element in the current injection vector.

Thus, a linear equation system results from which the injection currents are determined. Inserting these values in (4), the voltage changes are obtained and consequently the corresponding line power flows are known.

B. New Method

For the derivation of the basic current injection method, it was assumed that the bus currents in the faulted case stay approximately the same as in the normal situation. But this does only hold for buses where there are no generators and loads connected. There, the bus current is always zero. But for other buses the current will change, otherwise it is not possible to keep a given voltage reference or given injected active and reactive power. Therefore, the injection method is extended by additional bus injection currents $\underline{I}_T$ where only the elements corresponding to buses with generators and loads are non-zero. With these injection currents given voltages and power injections are enforced. The bus currents for the faulted situation are then

$$\underline{I}_B = \underline{I}_0 + \underline{I}_T \tag{8}$$

From this, equation (4) results in

$$\Delta\underline{U} = \underline{U}_0 - \underline{U}_B = -\underline{Z}_0 \cdot (\underline{I}_S + \underline{I}_T) \tag{9}$$

The next step is to set up the equations for the different bus types. In the following derivations, a single index added to a matrix corresponds to the row in the matrix given by this index and an index added to a vector indicates the element at this index, i.e. $\underline{Z}_{0_k}$ is the k-th row of matrix $\underline{Z}_0$ and $\underline{U}_{B_m}$ is the m-th element in vector $\underline{U}_B$, respectively.

- slack bus k: voltage angle as well as voltage magnitude stay unchanged, therefore

$$\Delta\underline{U}_k = -\underline{Z}_{0_k} \cdot (\underline{I}_S + \underline{I}_T) = 0. \tag{10}$$

- PU bus m: voltage magnitude and active power injection are given and therefore the same before and during the outage, thus

$$|\underline{U}_{B_m}| = |\underline{Z}_{B_m} \cdot (\underline{I}_0 + \underline{I}_T)| = |\underline{U}_{0_m}| \tag{11}$$

$$\Re\{\underline{S}_{B_m}\} = \Re\{\underline{U}_{B_m} \cdot (\underline{I}_{0_m} + \underline{I}_{T_m})^*\} = \Re\{\underline{U}_{0_m} \cdot \underline{I}_{0_m}^*\} \tag{12}$$

- PQ bus n with load: here, the active and the reactive power injections are predefined which is expressed by

$$\underline{S}_{B_n} = \underline{U}_{B_n} \cdot (\underline{I}_{0_n} + \underline{I}_{T_n})^* = \underline{U}_{0_n} \cdot \underline{I}_{0_n}^* \tag{13}$$

- buses adjacent to the faulted line: the additional injection currents $\underline{I}_T$ also have influence on the virtual current on the faulted line. Therefore, (6) and (7) are extended to

$$\underline{I}_{S_i} = \underline{I}_{ij0} - \underline{D}_{i,i} \cdot \underline{I}_{S_i} - \underline{D}_{i,j} \cdot \underline{I}_{S_j} - \underline{D}_i \cdot \underline{I}_T \tag{14}$$

$$\underline{I}_{S_j} = \underline{I}_{ji0} - \underline{D}_{j,i} \cdot \underline{I}_{S_i} - \underline{D}_{j,j} \cdot \underline{I}_{S_j} - \underline{D}_j \cdot \underline{I}_T \tag{15}$$

Now, equations (10) – (13) are written out and $\underline{U}_B$ is substituted by the formula given in (9). When the terms including the injection currents squared are neglected in (12) and (13) and split into real and imaginary parts, again a linear equation system results

$$A \cdot \begin{pmatrix} \Re\{\underline{I}_S\} \\ \Im\{\underline{I}_S\} \\ \Re\{\underline{I}_T\} \\ \Im\{\underline{I}_T\} \end{pmatrix} = b. \qquad (16)$$

Once the injection currents are calculated, then the changes in voltages are determined by (9) and from this the power flows on the lines.

C. Simulation Results

As in the extended current injection method the quadratic terms of the injection currents have been neglected, the line loadings determined with this method are not exact but approximated values. The grid used as test system is the IEEE 57 Bus grid [9] with an additional generator at bus 30 shown in Fig. 2.

