Improvement of OPF Decomposition Methods Applied to Multi-Area Power Systems

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***Abstract*— Large power systems are nowadays mostly operated as interconnected subsystems, each not always based on identical legislative, historical and geographical regulations. Within this interconnection, each power system is controlled by its respective control authority, forming a decentralized structure. In this paper methods are presented, decomposing a central optimal power flow (OPF) problem into distributed subproblems. These subproblems are then solved in an iterative way, independently but coordinated. Based on an available decomposition method, an improved decomposition procedure is proposed. With the new method, the convergence rate is considerably enhanced and no parameter tuning is required. Simulation results are presented, applying the procedures to an illustrative 9-bus as well as to the IEEE 39-bus system.**

I. Introduction

The UCTE [1] is an aggregation of Transmission System Operators (TSO), where each TSO is responsible for a dedicated part of the power grid. These various parts are regulated in an autonomous way and thus organized in a decentralized structure. Concerning the adjustment between these parts, the organization suffers from various disadvantages. For the coordination among the entities, no continuous self-acting procedure is provided but adjustments between the areas are obtained by predefined power flows arranged once a day (UCTE Operation Handbook [1]). In emergency cases, human (i.e. operators) judgment is employed for the coordination of control actions among areas, increasing the risk of faults. To overcome this limited coordination, a control procedure needs to be implemented which is suitable for distributed and coordinated control enabling a fast and automatic adaption between entities.

Today, the coordination between areas is obtained by measuring the in- and outflows between areas. Thereby, the impact of a neighboring area is incorporated by approximating the neighbor as an additional load [2]. The insufficiency of this coordination is demonstrated in [3] where a simulation on the IEEE 39-bus system is carried out. Due to a deficiency in today's coordination, the system wide optimal solution which is obtained by applying centralized control is not provided. In a centralized control scheme, one optimization problem including all areas is solved by a single central controller. Applying decentralized control, the overall optimization problem is divided into subproblems which are solved in an iterative way, separately but coordinated.

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The major reason to implement a decentralized control procedure arises from the adaption to the current situation. As already mentioned, the UCTE is established by distributed control entities which not always have identical operating goals and thus base their optimization on non-coherent criteria. Furthermore, less data transfer is needed as well as higher robustness and faster computation times are provided.

Two basic approaches for OPF decomposition are established which enable the coordination within a multi-area system. A first method is proposed in [4], where a coordination is achieved by adjustment at an existing or fictitious border bus. Different methods for decomposability are compared in [5], [6]. A second method is presented in [7],[8], where the coordination is carried out by exchanging dedicated variables between adjacent areas. The methods presented in [7], [8], [3] are extended for overlapping areas in [9]. In this paper, a new procedure for decomposing OPF problems with a faster convergence rate is proposed, where no tuning of parameters is required. The improved method applies a master-slave principle to interconnected areas in order to enhance the coordination among them.

II. Decomposition Techniques

Multi-area control is illustrated on two interconnected areas, but the presented approaches can be extended to an arbitrary number of areas. Considering two areas A and B, the overall OPF is given by

$$\min_{x_A,\ x_B} \quad f(x_A, x_B) = c(x_A, x_B) \tag{1}$$

$$\text{subject to} \quad g(x_A, x_B) = 0 \tag{2}$$

$$h(x_A, x_B) \leq 0 \tag{3}$$

where x_A and x_B represent the OPF variables relevant for area A and B, respectively. Generally, the optimization vectors comprise the voltage magnitudes and angles of all buses and the control variables, e.g., yielding $x_A = [\theta_A, V_A, u_A]$ for area A. The objective function comprehends various application areas such as active power control, relieving overloaded lines or improving the voltage profile. The equality constraints correspond to the power flow equations and the inequality constraints include constraints on voltages, power flows and transmission capacity. In general, the problem defined above represents a nonlinear optimization problem. The solution space is non convex, thus, finding the global optimum cannot be guaranteed with numerical methods.

In order to decompose the optimization problem the objective function (1) as well as the equality and inequality constraints (2), (3) need to be separable. Depending on the decomposition procedure, the objective and the constraints are divided differently to the areas. Both approaches are sketched in Fig. 1 and illustrated in the following sections.