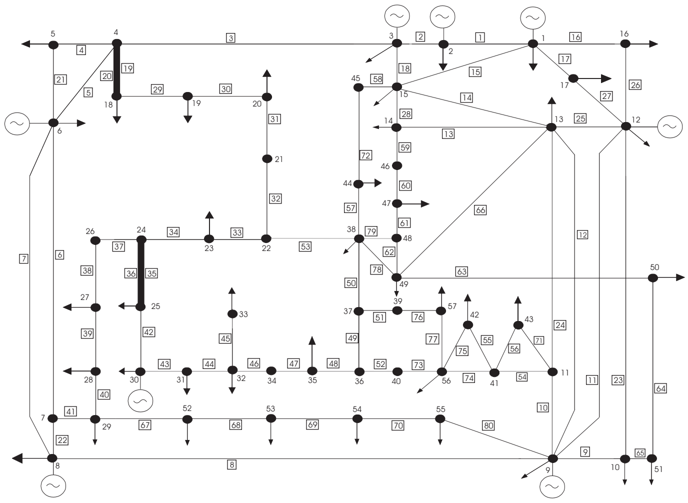

Fig. 2. IEEE 57 Bus grid with additional generator at bus 30

To investigate the validity of the method, the apparent power flows on the lines are determined once with the extended current injection method and once with exact power flow calculations and the results are compared. In Fig. 3 and 4, the apparent power flows for an outage of line 8 and an outage of line 28 are given, respectively. For a convenient demonstration of the results, only the values for lines 1 to 40 are shown, but the accuracies for lines 41 to 80 are very similar.

Line 8 is the line with the highest power flow, therefore, an outage of this line is the most difficult case to determine the power flows as there are the most changes. But it can be seen in Fig. 3 that the power flows determined with the extended current injection method are quite close to the exact values. The maximal error is 0.0456 p.u. in line 22. In case of an outage of line 28, which is a line with average power flow, the approximated values almost coincide with the exact values. The maximal error occurs in line 1 with a value of 0.0025 p.u.

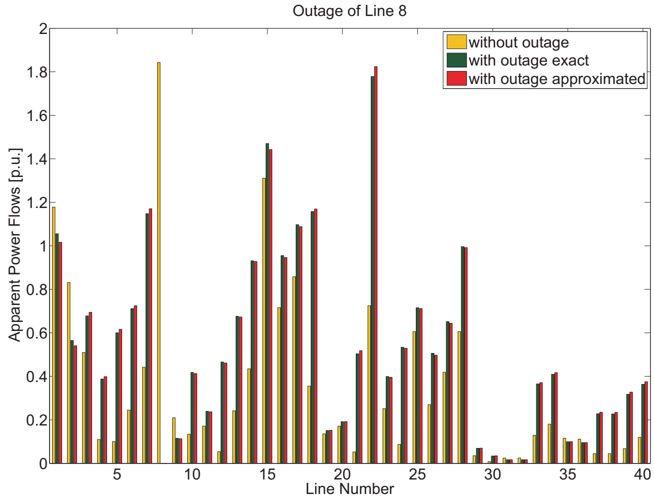

Fig. 3. Exact and approximated apparent power flows in case of an outage of line 8

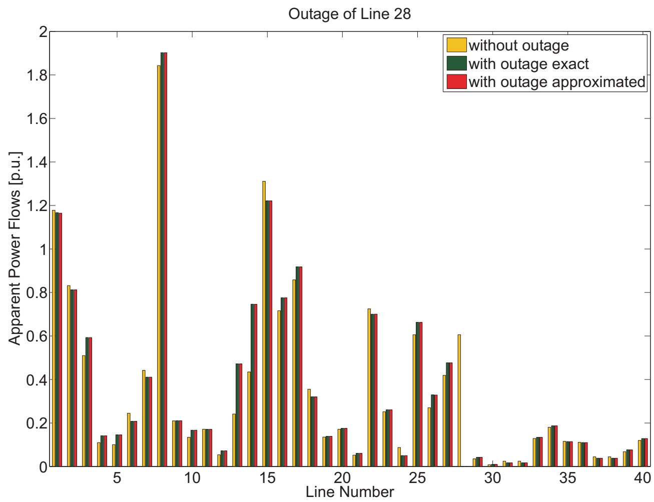

Fig. 4. Exact and approximated apparent power flows in case of an outage of line 28