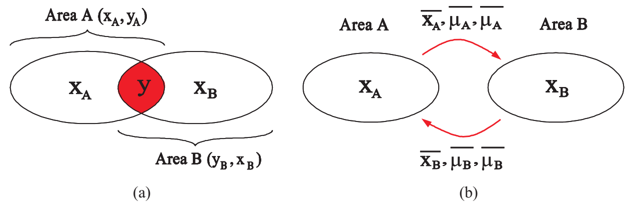


Fig. 1. Coordination between two areas A and B: (a) adjustment at interface and (b) passing adjacent variables.

A. Adjustment at the Interface

The first approach to decompose an optimization problem is operating with an adjustment procedure at a common interface. The procedure, further referred to as procedure II-A, is illustrated on the overlapping two-area system depicted in Fig. 1(a). Next to the own variables x_A and x_B, both areas share some variables at the common border, denoted by y. To decompose the overall optimization problem, the border variables y, also referred to as interface variables, are duplicated in y_A and y_B and assigned to the areas A and B. Now, both areas are enabled to solve their subproblems with the vectors (x_A, y_A) and (x_B, y_B), respectively. The original equalities (2) and inequalities (3) are split according to the described vectors, i.e. involving only the own variables and the duplicated border variables, and assigned to the particular area. Usually, the border vector y is much smaller than the core vectors x_A and x_B. The smaller this ratio the looser the coupling between the areas is and the more individual acting is enabled. Furthermore, better convergence of the algorithm is expected due to the minor coupling between the regional power flows.

To recover the original optimization problem, a *coupling constraint* is imposed to enforce equality between the two copies ($y_A - y_B \overset{!}{=} 0$). Generally, standard Lagrangian approaches to relax the coupling constraints are converging slowly to the solution, thus the augmented Lagrangian procedure is applied to improve convergence. The linear equality constraint is relaxed and added to the original objective resulting in

$$c_A(x_A, y_A) + c_B(x_B, y_B) + \lambda^T(y_A - y_B) + \underbrace{\frac{\gamma}{2}\|y_A - y_B\|^2}_{a} \quad (4)$$

where λ^T denotes the transpose of the Lagrangian multiplier and γ is a positive constant. This objective function is not decomposable anymore as it contains the inseparable cross term marked by a, whose variables belong to both areas. To make this term separable, it is linearized using the Auxiliary Problem Principle (APP) [10]. Applying this algorithm, the solution of the optimization problem with the objective (4) is obtained by solving a sequence of auxiliary problems. These auxiliary problems are now decomposed into subproblems for each area and solved iteratively. In the case of area A, the minimization yields at each iteration step k the updated values of the core variables x_A^{k+1} and the interface variables y_A^{k+1}:

$$\begin{aligned} (x_A^{k+1}, y_A^{k+1}) = \underset{(x_A,\ y_A)\in A}{\arg\min}\ & c_A(x_A, y_A) + \frac{\beta}{2}\|y_A - y_A^k\|^2 + && (5)\\ & \gamma y_A^T(y_A^k - y_B^k) + {\lambda^k}^T y_A && \\ \text{subject to}\ \ & g_A(x_A, y_A) = 0 && (6)\\ & h_A(x_A, y_A) \le 0 && (7) \end{aligned}$$

where β is an additional positive constant introduced by the APP - algorithm. Similar, the optimization of area B provides updated values of x_B^{k+1} and y_B^{k+1}:

$$\begin{aligned} (x_B^{k+1}, y_B^{k+1}) = \underset{(x_B,\ y_B)\in B}{\arg\min}\ & c_B(x_B, y_B) + \frac{\beta}{2}\|y_B - y_B^k\|^2 - && (8)\\ & \gamma y_B^T(y_A^k - y_B^k) - {\lambda^k}^T y_B && \\ \text{subject to}\ \ & g_B(x_B, y_B) = 0 && (9)\\ & h_B(x_B, y_B) \le 0. && (10) \end{aligned}$$

The objective function of both areas consists of two parts. The first term describes the main objective originating from the centralized optimization (1), while the second part composed of three terms is responsible for the adaption between the adjacent areas. Solving the optimization problem for each area is alternated with an update of the Lagrangian multiplier λ

$$\lambda^{k+1} = \lambda^k + \alpha(y_A^{k+1} - y_B^{k+1}). \quad (11)$$

The update is arranged by a sub-gradient method incorporating the linear coupling constraint, whereas the weighting of the update is defined by the positive constant α. The value of the Lagrangian multiplier λ^k is an estimate of the cost to maintain the constraint $y_A^k - y_B^k = 0$ at iteration step k. The iteration procedure terminates as soon as the mismatch between the coupling variables is smaller than a specified threshold value. Thereby, the convergence is quite sensitive to the values of the positive constants α, β and γ which represent the weighting factors for the adaption procedure. Appropriate tuning of these parameters is necessary to obtain proper convergence rates.

The procedure can either be executed in a synchronized or in sequential way. In the first case, the optimizations are performed in parallel with equal starting values where as in the second case the optimization problems are solved after each other using updated starting values. The only information to be exchanged between the areas are the updated border variables y_A and y_B. Therewith, each area can perform the update of the Lagrangian multiplier by itself and proceed for the next iteration step.

Other decomposition methods based on the coordination using shared variables are presented in [5] which differ by the way of handling the non-separable term in (4). In [6], the coupling constraint is incorporated into the objective by building the Lagrangian instead of the Augmented Lagrangian, while decomposability is achieved applying the alternating Direction Method (ADM).

Implementing this procedure within electric power systems, the border region including its variables needs to be defined.

In principle, for each tie-line between two adjacent areas a bus within the overlap region needs to be identified. If no interface bus exists, a fictitious *dummy bus* is created. With each dummy bus voltage and angle at the bus as well as active are reactive power flow through the bus are associated. Hence, the shared border vector y contains four entries for each tie-line. Considering the overlapping areas A and B to be interconnected by one tie-line (Fig.1(a)), the border variables are defined as $y_A = [\theta_A \ V_A \ P_A \ Q_A]$ for area A and $y_B = [\theta_B \ V_B \ P_B \ Q_B]$ for area B.

B. Passing Adjacent Variables

The second decomposition approach is carried out on strictly separated areas rather than on areas sharing a common border (Fig. 1(b)). This approach, referred to as approach II-B, achieves coordination between adjacent areas by passing dedicated peripheral variables to the neighbor. As no duplicated shared variables are incorporated in this approach, constraints including variables of area A as well as of area B appear. These so-called *complicating constraints* are related to both areas and thus prevent each subsystem from operating independently. Concurrently, these constraints enable the coordination between the areas. Regarding the subproblems, the equality constraints (2) are divided into constraints related to only one area $g_A(x_A)$, $g_B(x_B)$ and into complicating constraints indicated by $\widetilde{g}(x_A, x_B)$. The complicating constraints are again split and assigned to the area which contains the majority of complicating variables. Complicating variables, incorporated in the complicating constraints, appear at the periphery of the different subsystems. The same classification holds for the inequality constraints (3). Fig. 2 delineates the assignment of the constraints to the corresponding areas.

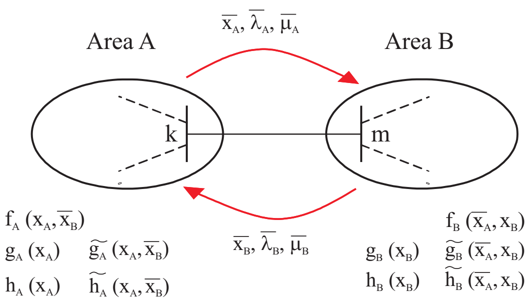


Fig. 2. Coordination procedure II-B between two areas A and B, with peripheral buses k, m, including the assignment of constraints.

One method to obtain the subproblems consists of relaxing the own complicating constraints and incorporating them into the objective, applying the standard Lagrangian relaxation procedure. These subproblems can now be solved independently by keeping in each case the variables of the foreign area constant. Thereby, the Lagrangian multipliers need to be updated by a sub-gradient method incorporating the nonlinear complicating constraint, similar to the approach presented in Section II-A. To avoid this, a new method is proposed in [7],[8]. This method establishes the subproblems by relaxing the complicating constraints assigned to the foreign area and adding them to the objective function while maintaining the own complicating constraints. For area A, the optimization problem is specified as