III. Inclusion of FACTS devices

In this paper, FACTS devices are used to control the state of the system. In order to determine the set values, an Optimal Power Flow control is applied with the objective to bring the system into a secure state. Therefore, the FACTS devices have to be modelled such that they can easily be included in the representation of the current injection method, thus, in the admittance matrix. This is fulfilled by the total susceptance or reactance model [10].

A. SVC

The SVC is shunt-connected to a bus. In Fig. 5(a), a possible structure is given. Depending on the firing angle of the thyristors, the SVC injects or absorbs a certain amount of reactive power. Therefore, an SVC is modelled as a variable susceptance shunt-connected to the bus (Fig. 5(b)) [10]. Given from the firing angle range of $0°$ to $90°$, the equivalent susceptance B_{SVC} has an upper and a lower limit. This yields the constraint

$$B_{SVC}^{min} \leq B_{SVC} \leq B_{SVC}^{max}. \quad (17)$$

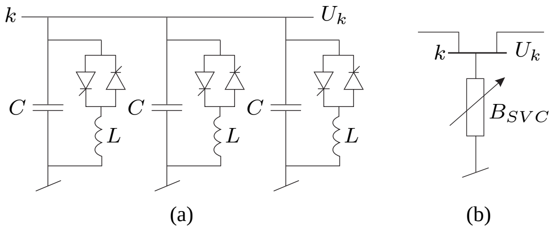

Fig. 5. (a) Possible structure and (b) model of an SVC

Concerning the current injection method, the SVC is included into the admittance matrix by an additional column. The only non-zero element in this column is in the row of the bus to which the SVC is connected and is equal to jB_{SVC}.

B. TCSC

A TCSC is connected in series with the line. The structure is shown in Fig 6(a). As this corresponds to a variable reactance whose value depends on the firing angle of the thyristors, it is able to influence the line reactance and therefore the active power flow through this line. Hence, the model of a TCSC is a variable reactance connected in series with the line (Fig. 6(b)) [11].

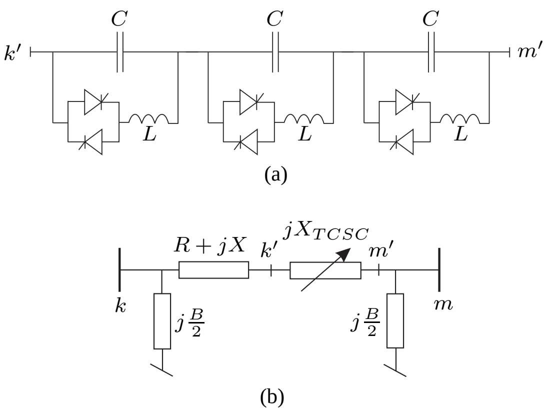

Fig. 6. (a) Possible structure and (b) model of a TCSC

There are two parameters which limit the possible values of the reactance. First, the range of the firing angle yielding an internal limit of a lower X_{TCSC}^{min} and an upper value X_{TCSC}^{max}. Additionally, the TCSC is not allowed to compensate the line reactance more than 20% inductive and 80% capacitive [11]. Therefore, the following constraint results

$$\begin{aligned} \max\left\{X_{TCSC}^{min}, -0.8 \cdot X_{line}\right\} &\leq X_{TCSC} \\ &\leq \min\left\{X_{TCSC}^{max}, 0.2 \cdot X_{line}\right\}. \quad (18) \end{aligned}$$

The inclusion into the admittance matrix used for the current injection method is straightforward. The reactance of the TCSC is simply added to the reactance of the line where it is placed and from this the admittance matrix is built.