$$\min_{x_A} \quad c_A(x_A, \overline{x_B}) + \overline{\lambda_B}^T \cdot \widetilde{g_B}(x_A, \overline{x_B}) \tag{12}$$
$$+ \overline{\mu_B}^T \cdot \widetilde{h_B}(x_A, \overline{x_B})$$
$$\text{subject to} \quad g_A(x_A) = 0 \qquad \widetilde{g_A}(x_A, \overline{x_B}) = 0 \tag{13}$$
$$h_A(x_A) \leq 0 \qquad \widetilde{h_A}(x_A, \overline{x_B}) \leq 0 \tag{14}$$

and for area B as

$$\min_{x_B} \quad c_B(\overline{x_A}, x_B) + \overline{\lambda}_A^T \cdot \widetilde{g_A}(\overline{x}_A, x_B) \tag{15}$$
$$+ \overline{\mu}_A^T \cdot \widetilde{h_A}(\overline{x}_A, x_B)$$
$$\text{subject to} \quad g_B(x_B) = 0 \qquad \widetilde{g_B}(\overline{x}_A, x_B) = 0 \tag{16}$$
$$h_B(x_B) \leq 0 \qquad \widetilde{h_B}(\overline{x}_A, x_B) \leq 0 \tag{17}$$

where $\overline{x_A}$, $\overline{x_B}$ denominate the state variables of the previous iteration step of area A and B, respectively. The variables $\overline{\lambda_A}$, $\overline{\lambda_B}$ and $\overline{\mu_A}$, $\overline{\mu_B}$ represent the Lagrangian multipliers of the corresponding equality and inequality constraints. At each iteration step, these values are sent to the other area (Fig. 2). Similar to approach II-A, the objective is composed of two terms. The second term is used for the coordination between the areas while minimizing the main objective, described by the first term. The optimization problems are solved in an iterative way and convergence is achieved if the variables do not change significantly in two consecutive iterations. Again, the optimization problems can either be solved in a synchronized or sequential way. In contrary to the standard Lagrangian relaxation procedure, the Lagrangian multipliers representing the weighting factors of the foreign complicating constraints result directly from the neighboring optimization problem. For example, the Lagrangian multipliers $\overline{\lambda_B}$, $\overline{\mu_B}$ included in the objective of area A are obtained from area B's optimization by keeping its own complicating constraints ((16), (17) on the right) .

Applying this mathematical procedure to electric power systems, the constraints are arranged in the following way. The equality constraints $g_A(x_A)$, $g_B(x_B)$ comprise the power flow equations of all inner buses of the particular area. The complicating equality constraints comprehend the power flow equations at the peripheral buses k, m (Fig. 2). A coupling between the areas is only enabled when these power flow equations include variables of both areas. This means, the constraints for active and reactive power balance serve as complicating constraints but not the constraints regarding voltage and angle reference settings. Hence, having PQ buses at the common tie-lines results in two complicating constraints per peripheral bus. A less tight coupling is achieved with PV buses, yielding only one complicating constraint. If the slack bus is situated at one of the peripheral buses between two areas, the procedure is not implementable. The inequality constraints comprise the limits on voltages, power generations and transmission capacities of each area. Complicating inequality constraints occur by transmission limits on tie-lines belonging to both areas. To classify the inequality constraints into own and foreign complicating constraints the tie-lines needs to be allocated to one area, arbitrarily. For more detailed information refer to [7], [8].

C. Comparison of Decomposition Methods

Basically, both approaches achieve a coordination between interconnected areas by adding additional terms into the objective function. The additional terms in procedure II-A enforce the matching between the interface variables whereas procedure II-B includes foreign constraints into the objectives. Table I provides a more detailed comparison of both methods. Objectives and constraints are delineated only for area A in each case. Further, the convergence of both decomposition techniques is presented in [8], where the procedures II-A, II-B as well as the standard Lagrangian relaxation procedure are applied to a mathematical example.