IV. N-1 in Optimal Power Flow

As the extended current injection method consisting of a linear equation system yields quite accurate results for power flows in case of line faults, it provides the opportunity to include N-1 considerations into Optimal Power Flow calculations with acceptable additional computation effort.

A. Optimal Power Flow control

The Optimal Power Flow control [5], [6] is applied in order to set the values of the FACTS devices. The general form of an Optimal Power Flow problem is

$$\min_{x,u} f(x,u) \quad (19)$$

$$\text{s.t.} \quad g(x,u) = 0 \quad (20)$$

$$h(x,u) \leq 0. \quad (21)$$

where $f(x,u)$ is the objective function which is minimized subject to the equality constraints $g(x,u)$ and the inequality constraints $h(x,u)$. The vector x is the state vector including voltage magnitudes and angles as well as possibly used slack variables and u is the vector of control variables, thus, the set values of the FACTS devices.

Here, the equality constraints correspond to the power flow equations. The inequality constraints include limits on the control variables, on line loadings and bus voltages. In order to avoid an unsolvable system, the latter two are defined as soft constraints given by

$$S_{ij}/S_{ij}^{max} \leq 1 + \mu_{ij} \quad (22)$$

$$|U_j - U_j^{ref}| \leq U^{lim} + \rho_j \quad (23)$$

$$\mu_{ij}, \rho_j \geq 0 \quad (24)$$

The variables have the following meaning

- S_{ij}: apparent power flow on line ij
- S_{ij}^{max}: thermal limit of apparent power flow on line ij
- U_j: voltage magnitude at bus j
- U_j^{ref}: reference voltage at bus j
- U^{lim}: allowed deviation of voltage from reference value
- μ_{ij}, ρ_j: slack variables to define soft constraints

The objective function is composed of three components:

1) minimizing active power losses
2) keeping line loadings below 100%
3) keeping voltages close to given reference values and within acceptable limits

The second component and the part concerning acceptable limits on voltages in component three are included by heavily penalizing the slack variables of inequalities (22) and (23) in the objective function. Thus, the objective function results in

$$\begin{aligned} f(x,u) = &\sum_{lines} \left(a \cdot P_{ij}^{loss} + b \cdot \mu_{ij}\right) \\ &+ \sum_{buses} \left(c \cdot |U_j - U_j^{ref}|^2 + d \cdot \rho_j\right) \quad (25) \end{aligned}$$

with control parameters a, b, c and d.

B. Inclusion of N-1

Theoretically, for each line an outage may occur and the consequences have to be taken into account. But especially in a heavily meshed grid, only some outages are critical and some lines are in danger to be overloaded in case of an outage. The operators are able to determine the critical lines by contingency scanning and therefore only these critical lines are considered in the optimization process.

For each of these critical line outages, the equation system (16) is set up. The voltage changes in case of an outage are functions of the determined injection currents, as given in (9), and therefore, also the resulting apparent power flows S_{ij}^m on the lines, where m refers to the failed line. Additional constraints are defined to keep the line loadings in case of a line failure below 100%, i.e.

$$S_{ij}^m / S_{ij}^{max} \leq 1 + \delta_m \quad (26)$$

$$\delta_m \geq 0 \quad (27)$$

The slack variables δ_m are in addition penalized in the objective function (25). The difference to (22) is that there is only one slack variable for all lines. Like this, the number of variables is kept low.

C. Simulation Results

The thermal limits of the test system (given in Fig. 2) are chosen such that in case of failures of lines 8, 60 or 66 at least one other line is overloaded. Thus, neglecting that a failure of line 45 causes the load at bus 33 to be unsupplied, the system is not in an N-1 secure state as defined at the beginning. To bring it into such a secure state, a TCSC is placed in line 72 and an SVC at bus 34. The reasons for these placements are given after the presentation of the obtained simulation results. The set values for the FACTS devices are determined by the Optimal Power Flow control described in the previous section. The reference value for the bus voltages U_j^{ref} is set to 1.01 p.u.

Figures 7 – 11 show the simulation results. In order to show the effect of the FACTS devices and their control, in each graph the situation is given assuming that the FACTS devices are out of operation and in addition the situation with FACTS.