TABLE I
COMPARISON OF DECOMPOSITION METHODS

	Adjustment at the Interface	**Passing Adjacent Variables**
Objective	$f_A(x_A, y_A, y_B) =$ $c_A(x_A, y_A)$ $+ \frac{\beta}{2}\|y_A - y_A^k\|^2$ $+ \gamma y_A^T(y_A^k - y_B^k)$ $+ {\lambda^k}^T y_A$	$f_A(x_A, \overline{x_B}) =$ $c_A(x_A, \overline{x_B})$ $+ \overline{\lambda_B}^T \cdot \widetilde{g_B}(x_A, \overline{x_B})$ $+ \overline{\mu_B}^T \cdot \widetilde{h_B}(x_A, \overline{x_B})$
Constraints	$g_A(x_A, y_A) = 0$ $h_A(x_A, y_A) \leq 0$	$g_A(x_A) = 0$, $h_A(x_A) \leq 0$ $\widetilde{g_A}(x_A, \overline{x_B}) = 0$ $\widetilde{h_A}(x_A, \overline{x_B}) \leq 0$
Coordination	Defined by an auxiliary linear coupling constraint, artificial introduced at the shared interface. $y_A - y_B = 0$	Defined by nonlinear complicating constraints, specified by original constraints at the periphery of each area. $\widetilde{g}(x_A, x_B) = 0$ $\widetilde{h}(x_A, x_B) \leq 0$
Exchanged variables	Interface variables, to enable the matching. y_A, y_B	Peripheral variables and Lagrangian multipliers, to evaluate the complicating constraints. $\overline{x_A}$, $\overline{x_B}$, λ_A, λ_B, μ_A, μ_B
Lagrangian Multiplier	Associated with coupling constraint. Common Lagrangian multiplier, obtained by a subgradient method. λ	Associated with complicating constraints. Different Lagrangian multipliers, obtained by foreign optimization problems. λ_A, λ_B, μ_A, μ_B
Weighting parameters of coordination terms	Coordination terms and Lagrangian update are weighted by tuned parameters. α, β, γ	All weighting parameters are represented by Lagrangian multipliers, obtained by foreign optimization. λ_A, λ_B, μ_A, μ_B

Both decomposition procedures yield similar convergence rates for the objective values and the control variables. However, the adjustment procedure presented in [4],[5] shows rather poor convergence properties of the the border variables between the adjacent areas. Especially small systems show a low damped oscillating behavior at the interface (Section III). Further, the convergence rate is highly sensitive to the values of the weighting factors α, β, γ and thus, appropriate tuning is required. Regarding the approach II-B, coordination problems occur if there is a lack of complicating constraints at the peripheral buses. Actually, this algorithm is depending on the borders of the particular areas. For the application area of power systems this can be handled by inserting an additional PQ bus in between the regions to obtain enough complicating constraints. Simulations with additional PQ buses are presented in [3] and yield the optimal values with slightly faster convergence rates.

D. Development of Improved Coordination Procedure

To enhance the convergence of the border variables, method II-A is extended by introducing a master-slave principle. Thereby, one area is designated as master and determines the values at the shared interconnection to the neighboring area. This adjacent area, referred to as slave, adapts its values at the interconnection to those defined by the master. The master-slave principle, referred to as procedure II-D, is implemented by involving specific constraints regarding the border enforcing the equality between certain interface variables. These additional constraints are in each case incorporated by the area taking the position of the slave. Basically, for each interconnection a master is designated which defines the values at the corresponding interface.

The question is, if this predetermination of the interface variables by one area still leads to the system wide optimal solution. The search direction of the slave is influenced by the master's prior decisions. The mathematical proof is out of the scope of this paper. However, the simulations have shown that the system wide optimal values are obtained. As the master-slave principle is only related to the fictitious dummy bus, the main objective of both areas is not affected. Furthermore, only one out of the four interface variables is transferred to the neighbor to allow enough freedom to its optimization. Thereby, it is crucial to leave one variable of the coupling pairs θ/P and V/Q undetermined to avoid infeasibility of the optimal power flow problem.

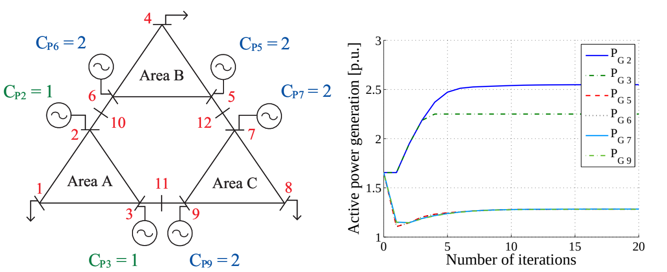
Improvements of the convergence rate mainly arise in small systems where the number of tie-lines, i.e. dummy buses, is large in comparison with the number of core buses. In these systems, the interface variables show a long ongoing counteracting behavior and hence the master-slave principle is highly effective. The improvement of the convergence rate is shown in Section III-A. Moreover, a much lower sensitivity of the convergence rate to the weighting factors is obtained when implementing the additional constraints.

III. Simulation Results

Simulations on two different systems are presented. A first example illustrates the coordination between three interconnected areas on a small 9-bus network. Within a second example, the procedures are carried out on the IEEE 39-bus system. As the optimization problems are non convex, the global optimum can not be guaranteed applying numerical methods. However, the system wide optimal values are assumed to be provided by the centralized control procedure in each case. These values serve as a reference of optimality and the subsequent simulation results obtained by the decentralized procedures are in each case compared with these values. The optimization problems are solved using the solver `fmincon` provided by the standard optimization toolbox of MATLAB.

A. Illustrative Example

The first simulation is carried out on the system setup given in Fig. 3(a), where a coordination between three interconnected areas A, B and C is investigated. Each area contains two generators where bus 2 is modelled as slack bus. Loads are attached at buses 1, 4 and 8. Buses 10, 11, 12 represent the border buses between the areas.



(a) (b)

Fig. 3. System of three interconnected areas A, B and C (a) single line diagram and (b) progression of control variables.

Active power control is applied to minimize the overall cost by an economic dispatch yielding the following objective function

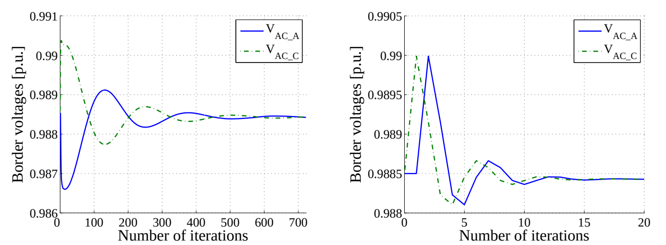
$$f(x,u) = \sum_{i \in \Omega_G} C_{P_i} P_{G_i}^2, \tag{18}$$

whereas the active power generations within the set of all generation buses Ω_G are defined as control variables

$$u = \left[\; P_{G_i} \; \right]. \tag{19}$$

The production costs for the generated active power are 1 p.u. for the generators at buses 2 and 3 (area A) and 2 p.u. for the remaining generators (area B and C).

The progression of the control variables is illustrated in Fig. 3(b) where procedure II-A is applied. Within the first 20 iteration steps, steady state values for the objective as well as for the control variables are acquired which correspond to those of the centralized procedure. The simulation is started from an equilibrium, which is defined by equal active power generation of all generators. As soon as control is applied, the generators adjust their production according to their assigned costs. The two generators of area A increase their production although not to an equal amount as P_{G_3} reaches its operation limit. Considering the convergence between the areas, the procedures II-A and II-D are compared. In Fig. 4, the adaption of the interface voltages between area A and C at border bus 11 is demonstrated. Applying the basic adjustment procedure II-A, these voltages counteract and finally adapt to each other after 723 iteration steps (Fig. 4(a)). The control variables as well as the objective values are not affected by these oscillations because the border variables only appear in the second part of the objective (5), responsible for the adaption. Applying the master-slave principle II-D by implementing additional constraints concerning the voltages, a convergence of the algorithm within 22 iteration steps is achieved (Fig. 4(b)). The constraints are arranged clockwise, e.g., area A decides about interconnection AB at bus 10. Thus, for each area an additional constraint is implemented such as $V_{AC\_A} \overset{!}{=} V_{AC\_C}$ for area A. The areas perform their optimization clockwise. Therefore, the voltage of area A follows that of area C with a delay of one iteration step because interconnection AC connects the areas first and last optimizing. As the master-slave procedure only affects the border adaption but not the main objective, the final values of the objective and the control variables correspond to those of procedure II-A but the progression is slightly different. The weighting factors are chosen as $\alpha = 1$, $\beta = 2$ and $\gamma = 1$ for both procedures II-A and II-D to illustrate the improved convergence.



(a) (b)

Fig. 4. Convergence of border voltages at interface between area A and C (bus 11): (a) procedure II-A, basic adjustment at interface and (b) procedure II-D, improved coordination procedure.

Applying procedure II-B to the original 9-bus system without dummy buses, a convergence to the optimal values within 27 iteration steps is received, although only a loose coupling by one complicating constraint per peripheral bus is provided. For the case of active power control the slack bus is modelled as PV bus with additional angle reference in order to obtain enough complicating constraints. Using the system setup in Fig. 3(a), a convergence within 17 iteration steps results as the additional PQ buses enable 2 complicating constraints per peripheral bus. Simulations are presented in [3].