Concerning the bus voltages given in Fig. 7, it can be seen that the voltages at buses located close to bus 34 where the SVC is placed are increased and mostly brought closer to their reference value. Therefore, the objective concerning voltages is improved.

Fig. 8 shows the line loadings without failures and also the value for the active power losses. No line is in danger to be overloaded in this situation and the active power losses are decreased minorly by the FACTS devices.

The line loadings for the critical outages of lines 8, 60 and 66 are shown in Figs. 9 – 11, respectively. The line loadings which would result in case of a line failure and no FACTS device is in operation show clearly that the system is not N-1 secure without FACTS. Applying the Optimal Power Flow control, overloads are avoided for all three critical outages.

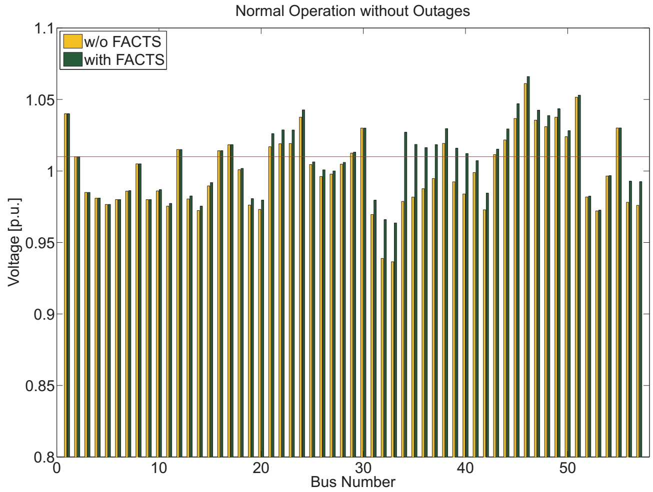

Fig. 7. Voltages without outages

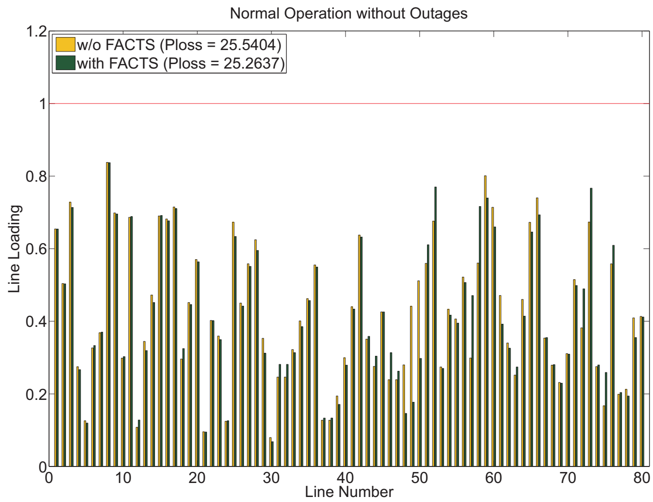

Fig. 8. Line loadings without outages

As the extended current injection method, used to determine the line loadings in case of a failure, includes some simplifications of the power flow calculations, there are some minor inaccuracies. Therefore, it might happen that the controller assumes that the line is loaded to exactly 100% when the line is actually already overloaded by a small amount. To avoid even such low overloads, line limits could be set a bit more conservative.

A problem which has not been addressed in this paper is the placement of the FACTS devices. Here, the TCSC has been placed such that it is able to influence the power flows on the lines which are overloaded in case of an outage. To find such a placement is of course often not simple. Sensitivity analysis may help and also the usage of more than one TCSC. The placement of the SVC has been chosen such that it is located at a bus whose voltage and the voltages around that bus are rather low and where it is able to help reaching the secure state.