B. Application to IEEE 39-bus system

In a further example, the decomposition approaches are carried out on the IEEE 39-bus grid which is partitioned into two areas (refer to [3] for the partition). The lack of coordination, as illuminated in Section I, is eliminated implementing the presented decomposition approaches. In Fig. 5, the active power generations of all generation buses obtained by three different methods are illustrated. Applying the limited coordination procedure operated today, a suboptimal solution with a overall active power production cost of C_{tot} = 518.89 p.u. is resulting (dark green bars). Implementing the decomposition approaches, the system wide optimal values are obtained, yielding an exact match between the centralized and decentralized procedures (C_{tot} = 481.36 p.u.).

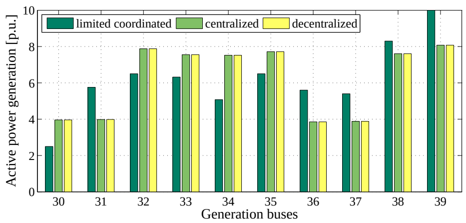
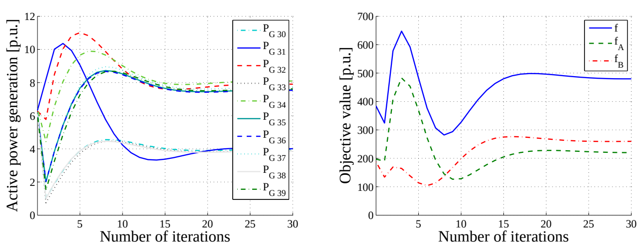


Fig. 5. Active power generation for different procedures:
(1) limited coordinated (dark green bars), C_{tot} = 518.89
(2) centralized coordinated (light green bars) C_{tot} = 481.36
(3) decentralized coordinated (yellow bars), C_{tot} = 481.36.

Fig. 6 presents the evolution of the control variables, i.e. the active power generations at all generation buses, and the values of the overall and the local objectives applying procedure II-B. A convergence within 48 iteration steps is reached, but only the relevant part is depicted above. For the progression of the complicating constraints during the simulation refer to [3].



(a) (b)

Fig. 6. Procedure II-B applied to IEEE 39-bus system: (a) active power generation at generator buses and (b) objective values of Area A, B and total objective value.

Applying procedure II-A yields slower convergence rates. A low damped oscillating behavior at the tie-lines occurs, but neither impacting the control variables nor the objectives. The convergence improves when the number of tie-lines compared to the number of core buses decreases. Running simulations on a two area system composed of two 39-bus systems, interconnected by two tie-lines, results in less oscillating interface variables. For both systems, the convergence is enhanced by implementing the master-slave principle II-D. With the coordination procedure II-B favorable convergence rates for both systems are achieved. Table II illustrates the convergence for the three different decomposition techniques applied on both systems. For procedures II-A and II-D the number of iteration steps regarding the convergence of the control variables (denoted by u) as well as of the interface variables (denoted by y) is stated. The simulation parameters are the same as in example III-A. The authors like to emphasize that the focus is not put on the performance when considering the rather slow convergence rates of the interface variables. The purpose is rather to demonstrate the achievement of the system wide optimal values. Nevertheless, the control variables converge within applicable iteration steps.

TABLE II
REQUIRED NUMBER OF ITERATIONS

Approach / System	II-A Interface Adjustment	II-D Master Slave	II-B Passing Variables
39-bus 2 area	u: 32 y: 347	u: 32 y: 108	u: 48
39-bus - 39-bus	u: 24 y: 258	u: 24 y: 85	u: 42

IV. Conclusion and Outlook

Different approaches of a decentralized control procedure are presented which do not only enable the coordination within multi-area power systems but provide the system wide optimal solution. A cooperative behavior is shown, where the neighbors help to support the system wide objective. No central coordinator is needed which follows the philosophy of large interconnected power systems, e.g., UCTE. A new decomposition procedure is developed by implementing a master-slave principle which enhances the convergence properties. Future work is dedicated to outages of control authorities and break downs of communication links as well as to the incorporation of discrepancies within computation times of adjacent areas.

V. Acknowledgment

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