A possible extension of the above control is to take into

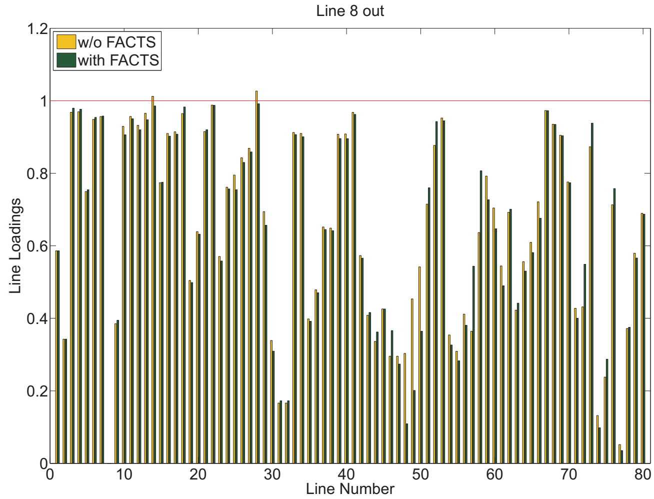

Fig. 9. Line loadings with outage of line 8

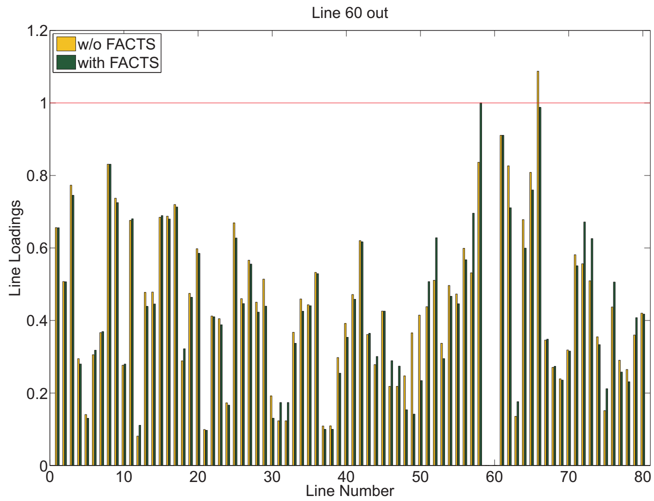

Fig. 10. Line loadings with outage of line 60

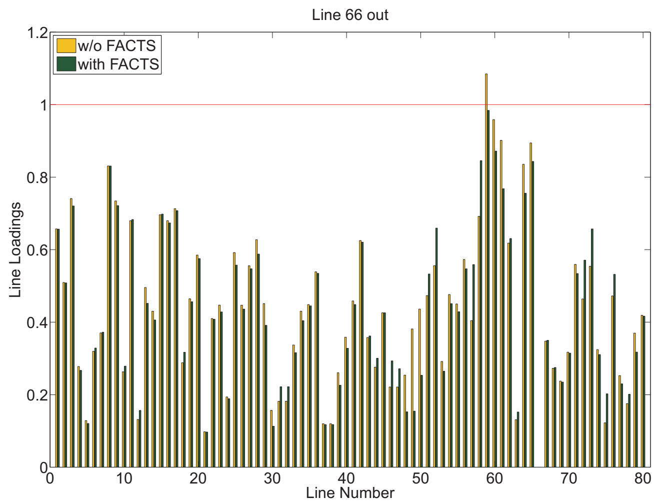

Fig. 11. Line loadings with outage of line 66

account the voltages in case of an outage. Due to the change in the grid topology they might change significantly and take unacceptable values. The inclusion of this objective is rather simple as the corresponding changes in voltages are already determined by (9) in the extended current injection method. But the inaccuracies for the approximation of the voltages are relatively larger as for power flows and if this additional objective is included even more trade offs have to be made between the different objectives. A solution is to use a larger number of FACTS devices which result in more variables which can be set by the controller.

V. Conclusion

In this paper, a possibility has been shown how the current injection method may be extended in order to determine power flows in case of line failures accurately. This extended method is applied in Optimal Power Flow control for the determination of the optimal settings for FACTS devices in a power grid. The objectives are to bring the system into a secure state, i.e. no line is overloaded when a line outage happens, to improve the voltage profile and to minimize active power losses in normal operation. Simulations in the adapted IEEE 57 Bus grid have shown that the derived control fulfills these requirements.

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